

Mathematics I
Unit 3 - Geometry
Concept 3 - Triangle Points of Concurrency
Plan for the Concept, Topic, or Skill – Not for the Day

Session #1**Essential Question:**

1. How can I find points of concurrency in triangles?
2. How can I use points of concurrency in triangles?

Activating Strategies: (Learners Mentally Active)

Using the **Construction Graphic Organizer**, students will construct a median, angle bisector, perpendicular bisector, and an altitude to review previously taught constructions from 7th grade GPS curriculum. This activity is teacher directed and should not be considered re-teaching these constructions, but refresh their memory of tasks previously mastered so the incenter, orthocenter, circumcenter, and centroid can be constructed later. Through class discussion students will define median, angle bisector, perpendicular bisector, and altitude, construct each, and then tell how they constructed it.

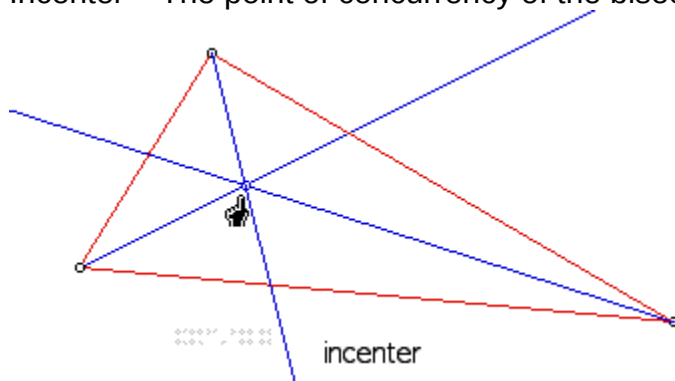
Acceleration/Previewing: (Key Vocabulary)

Vocabulary: incenter, orthocenter, circumcenter, centroid

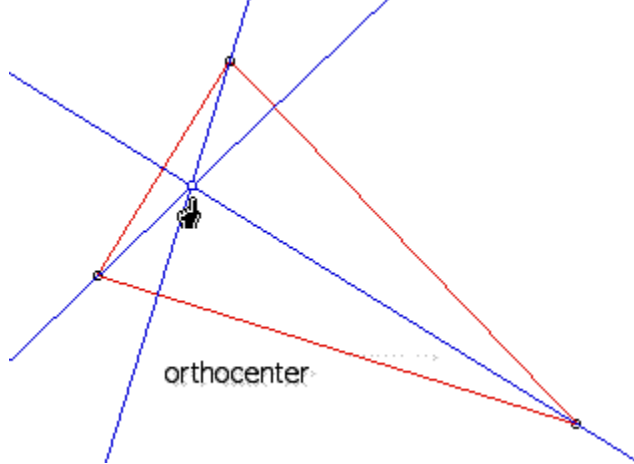
These words will be defined through discovery via the Amusement Park Task (Questions # 1 thru 4).

Below pictures are courtesy of <http://intermath.coe.uga.edu/dictionary>

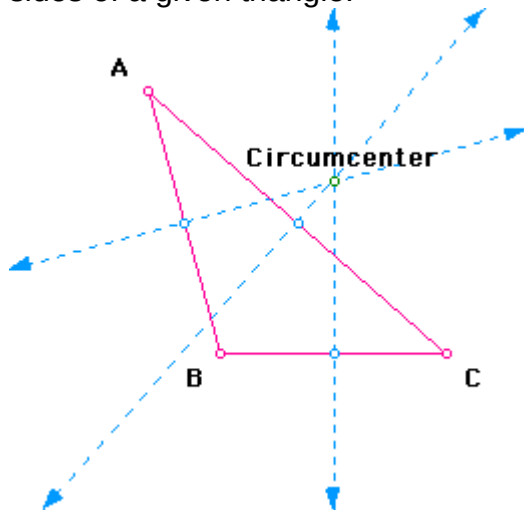
Incenter – The point of concurrency of the bisectors of the angles of a triangle.



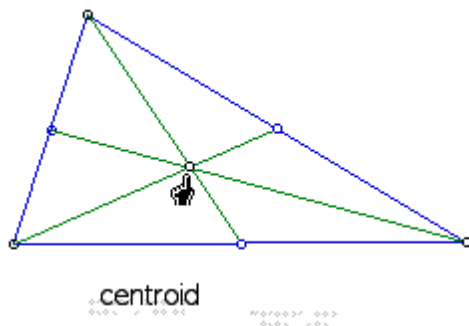
Orthocenter - The point of concurrency of the altitudes of a triangle.



Circumcenter - The point of concurrency of the perpendicular bisectors of the sides of a given triangle.



Centroid - The point of concurrency of the medians of a triangle.



Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

Amusement Park Task

Students will work in groups of three and designate themselves as Student A, Student B, and Student C. Lead class discussion of Amusement Park Handout #1.

Next, distribute all 4 triangle worksheets (Amusement Park Handouts 2 thru 5). Students will follow the directions on each triangle worksheet to make the following constructions: median, angle bisector, perpendicular bisector, and an altitude. They should incorporate the previously completed Construction Graphic Organizer and the teacher role should be that of a facilitator during this task.

Student are completing these constructions simultaneously within their groups, please direct their attention to the directions so that each student understands which triangle he/she will lead. Give students an opportunity to choose between various tools – patty paper, MIRA, compass and straight edge, and Geometer's Sketchpad. Students can use a different tool for each of your construction.

Once constructions are completed, a whole class discussion should be lead as follows:

Hopefully in each case you have noticed that there is a point of intersection (a point of concurrency) that occurs. This is an important property of constructions within triangles and one that solves many problems in mathematics.

Distribute Handout #6 to complete this task. It may be necessary for students to complete as homework.

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

How is a perpendicular bisector like a median? How is it different?

How is a perpendicular bisector like an altitude? How is it different?

How is a perpendicular bisector like an angle bisector? How is it different?

Summarizing Strategies: Learners Summarize & Answer Essential Question

As a ticket out the door (or homework assignment if time does not permit), complete the COMPARE AND CONTRAST Segments Involved in Triangle Constructions.

Give students an example of what should go on the sheet, " The perpendicular bisector is similar to the median because it intersects a side of the triangle at its midpoint." "A perpendicular bisector is different from an altitude because it

intersects a side of the triangle at its midpoint while an altitude may not.”

Session #1: Materials Needed:

Needed – Compass and Straight Edge, color pencils

Suggested – MIRA, patty paper, Geometer's Sketchpad

COMPARE AND CONTRAST Segments Involved in Triangle Constructions

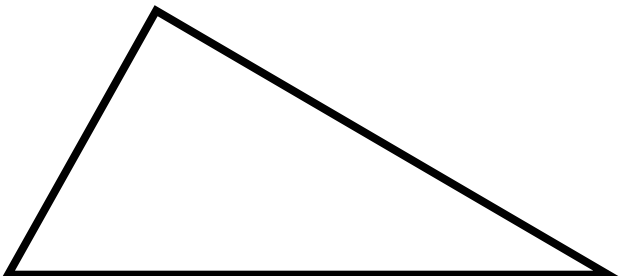
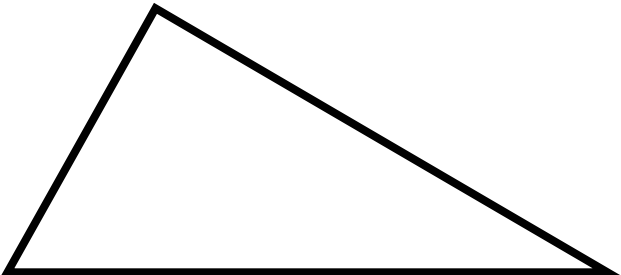
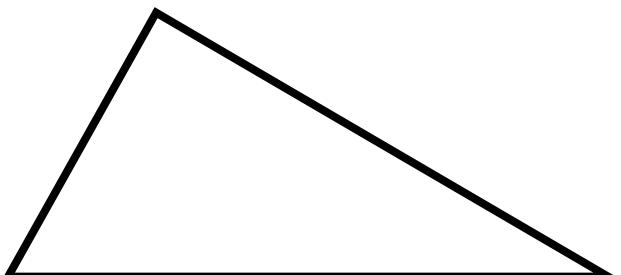
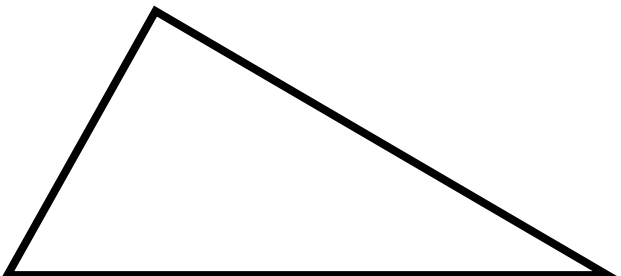
Based on the constructions and the Construction Graphic Organizer completed in class, how are all these construction alike and how are they all different?

<u>Perpendicular Bisector</u>	Altitude	Median	Angle Bisector
Characteristics that are similar:			
1.	1.	1.	1.
2.	2.	2.	2.
3.	3.	3.	3.
4.	4.	4.	4.

Characteristics that are different:

1.	1.	1.	1.
2.	2.	2.	2.
3.	3.	3.	3.
4.	4.	4.	4.

Can one segment have all the characteristics? Under what circumstances?

Type of Construction	Definition	Construction	How I constructed it
Median			
Angle Bisector			
Perpendicular Bisector			
Altitude			

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Session #2**Essential Question:**

3. How can I find points of concurrency in triangles?
4. How can I use points of concurrency in triangles?

Activating Strategies: (Learners Mentally Active)

Warm-Up:

Please match the following prefixes to their correct meaning:

- | | |
|-----------|----------------------|
| 1. In | a. center |
| 2. Centr | b. around |
| 3. Circum | c. in, into |
| 4. Ortho | d. straight, correct |

Answers: 1 – C, 2 – A, 3 – B, 4 – D

Acceleration/Previewing: (Key Vocabulary)

Vocabulary: incenter, orthocenter, circumcenter, centroid

Notes for the teacher about this vocabulary: (keep this in mind throughout today's lesson)

- Orthocenter can lie inside and outside of a triangle.
- Incenter always lies inside the triangle.
- Centroid always lies inside the triangle. Also note that this is the point of balance for the triangle.
- Circumcenter can lie inside the triangle, on the triangle, or outside the triangle.
- Anything that involves an altitude may lie outside of the triangle.

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)**Amusement Park Task (Continued)**

Reassemble the same groups from the previous lesson and ask them to pull out the handouts and constructions from the previous day to refer to.

Distribute Amusement Park Handout page # 7.

Have students answer question #6 and then discuss it.

Have students fill in the blanks on the worksheet using the word bank. Remind them to think about the Warm-Up and what the prefixes meant.

The teacher should summarize and clarify these definitions.

Have students answer questions #7, 8, and 9. Discuss their answers after students have had time to complete the questions.

(You may have to assign #9 as homework if you run out of time)

4 Segments in a Triangle Graphic Organizer

Please pass this graphic organizer out to students. Think-Pair-Share this graphic organizer. Give students five to six minutes to fill in their organizer and then four to five minutes to share with their partner. If students did not draw examples of each point of concurrency in the appropriate area,

please allow them time to do so.

Cell Phone Tower Task

Pass out either Cell Phone Tower Task Option A or Option B to each student. (Option B has a little more direction about what they need to do to solve the problem.)

Allow students to work in groups, pairs, or individually on this task. The teacher should serve as facilitator as students work. Students should finish the task for homework if not completed in class.

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

What does the word concurrency mean?

What is the difference between the centroid, circumcenter, incenter, and orthocenter of a triangle?

How are the centroid, circumcenter, incenter, and orthocenter of a triangle alike?

Summarizing Strategies: Learners Summarize & Answer Essential Question

As a ticket out the door, have students answer the following questions in complete sentences:

1. Does the centroid always lie inside the triangle?
2. Does the incenter always lie inside the triangle?
3. Does the circumcenter always lie inside the triangle?
4. Does the orthocenter always lie inside the triangle?
5. Can these points ever coincide?

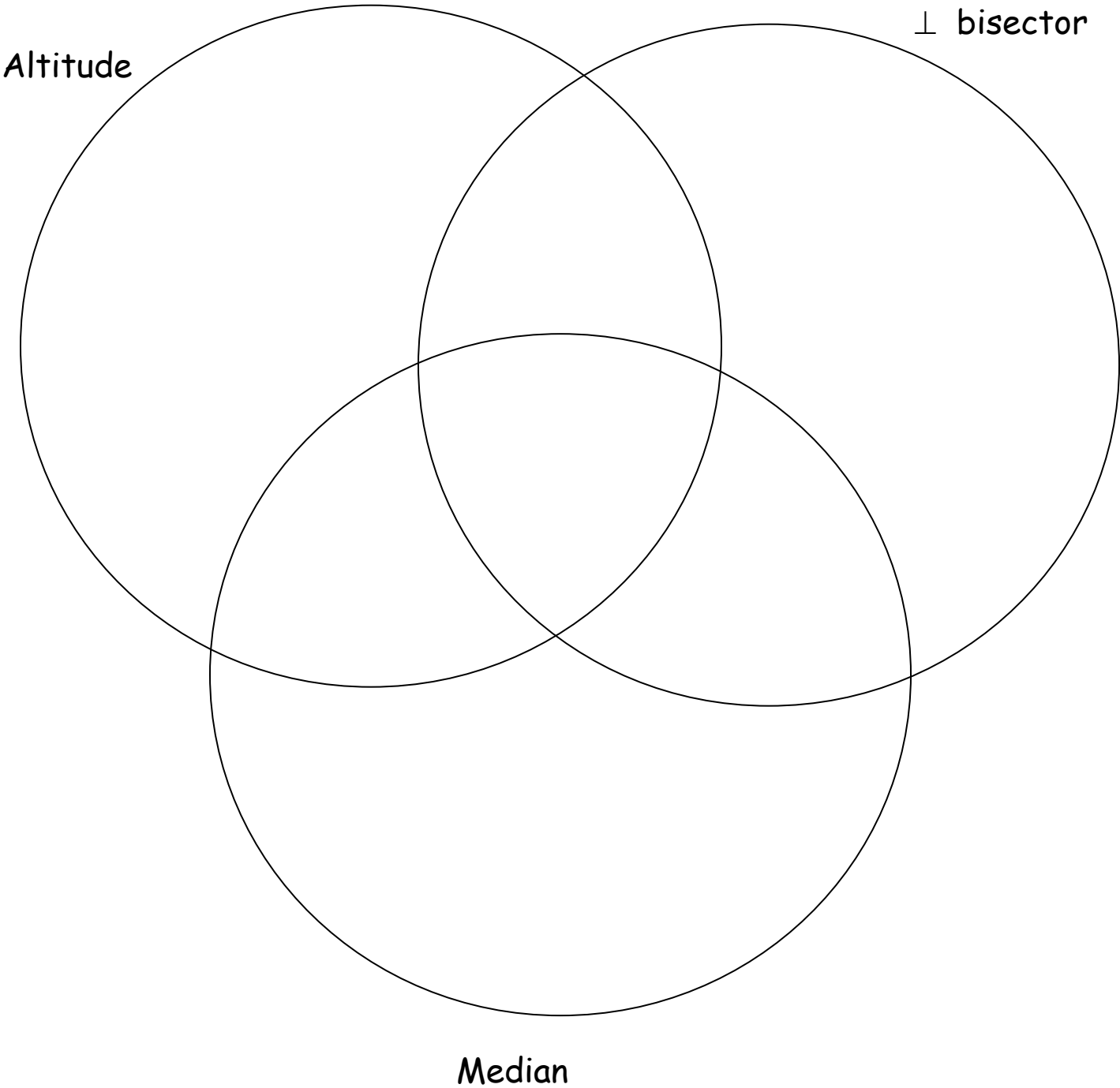
Session #2: Materials Needed:

Needed – Compass and Straight Edge, color pencils

Suggested – MIRA, patty paper, Geometer's Sketchpad

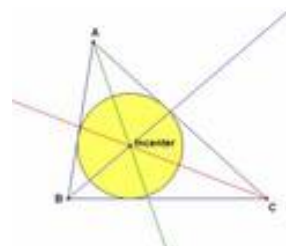
Four Segments in Triangle

Name _____



Angle Bisectors

-
-

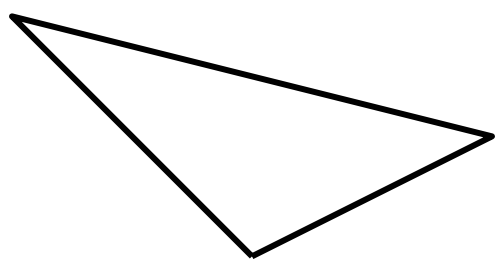
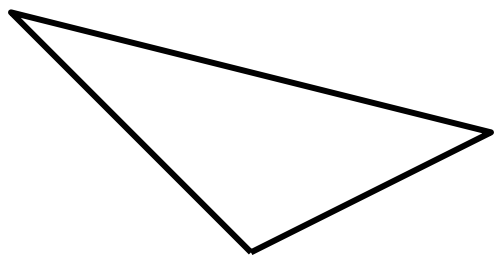
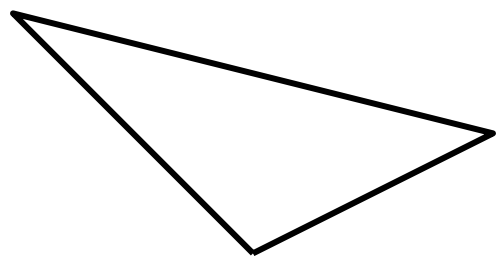
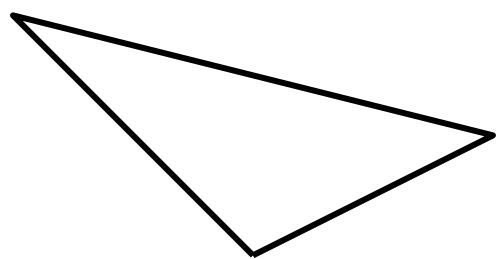


Cell Phone Tower Task Option A

A cell service operator plans to build an additional tower so that more of the southern part of Georgia has stronger service. People have complained that they are losing service, so the operator wants to remedy the situation before they lose customers. The service provider looked at the map of Georgia below and decided that the three cities: Albany, Valdosta, or Waycross, were good candidates for the tower. However, some of the planners argued that the cell tower would provide a more powerful signal within the entire area if it were placed somewhere between those three cities. Help the service operator decide on the best location for the cell tower.

Compose a memo to the president of the cell company justifying your final choice for the location of the tower. Use appropriate mathematical vocabulary and reasoning your justification.





Cell Phone Tower Task Option B

A cell service operator plans to build an additional tower so that more of the southern part of Georgia has stronger service. People have complained that they are losing service, so the operator wants to remedy the situation before they lose customers. The service provider looked at the map of Georgia below and decided that the three cities: Albany, Valdosta, or Waycross, were good candidates for the tower. However, some of the planners argued that the cell tower would provide a more powerful signal within the entire area if it were placed somewhere between those three cities. Help the service operator decide on the best location for the cell tower.

1. Just by looking at the map, choose the location that you think will be best for building the tower. Explain your thinking.

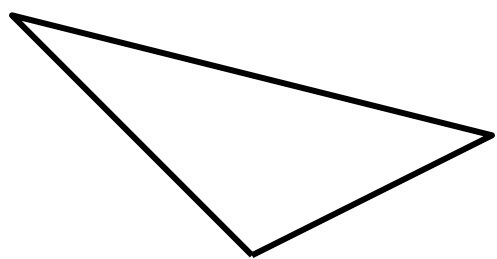
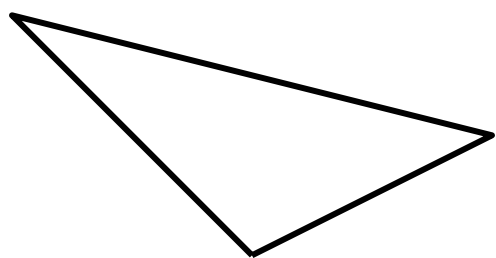
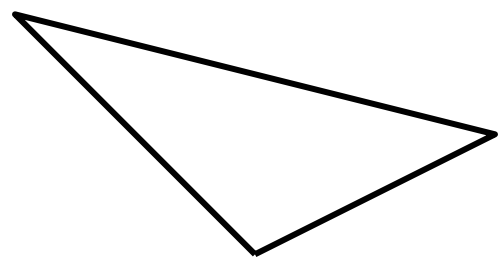
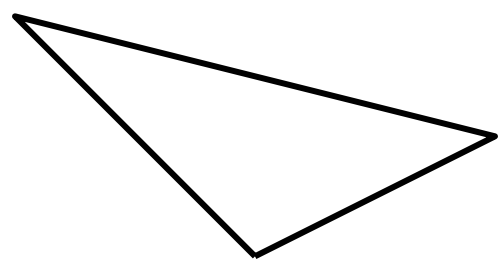
2. Now you are going to use some mathematical concepts to help you choose a location for the tower. Using the 4 triangles attached that approximately represent the triangle formed by Albany, Valdosta and Waycross, find the centroid, incenter, circumcenter, and the orthocenter. You may choose from patty paper, MIRA, compass and straight edge, and Geometer's Sketchpad to make your constructions.

3. Choose a location for the tower based on the work you did for question #2. Explain why you choose this point.

4. Compare the point you chose in question #3, based on mathematics, to the point you chose in question #1, based on observation?

5. Compose a memo to the president of the cell company justifying your final choice for the location of the tower. Use appropriate mathematical vocabulary and reasoning your justification.





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Session #3**Essential Question:**

5. How can I find points of concurrency in triangles?
6. How can I use points of concurrency in triangles?

Activating Strategies: (Learners Mentally Active)Option 1:

Fill in each blank with always, sometimes, or never.

1. The median of a triangle is _____ the perpendicular bisector.
2. The altitude of a triangle is _____ the perpendicular bisector.
3. The medians of a triangle _____ intersect inside the triangle.
4. The altitudes of a triangle _____ intersect inside the triangle.
5. The angle bisectors of a triangle _____ intersect inside the triangle.
6. The perpendicular bisectors of a triangle _____ intersect inside the triangle.

Option 2

Answer the following questions in complete sentences.

1. What is the difference between the centroid, circumcenter, incenter, and orthocenter of a triangle?
2. How are the centroid, circumcenter, incenter, and orthocenter of a triangle alike?

Acceleration/Previewing: (Key Vocabulary)

Vocabulary: incenter, orthocenter, circumcenter, centroid

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

Pass out the review worksheet and have students complete individually. After students have completed the review, have students team up into pairs or small groups to check their work.

The Airport Problem Task

Pass out the Airport Problem Task to students. Allow them to pair up and complete the task. Time may not permit students to finish this task in class. Direct students to finish this at home tonight. They will be allowed approximately 10 minutes tomorrow to get back with their partner to Think-Pair-Share their solutions to this task and then turn in.

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Will the orthocenter be a possible point we need to find for the Airport Problem? Why or why not?

Will the centroid be a possible point we need to find for the Airport Problem? Why or why not?

Summarizing Strategies: Learners Summarize & Answer Essential Question

As a ticket out the door, have students answer the following in complete sentences:
How do the points of concurrency in triangles solve problems?

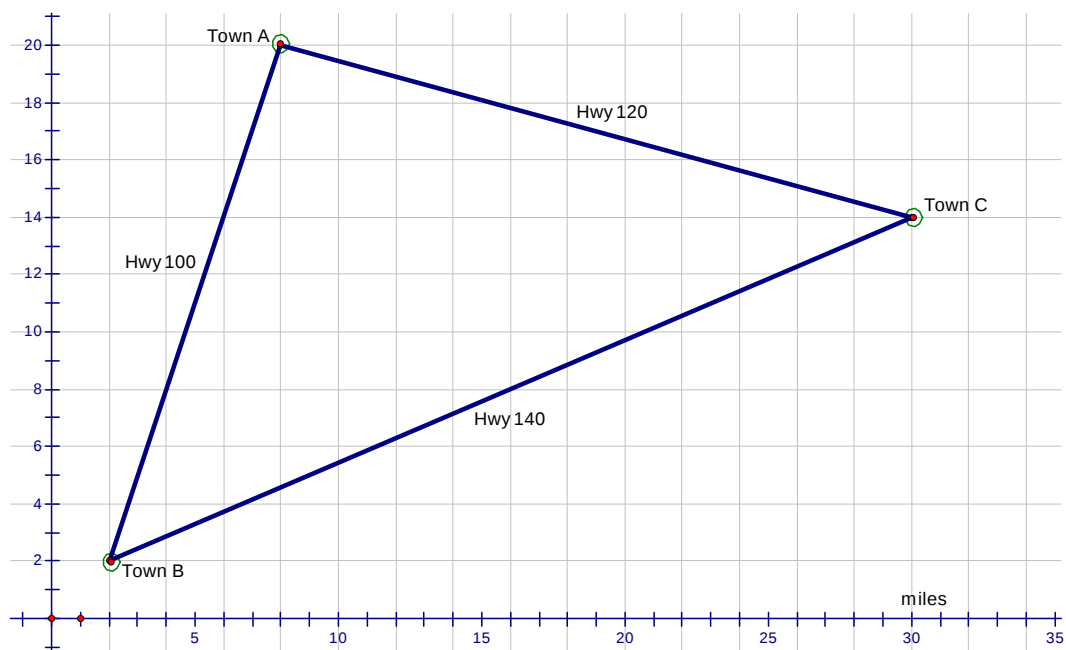
Session #3: Materials Needed

Needed – Compass and Straight Edge, color pencils

Suggested – MIRA, patty paper, Geometer's Sketchpad

The Airport Problem

A county plans to build a regional airport to serve its citizens and wants to locate it within easy access of its three largest towns as shown on the map below. The county has two options for location of the new airport and is working with the airport construction company to minimize costs wherever possible. No matter where the airport is located, roads will have to be built for access directly to the towns or to the existing highways.



Option A: Build the airport at a location that is equidistant from each of the three towns. If this option is selected the county will have to pay for building new roads connecting the airport to the three towns.

Option B: Build the airport at a location that is the shortest distance from each existing highway. If this option is selected the county will have to pay for building new roads to each existing highway and pay for resurfacing each existing highway from the point of intersection leading into each town.

Cost for building new roads is \$125,000 per mile.

Cost for resurfacing existing highways is \$50,000 per mile.

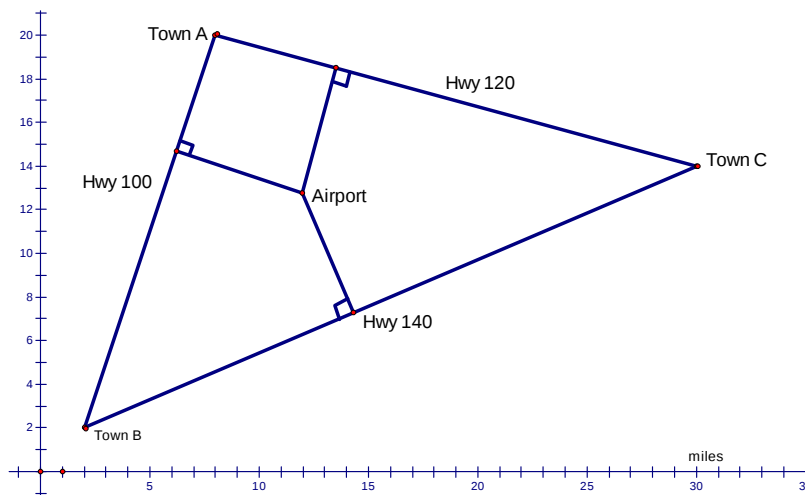
Where is the most cost efficient location for the new airport?

Option A

1. If Option A is selected, what construction would locate the point that is equidistant from the three towns?
2. Construct the point and determine (estimate) the coordinates of the airport location.
3. New roads will need to be constructed from this location directly to each of the three towns. Determine the distance from the airport location to each town.
4. Determine the cost for building the new roads.

Option B

1. If Option B is selected, what construction would locate the point that is the shortest distance to each of the three existing highways? The shortest distance from a point to a line is the perpendicular distance.



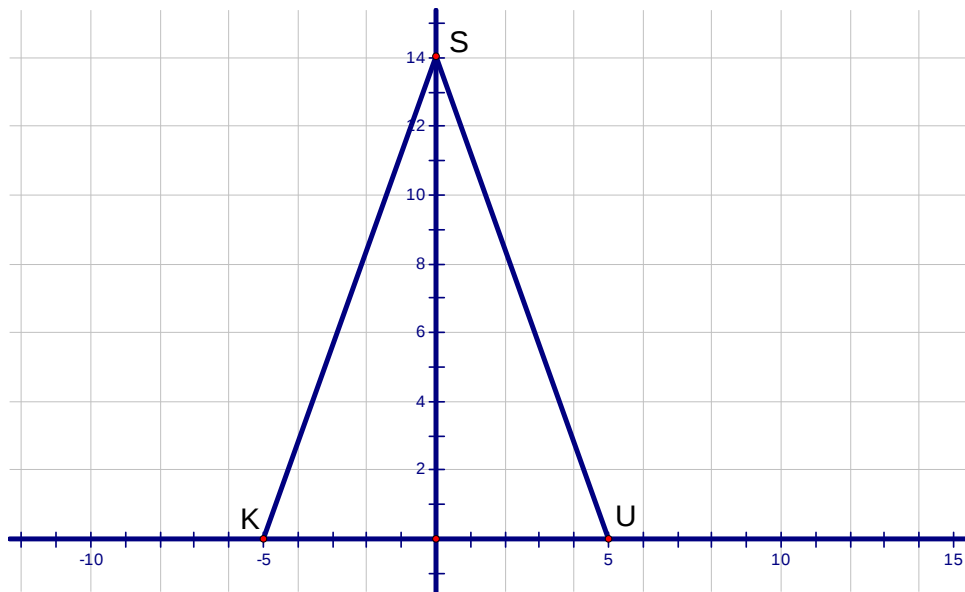
2. Construct the point and determine (estimate) the coordinates of the airport location.
3. New roads will need to be constructed from this location directly to each existing highway.
 - a. Determine the points of intersection for each new road and existing highway.
 - b. Find the length of each new road.
 - c. Determine the cost for building the new roads.

4. If Option B is selected the county has also agreed to resurface the existing highways leading from the airport roads to each town. Find the cost of resurfacing the existing highways.
5. What is the cost of Option B?
6. Which option is the most cost efficient for the airport location?
7. Based on your investigation can you suggest a better solution for the county in terms of saving money on road construction? If so, how could you prove this to the county as a money saving option?

Airport 2 (Alternate Task)

The same airport construction company has been hired in a neighboring county to construct a regional airport, this time with some new restrictions. The county wants the airport to be located as close as possible to all three towns, but closest to the largest town, Town S. Three engineers (who happen to be very good mathematicians) suggest that they build the site on the center of concurrency of the triangle formed by the 3 towns, but arguments ensue.

One engineer thinks that the centroid of the triangular region would be best, another says that the orthocenter would be optimal, the third said that that the circumcenter of the triangular region would be the best location.



1. Which of these airport location points would be closest to Town S?
2. Estimate the distance from each of the three towns
3. What do you notice about all 3 points of concurrency in this triangle? Will this always be true? Draw another triangle and construct the orthocenter, centroid, and circumcenter. Are the results the same? It should be! The line that passes thru those 3 points in a triangle are called the Euler (pronounced "Oiler") line.
4. In this particular situation which point is closest to Town S? Will this always be true? Explore some different triangles to see if there is a rule that will allow you to know which point of concurrency will be closer to a specific point? Try a triangle with coordinates the same as K and U but with S(0,20). Then try changing the coordinates of K and U.
5. Based upon road construction/resurfacing prices mentioned previously which site would be the most cost efficient for the county?

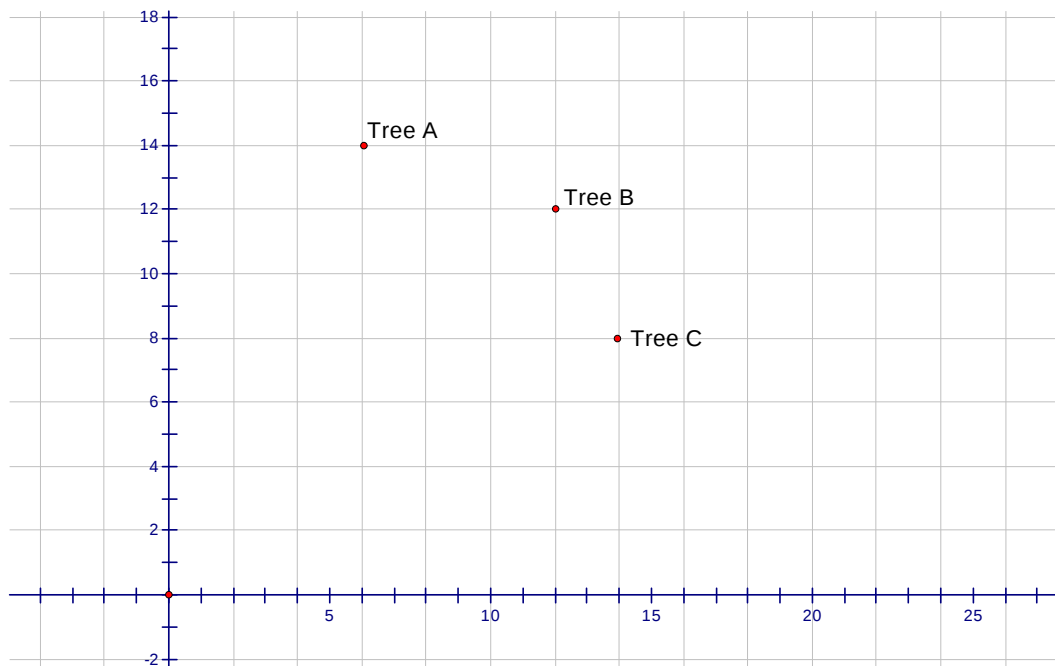
Conclusions:

What are the centroid, the incenter, the circumcenter, and the orthocenter of a triangle?

How do the points of concurrency in triangles solve problems?

Homework:**The Circle Challenge**

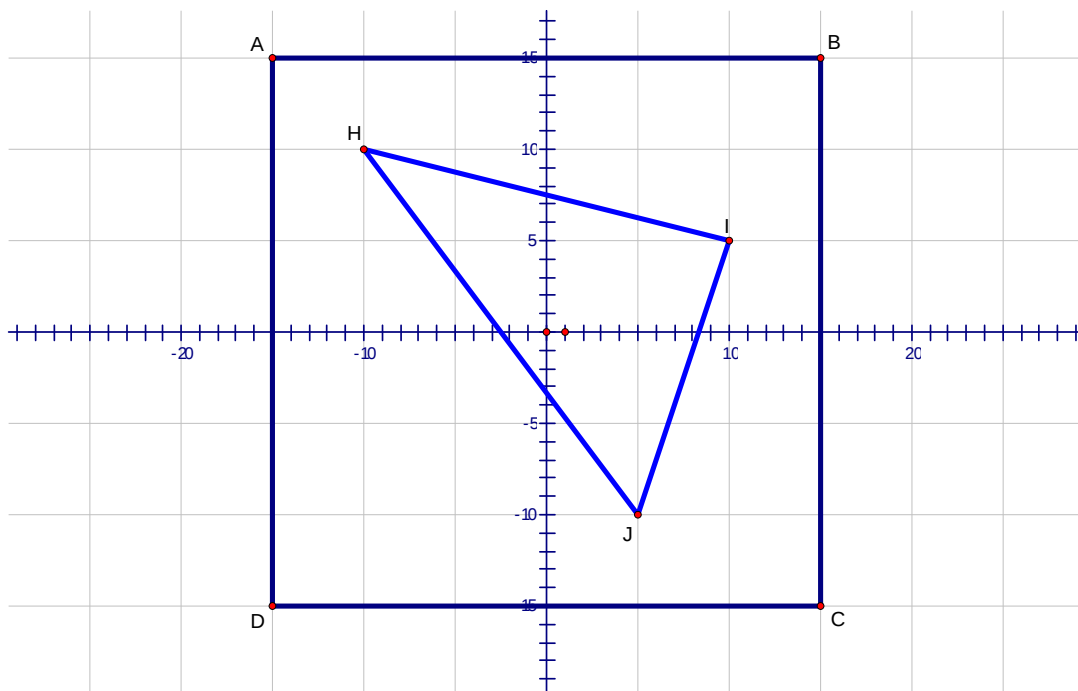
Dutch is creating a circular garden to be surrounded by a stone fence in his backyard. He has just planted 3 trees that will stand at the edge of the stone fence as shown. He also needs to determine the size of the circular garden so he can plan a budget for the project. Dutch decided to make a scale drawing of his plans as shown in the diagram. Each unit represents one meter.



1. How can Dutch find the center of the circle for his garden?
2. Construct the circle that would represent the stone fence.
3. What coordinate point represents the center of the circle? (Estimate)
4. What is the radius of the circle?
5. Dutch knows that it will cost \$8.00 per meter to build his fence. How much should this cost?

Target Practice

Michael is in charge of this year's dart tournament at the local recreation center. Instead of using a circular dartboard he decided that it would be interesting to use a square dartboard with a triangle shaped center as shown below. Points are awarded based upon how close the dart is to the centroid of the triangle. The centroid is the point of concurrency of the medians of a triangle.



1. What coordinate point represents the centroid of the triangle? Explain how you determined this.
2. The following table shows how points are awarded in this game:

Position of dart	Points
Inside square	5
Inside triangle	10
Within a 5 unit radius from the centroid of the triangle	25

What is the probability of hitting:

- a. inside the triangle?
 - b. within a 5 unit radius from the centroid of the triangle?
3. Michael scored 70 points during his first time up. Assuming he hit the dart board each time, what possible hits could he have made?

See also **Burn Center** task from GPS Math 1 Framework.

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Session #3 1/2**Essential Question:**

7. How can I find points of concurrency in triangles?
8. How can I use points of concurrency in triangles?

Activating Strategies: (Learners Mentally Active)

Students are to pair up into their pairs from the previous day. Allow them 10 minutes to Think-Pair-Share their solutions to the Airport Problem Task. When time is up, have students turn their papers in together as a pair.

Acceleration/Previewing: (Key Vocabulary)

Vocabulary: incenter, orthocenter, circumcenter, centroid

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

Pass out quiz over Triangle Points of Concurrency.
 Allow students approximately 20 minutes to complete.

Begin the next concept in this unit.

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Not applicable.

Summarizing Strategies: Learners Summarize & Answer Essential Question

Not applicable.

Session # 3 1/2: Materials Needed

None Needed

Points of Concurrency in Triangles Quiz
Unit 3

1. The incenter of a triangle can be found by constructing the _____ of that triangle.
 - a. Medians
 - b. Perpendicular bisectors
 - c. Angle bisectors
 - d. Altitudes

2. The orthocenter of a triangle can be found by constructing the _____ of that triangle.
 - a. Medians
 - b. Perpendicular bisectors
 - c. Angle bisectors
 - d. Altitudes

3. The centroid of a triangle can be found by constructing the _____ of that triangle.
 - a. Medians
 - b. Perpendicular bisectors
 - c. Angle bisectors
 - d. Altitudes

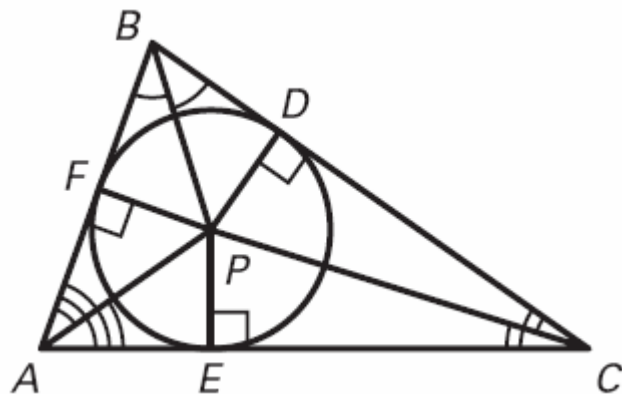
4. The circumcenter of a triangle can be found by constructing the _____ of that triangle.
 - a. Medians
 - b. Perpendicular bisectors
 - c. Angle bisectors
 - d. Altitudes

Triangle Points of Concurrency Teacher Notes

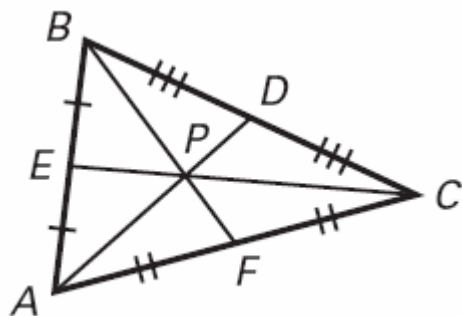
- Orthocenter can lie inside and outside of a triangle.
- Incenter always lies inside the triangle.
- Centroid always lies inside the triangle. Also note that this is the point of balance for the triangle.
- Circumcenter can lie inside the triangle, on the triangle, or outside the triangle.
- Anything that involves an altitude may lie outside of the triangle.

Helpful Theorems

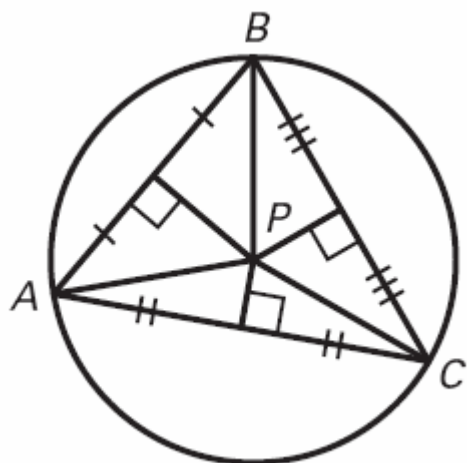
- Concurrency of Angle Bisectors of a Triangle – The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle. $PD = PE = PF$



- Concurrency of Medians of a Triangle – The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side. If P is the centroid of triangle ABC, then $AP = \frac{2}{3} AD$, $BP = \frac{2}{3} BF$, and $CP = \frac{2}{3} CE$.



- Concurrency of Perpendicular Bisectors of a Triangle – The perpendicular bisectors of triangle intersect at a point that is equidistant from the vertices of the triangle. $PA = PB = PC$



Student A: _____

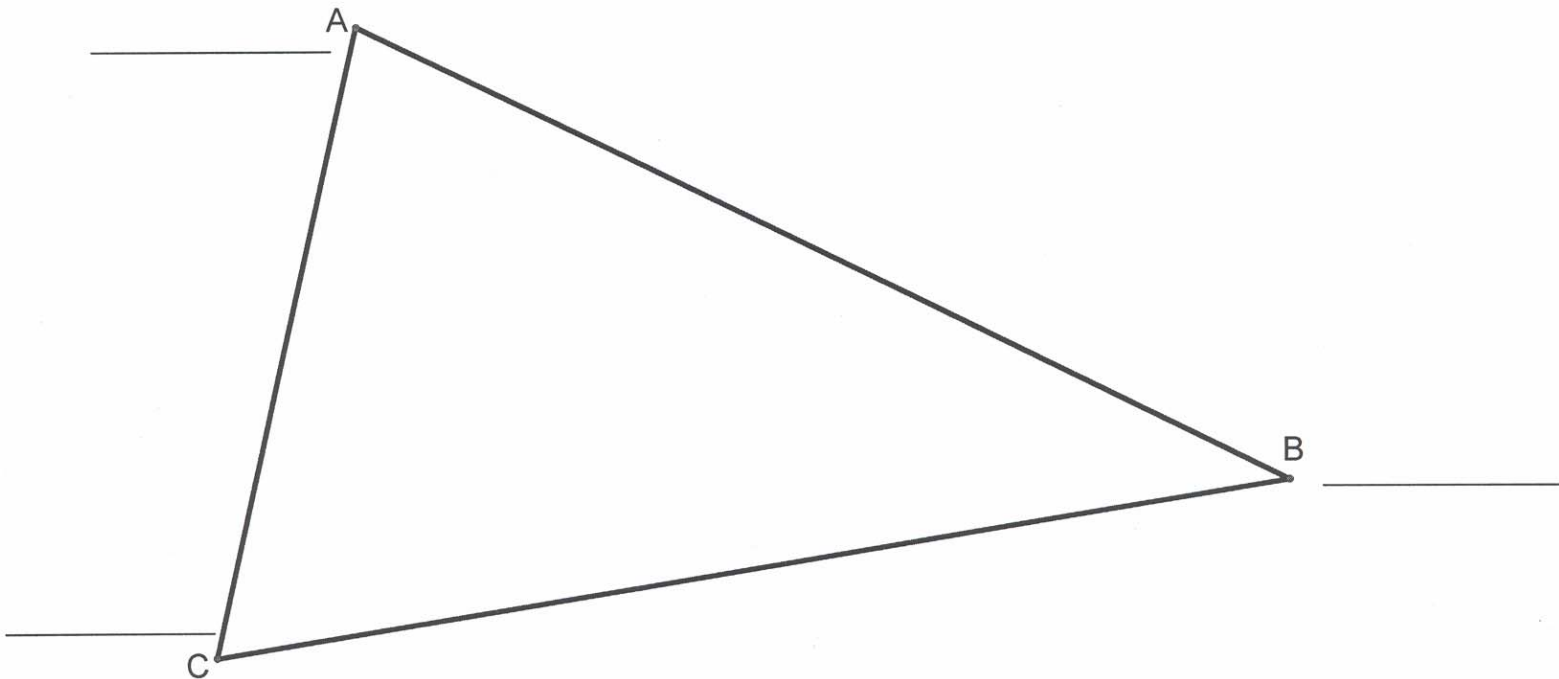
Student B: _____

Student C: _____

Amusement Park Task

A developer plans to build an amusement park but wants to locate it within easy access of the three largest towns in the area as shown on the map below. The developer has to decide on the best location and is working with the ABC Construction Company to minimize costs wherever possible. No matter where the amusement park is located, roads will have to be built for access directly to the towns or the existing highways.

1. Create a name for towns A, B and C on the map below.
2. Name the three highways connecting each town.



3. Just by looking at the map, choose the location that you think will be best for building the amusement park. Explain your thinking.

Student A: _____

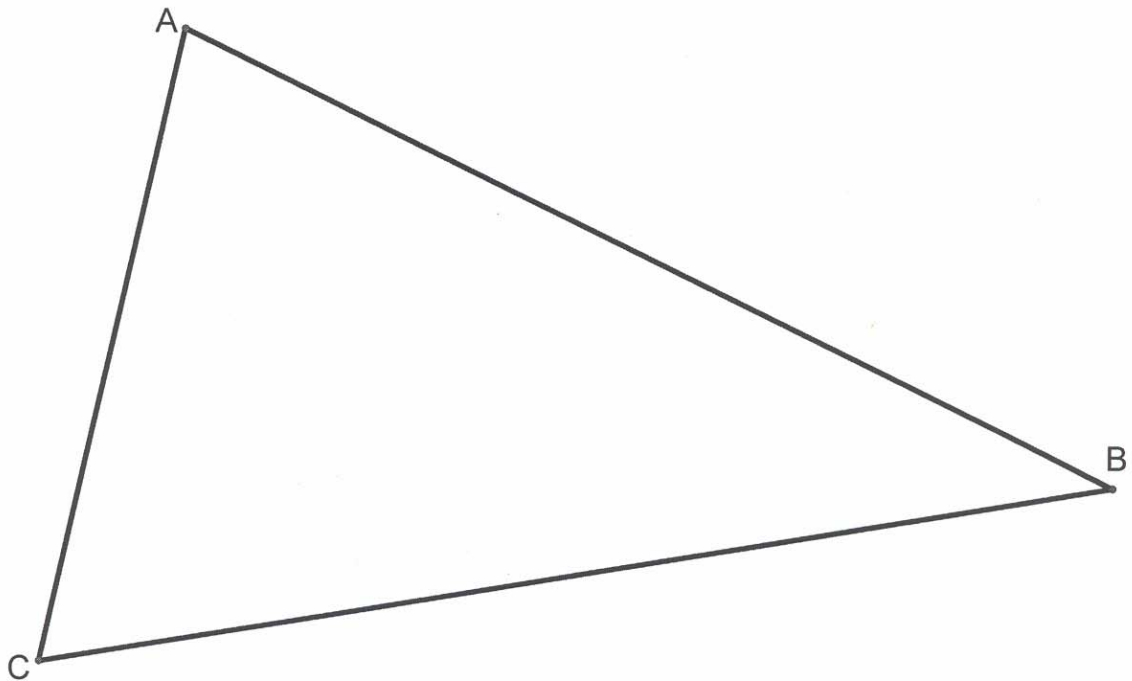
Student B: _____

Student C: _____

Triangle #1 Lead by Student A

Using your Construction Graphic Organizer, construct :

- Student A construct the median of segment AB, then connect that median to the opposite vertex. Pass paper to Student B.
- Student B construct the median of segment BC, then connect that median to the opposite vertex. Pass paper to Student C.
- Student C construct the median of segment CA, then connect that median to the opposite vertex. Keep this paper.



As a group now answer:

What can you conclude about this construction?

Student A: _____

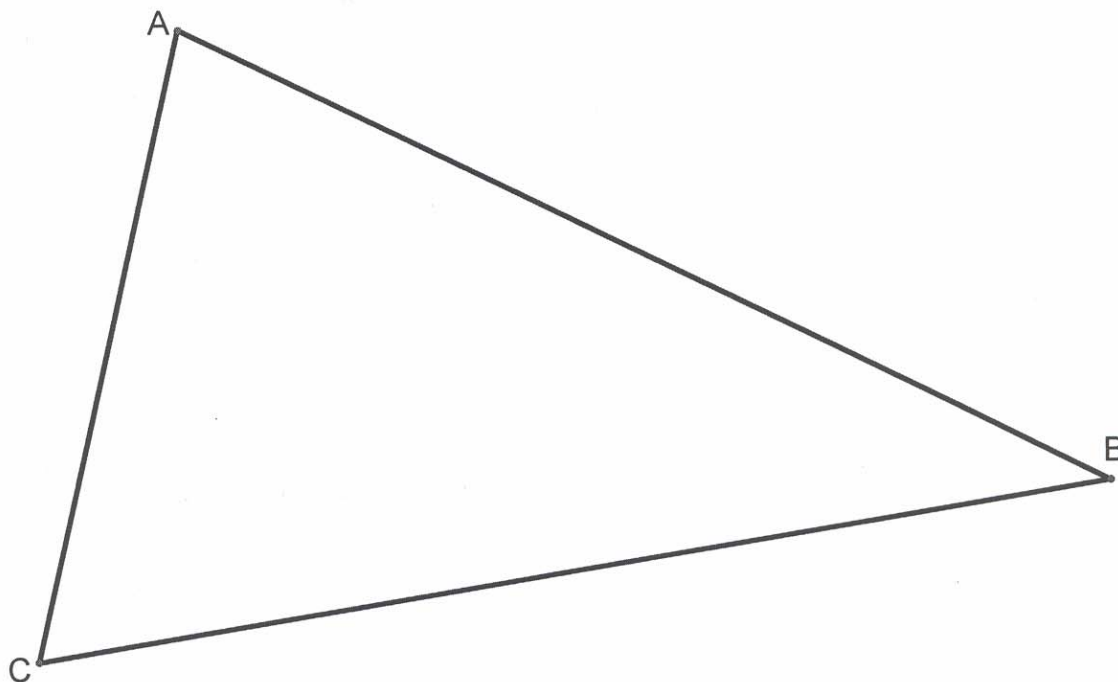
Student B: _____

Student C: _____

Triangle #2 Lead by Student B

Using your Construction Graphic Organizer, construct :

- Student B construct the angle bisector of angle B. Extend your angle bisector beyond the opposite side of your angle. Pass paper to Student C.
- Student C construct the angle bisector of angle C. Extend your angle bisector beyond the opposite side of your angle. Pass paper to Student A.
- Student A construct the angle bisector of angle A. Extend your angle bisector beyond the opposite side of your angle. Keep this paper.



As a group now answer:

What can you conclude about this construction?

Student A: _____

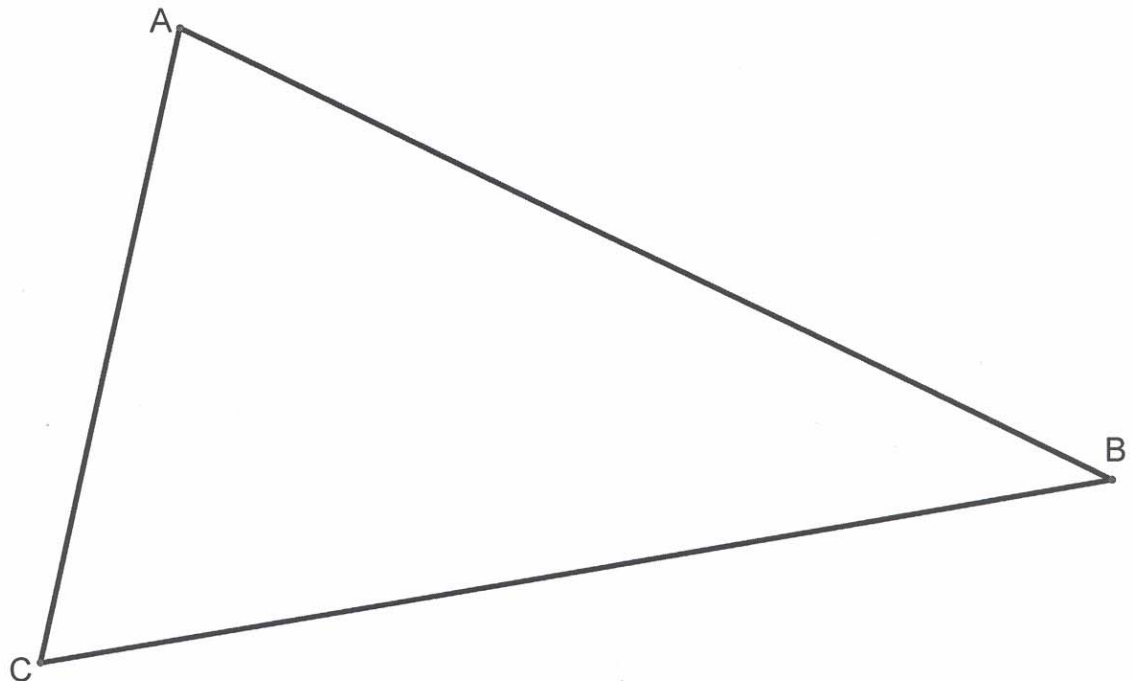
Student B: _____

Student C: _____

Triangle #3 Lead by Student C

Using your Construction Graphic Organizer, construct :

- Student C construct the perpendicular bisector of segment CA. Make sure your perpendicular bisector extends to the outside of the triangle. Pass paper to Student A.
- Student A construct the perpendicular bisector of segment AB. Make sure your perpendicular bisector extends to the outside of the triangle. Pass paper to Student B.
- Student B construct the perpendicular bisector of segment BC. Make sure your perpendicular bisector extends to the outside of the triangle. Keep this paper.



As a group now answer:

What can you conclude about this construction?

Student A: _____

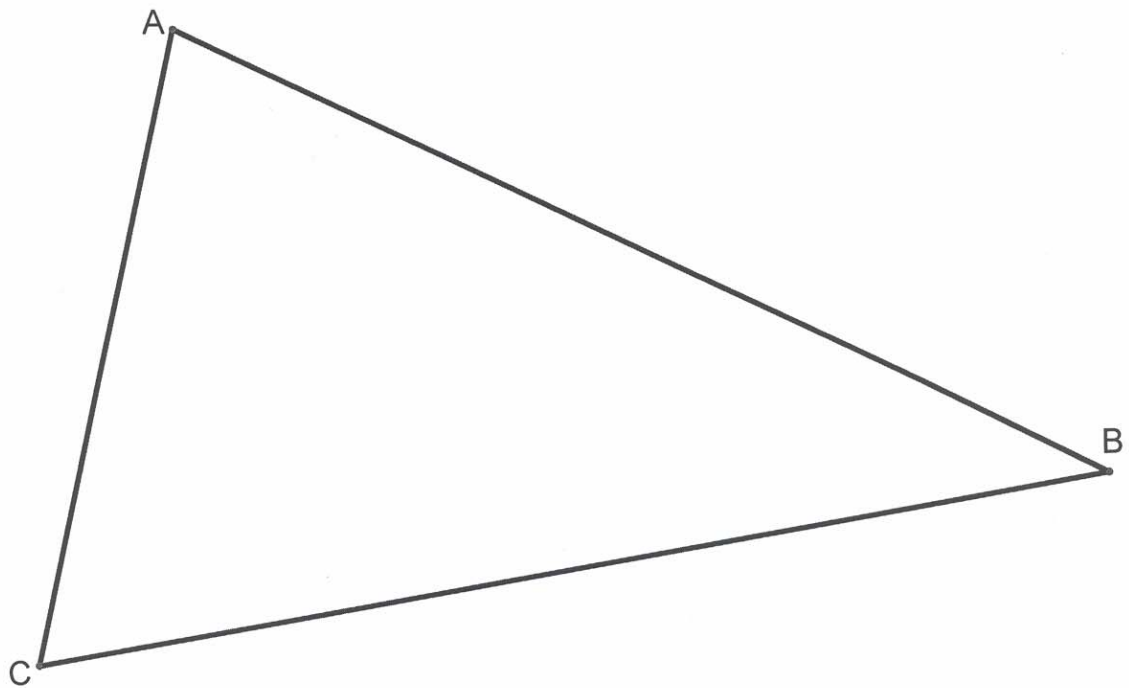
Student B: _____

Student C: _____

Triangle #4 Lead by Student A

Using your Construction Graphic Organizer, construct :

- Student A construct the altitude from vertex A to the line containing the segment CB. Pass paper to Student B.
- Student B construct the altitude from vertex B to the line containing the segment AC. Pass paper to Student C.
- Student C the altitude from vertex C to the line containing the segment AB. Keep this paper.



As a group now answer:
What can you conclude about this construction?

Student A: _____

Student B: _____

Student C: _____

4. Choose a location for the amusement park based on the work you did based on all 4 triangle constructions. Explain why you chose this point.

5. How close is the point you chose in question 4, based on mathematics, to the point you chose by observation?

Student A: _____

Student B: _____

Student C: _____

6. What do you notice about each set of segments resulting from your constructions on the triangles?

- These four points of intersection are called the _____.
- The point of concurrency of the medians of a triangle is called the _____. This is sometimes referred to as the “weighted center” or balancing point of a triangle.
- The point of concurrency of the angle bisectors of a triangle is called the _____. This point is the center of a circle that inscribes the triangle.
- The point of concurrency of the perpendicular bisectors of a triangle is called the _____. This point is the center of a circle and circumscribes the triangle.
- The point of concurrency of the altitudes of a triangle is called the _____.

Word Bank: Centroid, Circumcenter, Incenter, Orthocenter, Points of Concurrency

7. Can you give a reasonable guess as to why the specific names were given to each point of concurrency?

Student A: _____

Student B: _____

Student C: _____

8. Which triangle center did you recommend for the location of the amusement park?

9. The president of the company building the park is concerned about the cost of building roads from the towns to the park. What recommendation would you give him? Write a memo to the president explaining your recommendation. Please write in a letter format.

Points of Concurrency in a Triangle Summary Activity / Review Unit 3

Concurrency of Altitudes of a Triangle: The lines containing the altitudes of a triangle are concurrent.

- The point of concurrency of the altitudes of a triangle is called the _____(1).

Concurrency of Angle Bisectors of a Triangle: The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

- The point of concurrency of the angle bisectors of a triangle is called the _____(2).

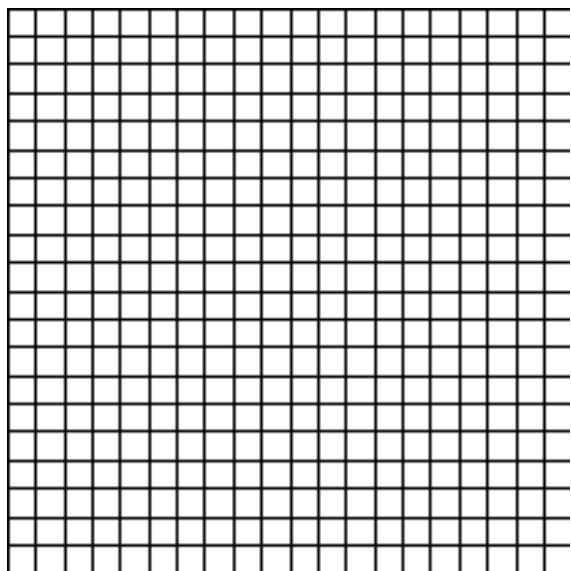
Concurrency of Medians of a Triangle: The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

- The point of concurrency of the medians of a triangle is called the _____(3). This point is sometimes referred to as the “weighted center” or balancing point of a triangle.

Concurrency of Perpendicular Bisectors of a Triangle: The perpendicular bisectors of a triangle are concurrent.

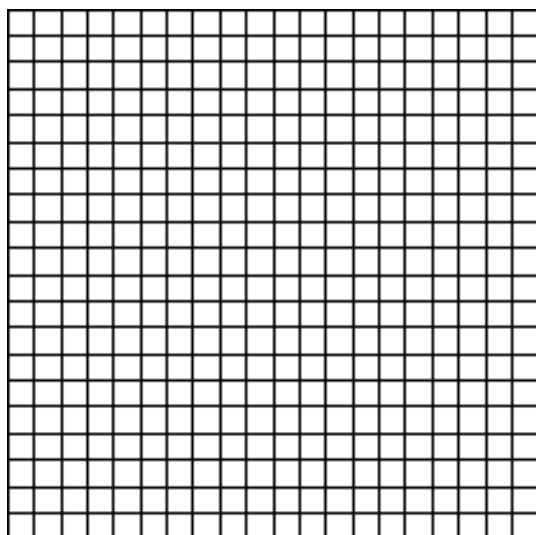
- The point of concurrency of the perpendicular bisectors of a triangle is called the _____(4).

(5). Find the coordinates of the centroid O of $\triangle ABC$ with coordinates $A(-7, -4)$, $B(-3, 5)$, $C(1, -4)$.



Centroid: _____

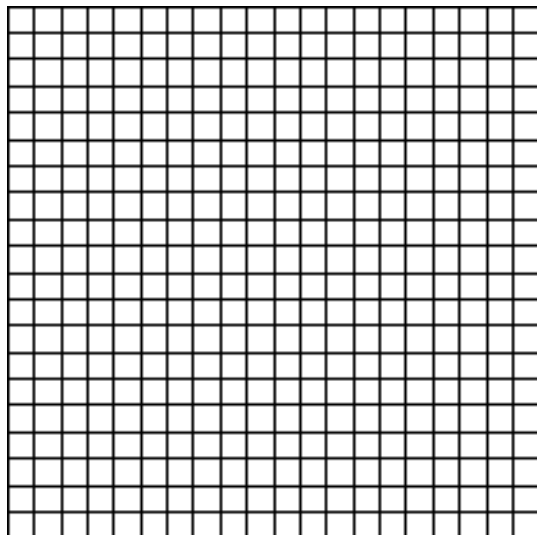
(6). Find the coordinates of the incenter P of $\triangle DEF$ with coordinates $D(0, 6)$, $E(4, -4)$, $F(-5, -5)$.



Incenter: _____

(7). Find the coordinates of the orthocenter Q of

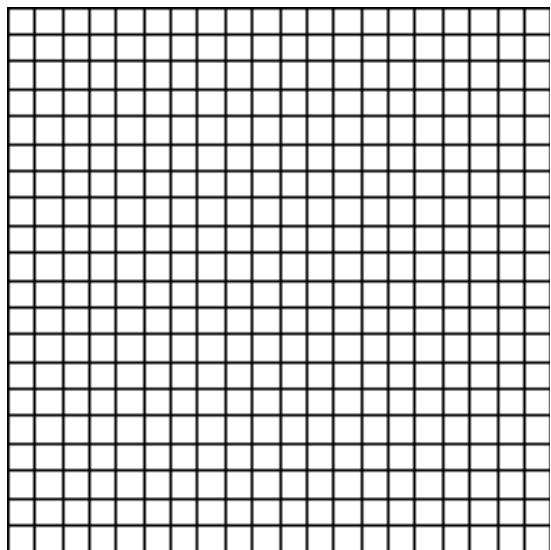
$\triangle GHI$ with coordinates $G(0, 0)$, $H(3, 7)$, $F(8, 3)$.



Orthocenter: _____

(8). Find the coordinates of the circumcenter P of

$\triangle JKL$ with coordinates $J(0, 1)$, $K(3, 4)$, $L(6, 0)$.



Circumcenter: _____