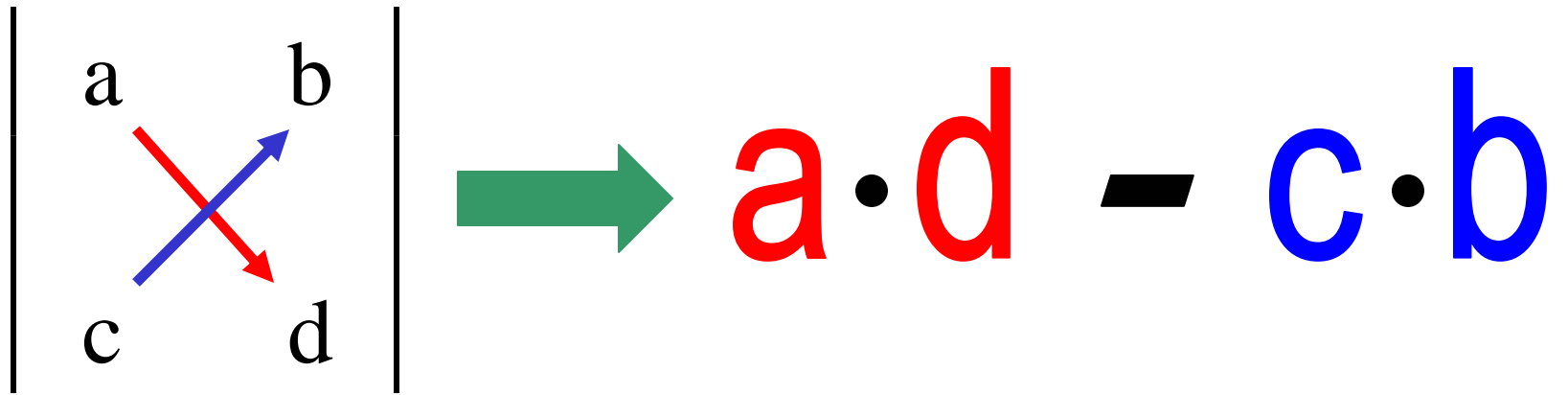


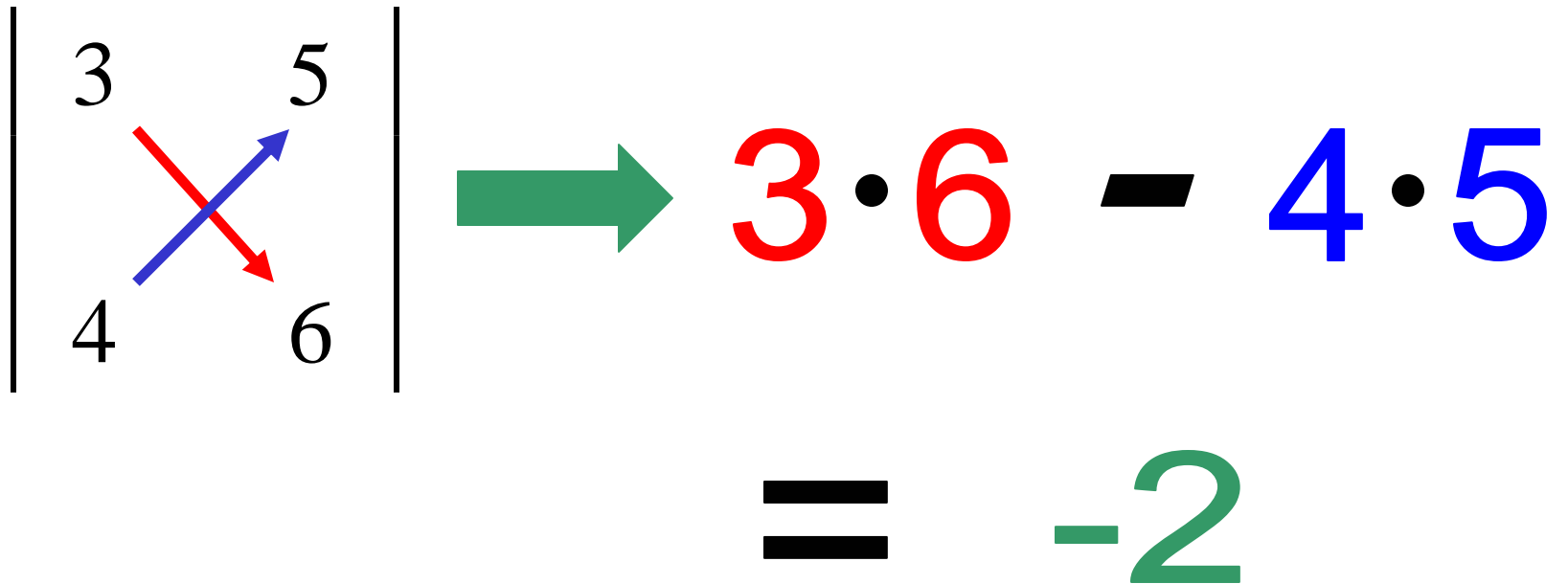
Determinants of 2 x 2 and 3 x 3 Matrices

Determinant of a 2 x 2 Matrix

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \rightarrow a \cdot d - c \cdot b$$


Example 1

Determinant of a 2 x 2 Matrix

$$\begin{vmatrix} 3 & 5 \\ 4 & 6 \end{vmatrix} \rightarrow 3 \cdot 6 - 4 \cdot 5 = -2$$


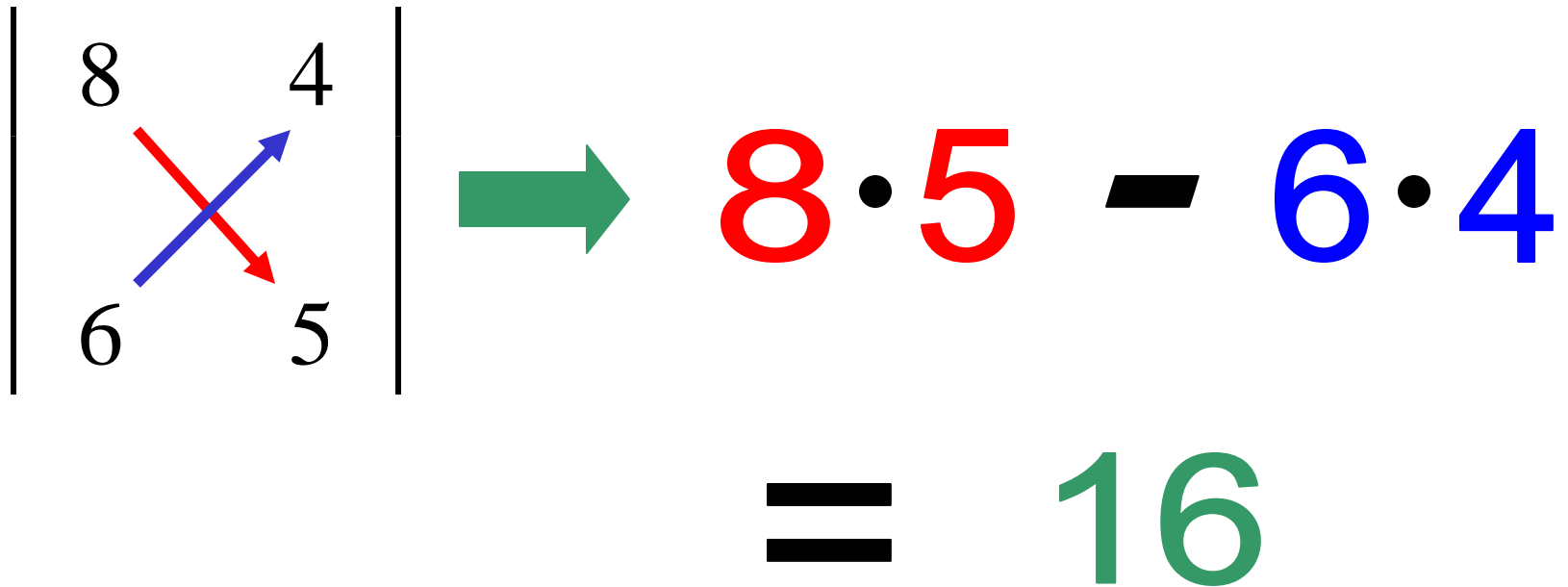
Example 2

Determinant of a 2 x 2 Matrix

$$\begin{vmatrix} -4 & 3 \\ 5 & 2 \end{vmatrix} \rightarrow -4 \cdot 2 - 5 \cdot 3 = -23$$

Example 3

Determinant of a 2 x 2 Matrix

$$\begin{vmatrix} 8 & 4 \\ 6 & 5 \end{vmatrix} \rightarrow 8 \cdot 5 - 6 \cdot 4 = 16$$


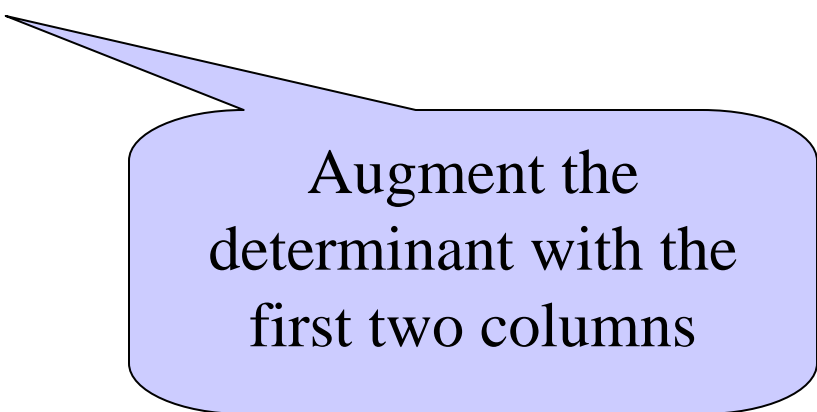
Example 4

Determinant of a 3 x 3 Matrix

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 6 & 7 \end{vmatrix}$$

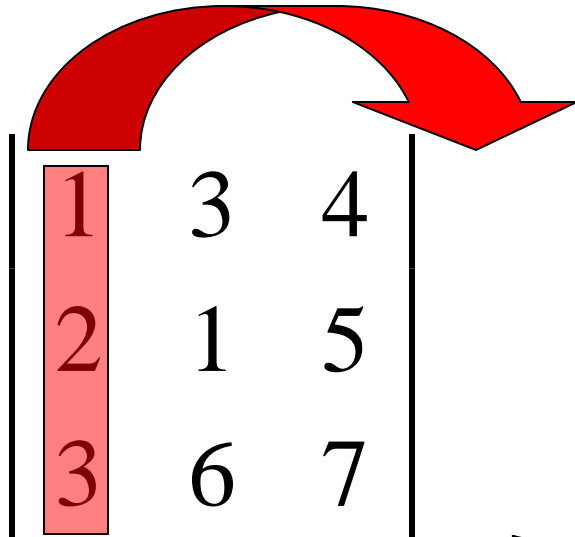
Determinant of a 3 x 3 Matrix

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 6 & 7 \end{vmatrix}$$



Augment the
determinant with the
first two columns

Determinant of a 3 x 3 Matrix

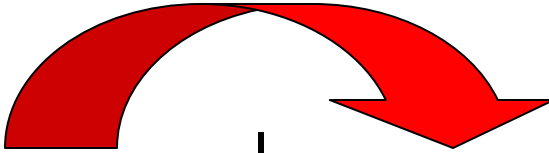


A 3x3 matrix is shown with a red arrow pointing from the first column to the second and third columns. The first column is highlighted in red.

1	3	4
2	1	5
3	6	7

Augment the determinant with the first two columns

Determinant of a 3 x 3 Matrix


$$\begin{vmatrix} 1 & 3 & 4 & | & 1 \\ 2 & 1 & 5 & | & 2 \\ 3 & 6 & 7 & | & 3 \end{vmatrix}$$

Augment the
determinant with the
first two columns

Determinant of a 3 x 3 Matrix

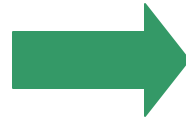
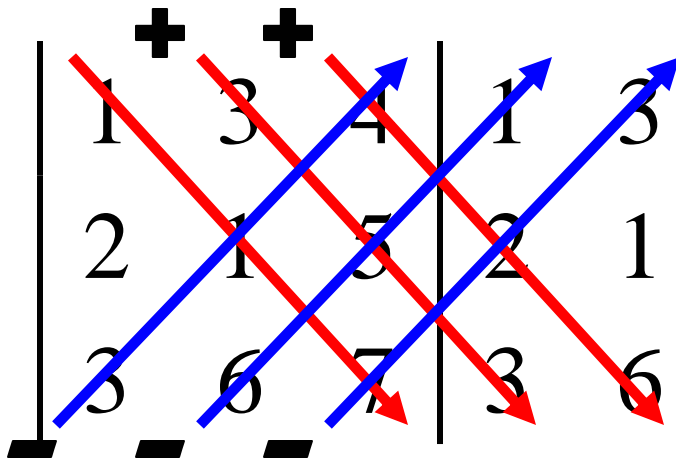
$$\left| \begin{array}{ccc|cc} 1 & 3 & 4 & 1 & 3 \\ 2 & 1 & 5 & 2 & 1 \\ 3 & 6 & 7 & 3 & 6 \end{array} \right. \rightarrow$$

Augment the determinant with the first two columns

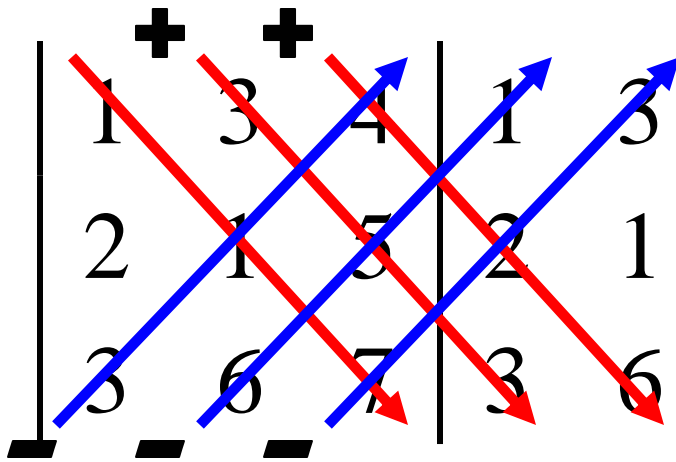
Determinant of a 3 x 3 Matrix

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 6 & 7 \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 2 & 1 \\ 3 & 6 \end{vmatrix} \rightarrow$$

Determinant of a 3 x 3 Matrix



Determinant of a 3 x 3 Matrix



Determinant of a 3 x 3 Matrix

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 6 & 7 \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 2 & 1 \\ 3 & 6 \end{vmatrix}$$



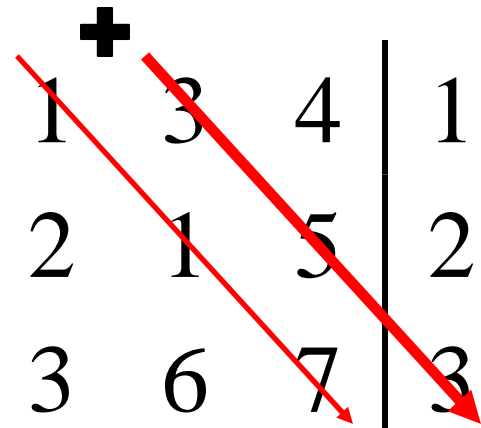
Determinant of a 3 x 3 Matrix

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 6 & 7 \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 2 & 1 \\ 3 & 6 \end{vmatrix}$$



$$1 \cdot 1 \cdot 7$$

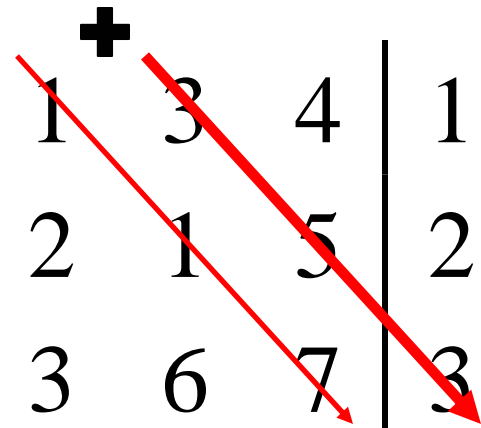
Determinant of a 3 x 3 Matrix

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 6 & 7 \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 2 & 1 \\ 3 & 6 \end{vmatrix}$$




$$1 \cdot 1 \cdot 7 +$$

Determinant of a 3 x 3 Matrix

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 6 & 7 \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 2 & 1 \\ 3 & 6 \end{vmatrix}$$




$$1 \cdot 1 \cdot 7 + 3 \cdot 5 \cdot 3$$

Determinant of a 3 x 3 Matrix

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 6 & 7 \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 2 & 1 \\ 3 & 6 \end{vmatrix}$$



$$1 \cdot 1 \cdot 7 + 3 \cdot 5 \cdot 3 +$$

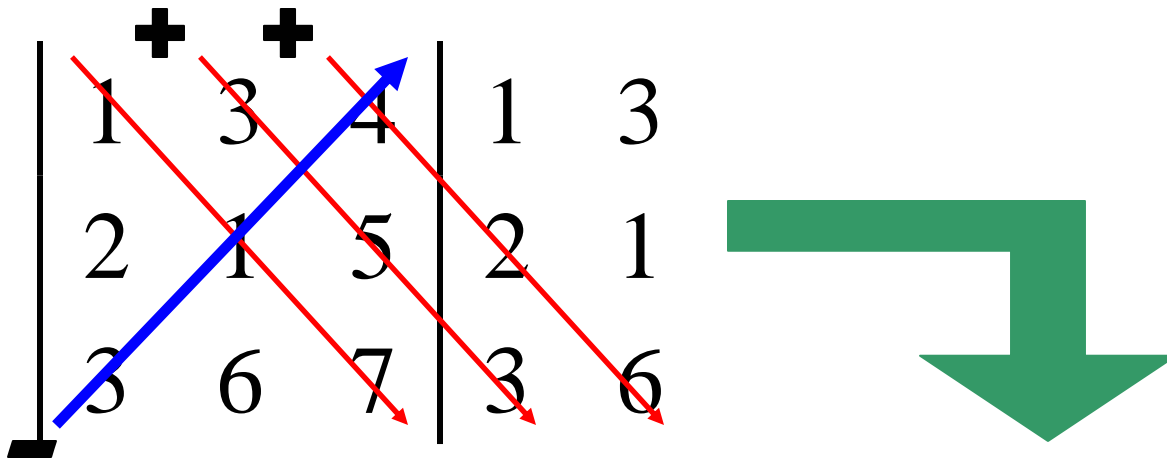
Determinant of a 3 x 3 Matrix

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 6 & 7 \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 2 & 1 \\ 3 & 6 \end{vmatrix}$$



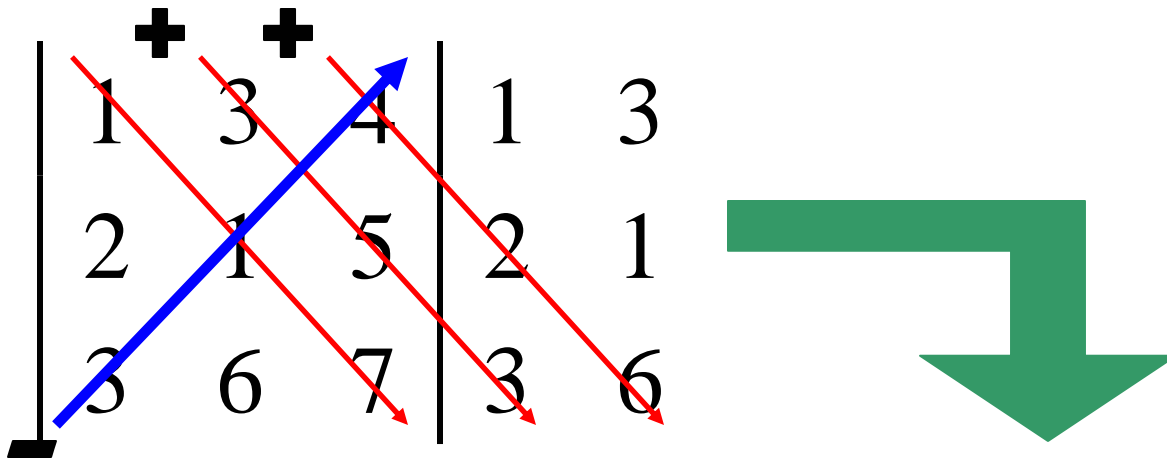
$$1 \cdot 1 \cdot 7 + 3 \cdot 5 \cdot 3 + 4 \cdot 2 \cdot 6$$

Determinant of a 3 x 3 Matrix



$$1 \cdot 1 \cdot 7 + 3 \cdot 5 \cdot 3 + 4 \cdot 2 \cdot 6 -$$

Determinant of a 3 x 3 Matrix



$$1 \cdot 1 \cdot 7 + 3 \cdot 5 \cdot 3 + 4 \cdot 2 \cdot 6 - 3 \cdot 1 \cdot 4$$

Determinant of a 3 x 3 Matrix

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 6 & 7 \end{vmatrix} \begin{matrix} 1 & 3 \\ 2 & 1 \\ 3 & 6 \end{matrix}$$

$$1 \cdot 1 \cdot 7 + 3 \cdot 5 \cdot 3 + 4 \cdot 2 \cdot 6 - 3 \cdot 1 \cdot 4 -$$

Determinant of a 3 x 3 Matrix

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 6 & 7 \end{vmatrix} \begin{matrix} 1 & 3 \\ 2 & 1 \\ 3 & 6 \end{matrix}$$

The diagram shows a 3x3 matrix with a vertical line to its right. The matrix elements are 1, 3, 4 in the first row; 2, 1, 5 in the second row; and 3, 6, 7 in the third row. To the right of the vertical line are the elements 1, 3 in the first row; 2, 1 in the second row; and 3, 6 in the third row. Red arrows indicate the subtraction of the products of the diagonals from the top-right to the bottom-left (1·1·7, 3·5·3, 4·2·6) and the addition of the products of the diagonals from the bottom-right to the top-left (3·1·4, 6·5·1). Blue arrows indicate the addition of the products of the diagonals from the top-left to the bottom-right (1·1·7, 3·5·3, 4·2·6) and the subtraction of the products of the diagonals from the bottom-left to the top-right (3·1·4, 6·5·1). Plus signs are placed above the 3 and 4 in the first row of the matrix. Minus signs are placed below the 1 and 3 in the first row of the matrix.



$$1 \cdot 1 \cdot 7 + 3 \cdot 5 \cdot 3 + 4 \cdot 2 \cdot 6 - 3 \cdot 1 \cdot 4 - 6 \cdot 5 \cdot 1$$

Determinant of a 3 x 3 Matrix

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 6 & 7 \end{vmatrix} \begin{matrix} + & + \\ - & - \\ + & + \end{matrix}$$



$$1 \cdot 1 \cdot 7 + 3 \cdot 5 \cdot 3 + 4 \cdot 2 \cdot 6 - 3 \cdot 1 \cdot 4 - 6 \cdot 5 \cdot 1 -$$

Determinant of a 3 x 3 Matrix

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 6 & 7 \end{vmatrix} \begin{matrix} 1 & 3 \\ 2 & 1 \\ 3 & 6 \end{matrix}$$

$$1 \cdot 1 \cdot 7 + 3 \cdot 5 \cdot 3 + 4 \cdot 2 \cdot 6 - 3 \cdot 1 \cdot 4 - 6 \cdot 5 \cdot 1 - 7 \cdot 2 \cdot 3$$

Determinant of a 3 x 3 Matrix

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 6 & 7 \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 2 & 1 \\ 3 & 6 \end{vmatrix}$$



$$1 \cdot 1 \cdot 7 + 3 \cdot 5 \cdot 3 + 4 \cdot 2 \cdot 6 - 3 \cdot 1 \cdot 4 - 6 \cdot 5 \cdot 1 - 7 \cdot 2 \cdot 3$$

Determinant of a 3 x 3 Matrix

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 6 & 7 \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 2 & 1 \\ 3 & 6 \end{vmatrix}$$

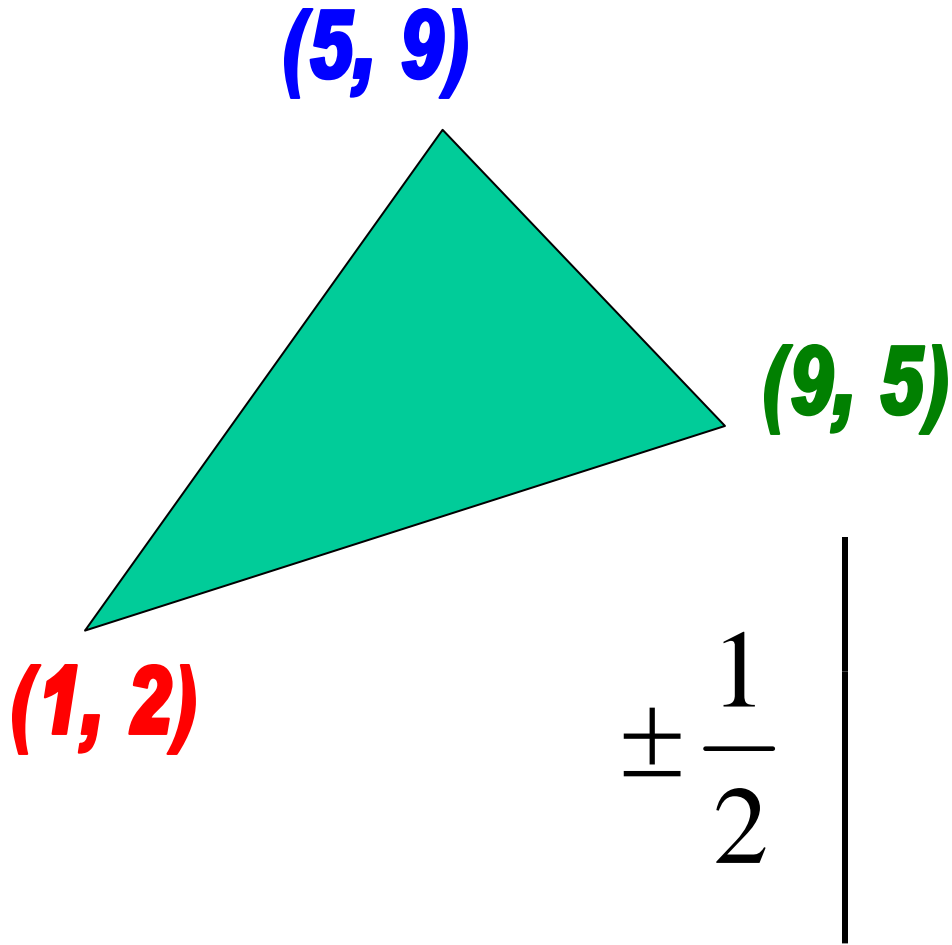
$$1 \cdot 1 \cdot 7 + 3 \cdot 5 \cdot 3 + 4 \cdot 2 \cdot 6 - 3 \cdot 1 \cdot 4 - 6 \cdot 5 \cdot 1 - 7 \cdot 2 \cdot 3$$

Determinant of a 3 x 3 Matrix

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 6 & 7 \end{vmatrix} \begin{matrix} + & + \\ - & - \\ + & + \end{matrix}$$

$$\begin{aligned} & 1 \cdot 1 \cdot 7 + 3 \cdot 5 \cdot 3 + 4 \cdot 2 \cdot 6 - 3 \cdot 1 \cdot 4 - 6 \cdot 5 \cdot 1 - 7 \cdot 2 \cdot 3 \\ & 7 + 45 + 48 - 12 - 30 - 42 \\ & = 16 \end{aligned}$$

Find the area of the Triangle



Use a
Determinant

Determinant of a 3 x 3 Matrix

$$\pm \frac{1}{2} \begin{vmatrix} 1 & 2 & 1 \\ 5 & 9 & 1 \\ 9 & 5 & 1 \end{vmatrix}$$

Diagram illustrating the determinant calculation of a 3x3 matrix. The matrix is shown with a vertical line separating the first two columns from the last two columns. Red arrows indicate the subtraction of the product of the top-right to bottom-left diagonal (1*9*1) from the product of the top-left to bottom-right diagonal (1*1*5). Blue arrows indicate the addition of the product of the top-left to middle-right to bottom-left diagonal (1*1*9) and the product of the top-middle to bottom-left to bottom-right diagonal (2*1*5). The signs of the terms are indicated by '+' above the 1 and 2 in the first row, and '-' below the 1, 5, and 9 in the first column.

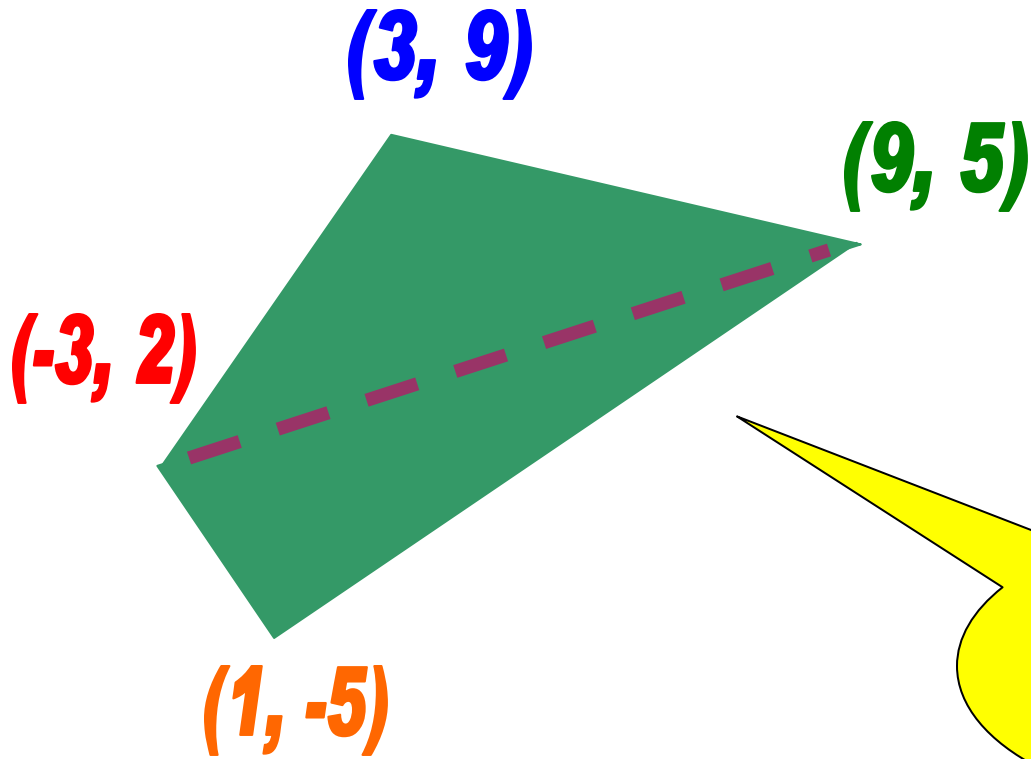


$$\pm \frac{1}{2} ($$

$$\pm \frac{1}{2} (9 + 18 + 25 - 81 - 5 - 10)$$

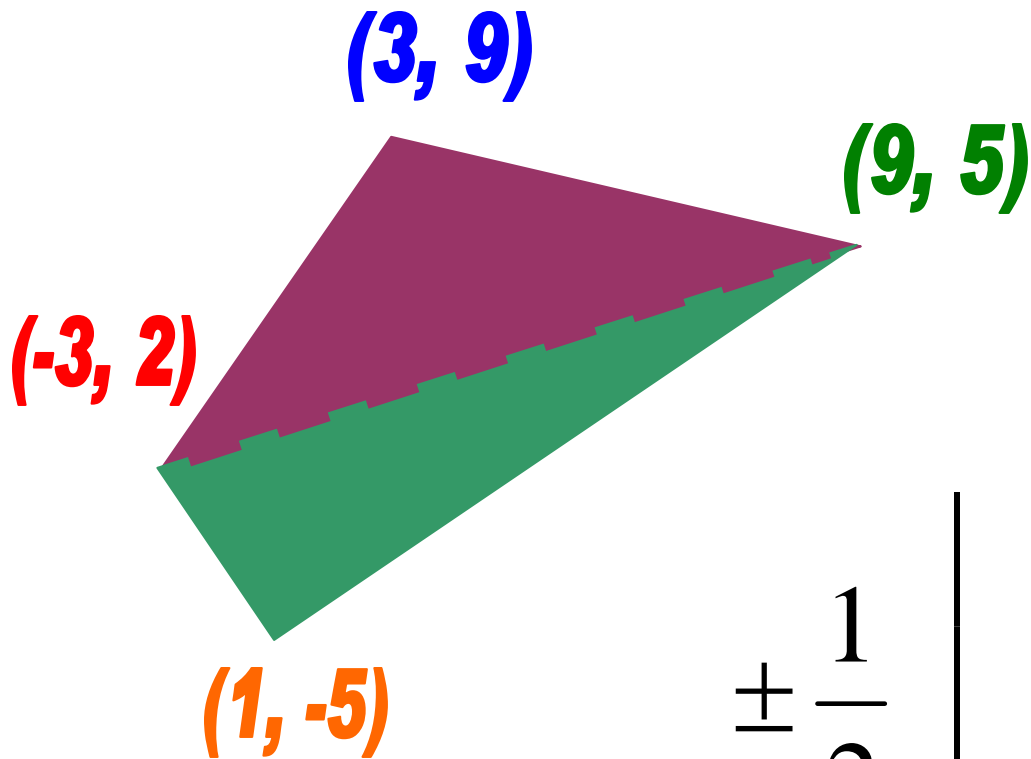
$$\pm \frac{1}{2} (-44) = \mathbf{22}$$

Find the area of the Quadrilateral



Divide into
two triangles

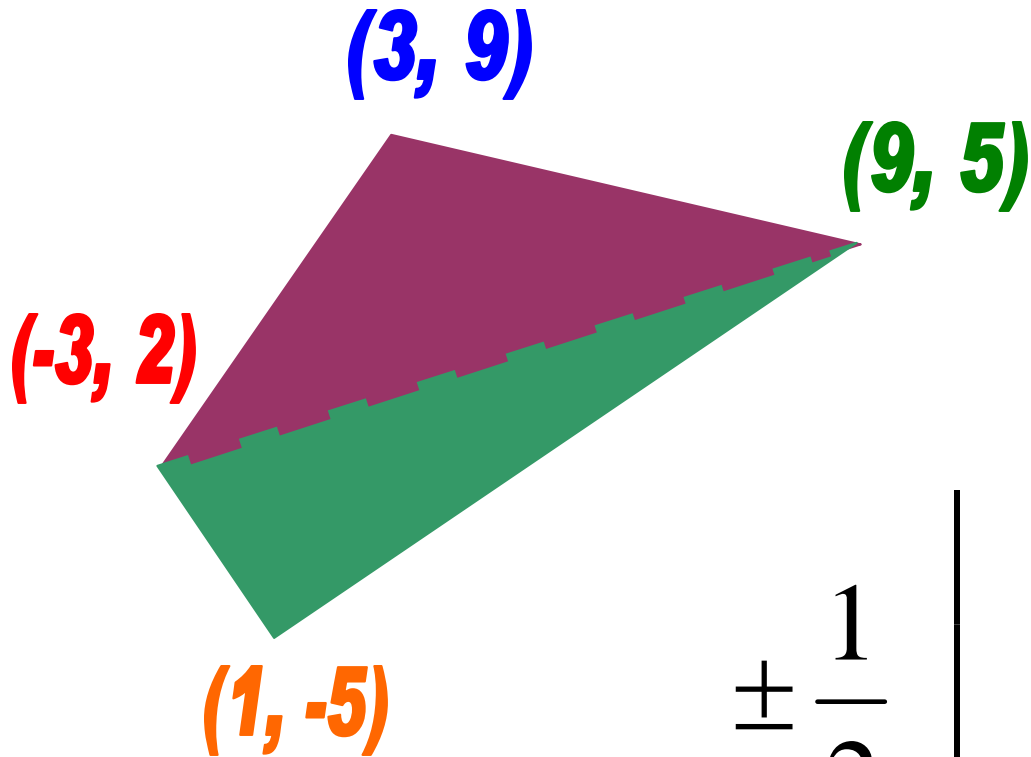
Find the area of the Quadrilateral



$$\pm \frac{1}{2} \left| \begin{array}{c} \\ \\ \end{array} \right| = \pm \frac{1}{2} (-66)$$

33

Find the area of the Quadrilateral

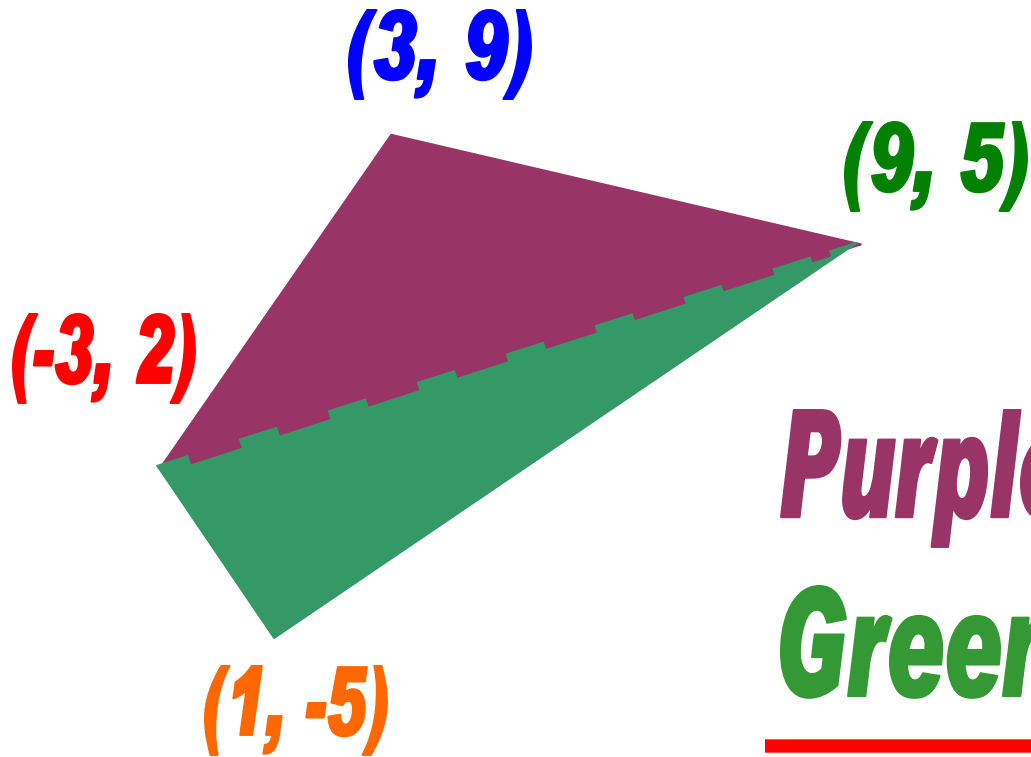


The bottom triangle

$$\pm \frac{1}{2} \left| \begin{array}{c} | \\ | \\ | \end{array} \right| = \pm \frac{1}{2} (-96)$$

48

Find the area of the Quadrilateral



$$\text{Purple Area} = 33$$

$$\text{Green Area} = 48$$

$$\text{Total Area} = 81$$