Student Learning Map
Unit 3

Topic: Geometry Gallery

Course: Mathematics 1

Key Learning(s):

1. The interior and exterior angles of a polygon can be determined by the number of sides of the polygon.
2. There are special inequalities that exist between the sides of a triangle, the angles of a triangle, and between angles and their opposite sides.
3. Congruency of triangles can be determined by special relationships of the sides and angles.
4. Triangles have four different points of concurrency or points of center.
5. Special quadrilaterals are related by their properties.

Unit Essential Question(s):

In what real life situations would it be necessary to determine the interior or exterior angles of a polygon?
How could the points of concurrency be used to solve a real life situation?
What relationships between sides and angles can be used to prove the congruency of triangles?
How are the parallelogram, rectangle, rhombus, square, trapezoid, and kite alike and different?

Optional Instructional Tools: Mira, patty paper, Geometer's Sketchpad, compass, straight edge, protractor, graphing calculator

Concept:

Measures of interior and exterior angles of polygons

Lesson Essential Questions
1. How do I find the sum of the measures of the interior angles of a polygon?
2. How do I find the measure of an interior angle of a regular polygon?
3. How do I find the sum of the measures of the exterior angles of a polygon and the measure of an exterior angle of a regular polygon?
4. What is the relationship between the exterior angle of a triangle and its 2 remote interior angles?

Vocabulary
1. convex polygon
2. concave polygon
3. remote interior angles

Concept:

Triangular Inequalities

Lesson Essential Questions

1. What does the sum of two sides of a triangle tell me about the third side?
2. In a triangle, what is the relationship of an angle of a triangle and its opposite side?
3. What is the relationship between the exterior angle of a triangle and either of its remote interior angles?
4. For two triangles with two pairs of congruent corresponding sides, what is the relationship between the included angle and the third side?
5. How do I use the Converse of the Pythagorean Theorem to determine if a triangle is a right triangle?

Vocabulary
1. triangle inequality
2. side-angle inequality
3. side-side-side inequality
4. exterior angle inequality

Concept:

Triangle Points of Concurrency

Lesson Essential Questions

1. How do I find points of concurrency in triangles?
2. How can I use points of concurrency in triangles?

Vocabulary

1. incenter
2. orthocenter
3. circumcenter
4. centroid

Concept:

Congruency Theorems for Triangles

Lesson Essential Questions

1. How can you justify that two triangles are congruent?

Vocabulary

1. SSS
2. SAS
3. ASA
4. AAS
5. LL
6. HA
7. LA
8. HL

Concept:

Relationships Between Special Quadrilaterals

Lesson Essential Questions

1. What characteristics differentiate the following quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid, and kite?
2. If 3 parallel lines cut off equal segments of one transversal, how would lengths of segments of any transversal be related?

Vocabulary:

1. Kite

Notes:
Vocabulary to Maintain interior angle, exterior angle, diagonal of a polygon, regular polygon

Notes:
Side-side-side inequality is the converse of the Hinge Theorem

Notes:

Notes:
Reference –corresponding parts of congruent triangles are congruent (CPCTC)

Notes:
Vocabulary to be maintained:
1. quadrilateral
2. parallelogram
3. rectangle
4. rhombus
5. square
6. trapezoid
7. diagonal

Mathematics I - Unit 3
Geometry
Teacher Notes

Robotic Gallery Guards Learning Task

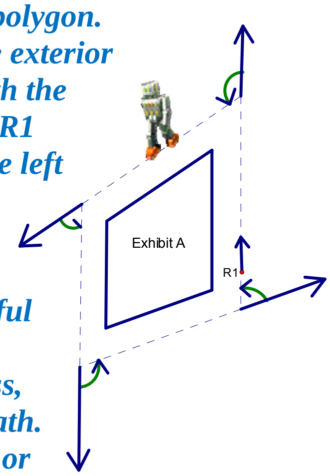
Notes:
This task provides a guided discovery for the following:

- ***Exterior Angle Sum Theorem: If a polygon is convex, ~~then~~ the sum of the measure of the exterior angles, one at each vertex, is 360º.***
- ***Sum of the measures of the interior angles of a convex polygon: 180º(n – 2).***
- ***Measure of each interior angle of a regular n-gon:*** $\frac{180°(n-2)}{n}$
- ***The measure of each exterior angle of a regular n-gon:*** $\frac{360°}{n}$.

Supplies Needed:
Protractors
If possible, enlarged copy of the museum floor

Solutions to Robotic Gallery Guards Learning Task

When a robot reaches a corner, it will stop, turn through a programmed angle, and then continue its patrol. Your job is to determine the angles that R1, R2, R3, R4 and RC will need to turn as they patrol their area. Keep in mind the direction in which the robot is traveling and make sure it always faces forward as it moves around the exhibits.



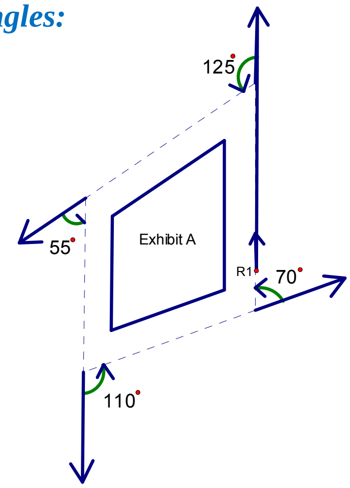
Comments: ***The students have to determine the angle in which the robots need to turn. As students begin this task they may struggle with determining the angle the robot will need to turn through. Students might initially measure the angles inside the polygon. But the correct directions for the movement of the robot require the use of the exterior angle. This may be hard for students to see initially. It helps to extend the path the robot would take if it did not turn when needed. Look to the right at the path R1 might follow. The arrows show the path R1 would take if he did not turn to the left at the top of the exhibit. The curved arrows show the turn the robot needs to make.***

If a student still has trouble visualizing or understanding this it could be helpful for them to act out the job of the robots. Create a path on the floor of the classroom and have a student volunteer to play the part of the robot. The class, or group members, must then give the student robot directions to follow the path. When the student reaches a turning point have them point in the direction he or she is heading and the direction he or she needs to go. The angle created by the student's arms will illustrate the needed turning angle. This process might need to be repeated for several turns.

The path the first robot needs to follow is illustrated below. The other paths are not shown but the angles are given in the correct order from the position of the robot.

Students' angle measures may be slightly different due to errors in measurement. However, the angle measures should be close to the measures indicated below.

1. What angles will R1 need to turn? What is the total of these turns?
   Angles: ***Sum: 360°***



2. What angles will R2 need to turn? What is the total of these turns?
   Angles: 90°, 73°, 17°, 45°, 135° ***Sum: 360°***

3. What angles will R3 need to turn? What is the total of these turns?
   Angles: 47°, 64°, 54°, 62°, 44°, 89° ***Sum: 360°***

4. What angles will R4 need to turn? What is the total of these turns?
   Angles: 56°, 20°, 90°, 26°, 50°, 104°, 14° ***Sum: 360°***

5. What do you notice about the sum of the angles? Do you think this will always be true?
   All the angles sum to 360°.
   Answers will vary.

6. Determine the measure of the interior angles of the polygons formed by Exhibits A – D route.
   Exhibit A: 55°, 125°, 70°, and 110° ***Sum: 360°***
   Exhibit B: 90°, 107°, 163°, 135°, and 45° ***Sum: 540°***

Exhibit C: 136°, 91°, 133°, 116°, 126°, and 118° ***Sum: 720°***
Exhibit D: 76°, 166°, 124°, 160°, 90°, 154°, and 130° ***Sum: 900°***

7. Look at the sum of the angles in the polygons. Do you notice a pattern?
The sums are increasing by 180° each time.

8. How could you determine the sum of the angles in the exhibits without using a protractor?
Comments: This problem gives students the chance to extend the pattern they noticed in #8. If they are having trouble, encourage them to relate the sum of the angles and the number of sides of the polygon. There are many simple investigations students can do here if needed.

Answers may vary.
Sample solutions:
a. I can divide the shape into triangles. Since the sum of the angles in a triangle is 180°, I can multiply 180° by the number of triangles in the figure.
b. The pattern seems to be related to the number of sides. If you subtract two from the number of sides of the polygon and multiply that by 180° it will give you the sum of the angles of the polygon.
c. By dividing the shapes into triangles, and comparing them, we found a pattern.

# Sides	# Triangles	Sum of Angles
3	1	180°
4	2	360°
5	3	540°
6	4	720°

The # of triangles is always 2 less than the number of sides.

9. Can you generalize your results from #9 so that you could find the sum of the interior angles of a decagon? Dodecagon? An n-gon?
Decagon: There are two fewer triangles than the 10 sides so,
(10 – 2)(180°) = 1440°
Dodecagon: There are two fewer triangles than the 12 sides so,
(12 – 2) (180°) = 1800°
n-gon: There are two fewer triangles than the n sides so,
(n – 2)(180°)

10. Choose one of the exhibits from the map and look at the interior and exterior angle found at a vertex. What do you notice about the sum of these two angles? Find this sum at each vertex. What do you notice?
The sum of the angles is 180°. The goal is for students to recognize the interior and exterior angle together form a linear pair. Even if they are relying on their measurements found using a protractor, they need to realize the sum is 180°.

11. Looking at your results from #10 and #11, can you find a way to prove your conjecture about the sum of the exterior angles from #6?
One possible solution:
There are n pairs of exterior and interior angles in an n-gon. The sum of all the interior and exterior angles is n(180°) = 180n°. Since I'm not interested in the interior angles I need to subtract the sum of all the interior angles. For an n-gon, the sum of the interior angles is (n – 2)(180). Putting this together I get:

$$180n° – [(n – 2)180°] =$$
$$180n° – [180n° – 360°] =$$
$$180n° – 180n° + 360° = 360°$$

So, for any n-gon the sum of the exterior angles is always 360°.

12. The museum intends to create regular polygons for its next exhibition, how can the directions for the robots be determined for a regular pentagon? Hexagon? Nonagon? N-gon?
The robot always turns the measure of the exterior angles. If a polygon is regular, the interior and exterior angles all have the same measure.

For the pentagon, there are 5 interior and exterior angles. The sum of the exterior angles is 360° and the sum of the interior angles is 540°.
Each exterior angle: 360°/5 = 72°
Each interior angle: 540°/5=108°

For the hexagon, there are 6 interior and exterior angles. The sum of the exterior angles is 360° and the sum of the interior angles is 720°.
Each exterior angle: 360°/6 = 60°
Each interior angle: 720°/6=120°

For the nonagon, there are n interior and exterior angles. The sum of the exterior angles is still 360° and the sum of the interior angles is 1260°.
Each exterior angle: 360°/9 = 60°
Each interior angle: 1260°/9=140°

For an n-gon there are find the measure of the exterior angles, divide 360° by the number of angles. To find the measure of the interior angles, divide the sum of the interior angles by the number of angles.
Each exterior angle: 360°/n
Each interior angle: 180°(n – 2) / n

13. A sixth exhibit was added to the museum. The robot patrolling this exhibit only makes 15º turns. What shape is the exhibit? What makes it possible for the robot to make the same turn each time?
The turns are the exterior angles of the polygon. Working backwards using the formula from above, we have:

$$15^\circ = \frac{360^\circ}{n}$$
$$15n^\circ = 360^\circ$$
$$n = \frac{360^\circ}{15^\circ}$$

$$n = 24$$

The exhibit has 24 sides.

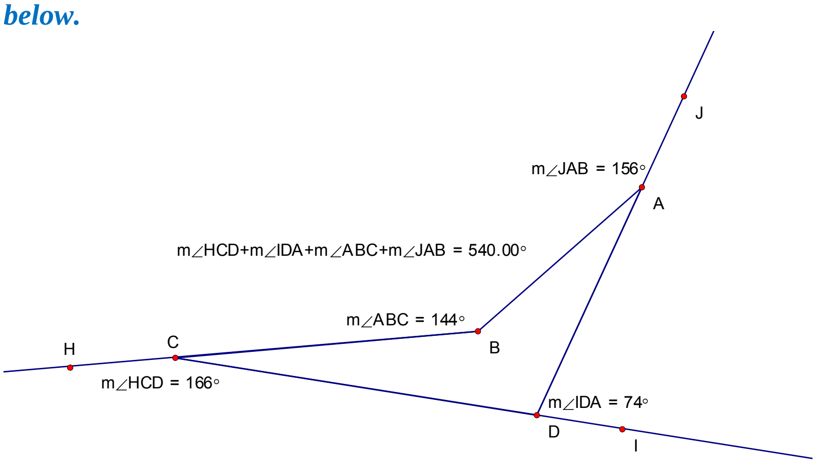
14. Robot 7 makes a total of 360 º during his circuit. What type of polygon does this exhibit create?
There is not enough information. All polygons have exterior angle sums of 360°.

15. The sum of the interior angles of Robot 7's exhibit is 3,420 º. What type of polygon does the exhibit create?
Working backwards using the formula for the sum of the interior angles of a polygon, we have:

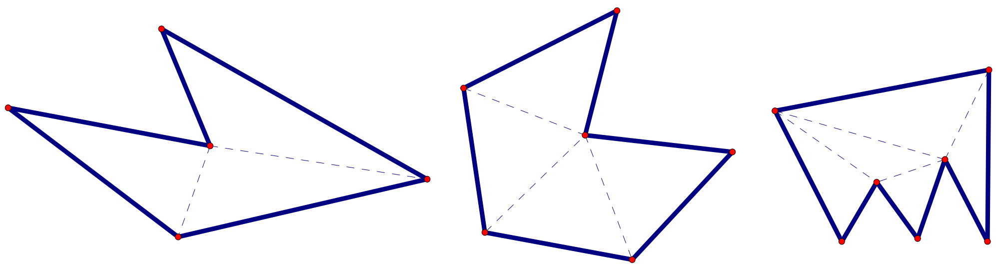
$$3{,}420^\circ = 180^\circ(n-2)$$
$$3420^\circ = 180n^\circ - 360$$
$$3780^\circ = 180n^\circ$$
$$21 = n$$

The exhibit has 21 sides.

16. All of the current exhibits and the robots are convex polygons. Would your generalizations hold for a concave polygon?
The generalization does not hold for the exterior angle sum. See the counter example below.



However, the generalization does hold for the interior angle sum. You can divide a concave polygon into 2 fewer triangles than the number of sides. See the examples below.



17. The museum is now using Exhibit Z. Complete a set of instructions for RZ that will allow the robot to make the best circuit of this circular exhibit. Defend why you think your instructions are the best.
    Comments: The robots will always travel along a polygonal path. However, by focusing on regular polygons students can use polygons that approach circular paths by using very small exterior angles.

    Solution: (answers may vary)
    If the robot is given instructions to make 5° turns it will walk a polygon with 72 sides. But if the robot is given instructions to make a 1° turn it will walk a polygon with 360 sides. The turn can be made increasingly smaller to make the polygon look more and more like a circle.

<u>Poor Captain Robot Learning Task</u>
<u>Notes:</u>
This task provides a guided discovery for the following:

- ***Measure of the exterior angle of triangle equals the sum of the measures of the two remote interior angles.***
- ***The sum of the lengths of any two sides of a triangle is greater than the length of the third side.***

<u>Supplies Needed:</u>
The supplies needed by students will depend upon the way in which you want them to investigate this problem. Students can use items such as straws, pipe cleaners or spaghetti noodles to create scaled examples to solve the tasks. Geometer's Sketchpad or similar software could also be used for this task.

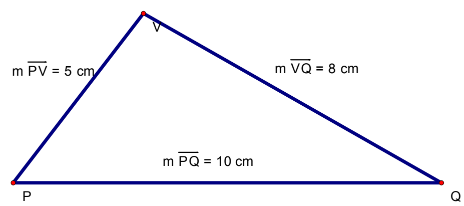
<u>Solutions to Poor Captain Robot Learning Task</u>
Captain Robot's positronic brain is misfiring and he will only take instructions to move three distances. He will no longer acknowledge angles given in directions and chooses all

of them himself. He will not travel the same path twice and refuses to move at all if he cannot end up back at his starting point.

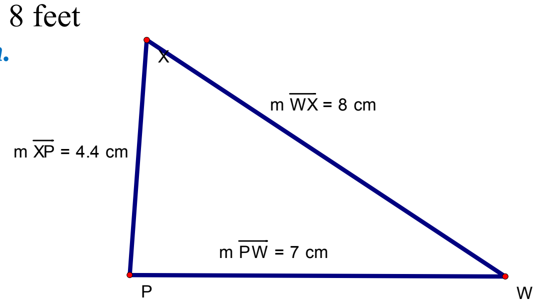
1. Captain Robot was given the following sets of instructions. Determine which instructions Captain Robot will use and which sets he will ignore. Be ready to defend your choices.

The robot will move if the path creates a triangle allowing Captain Robot to return to the original position.
If the numbers will not create a triangular path, Captain Robot will not move.

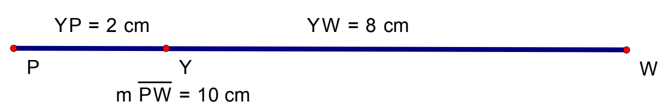
Instruction Set #1: 10 feet, 5 feet, 8 feet
Yes, this will create a triangular path.



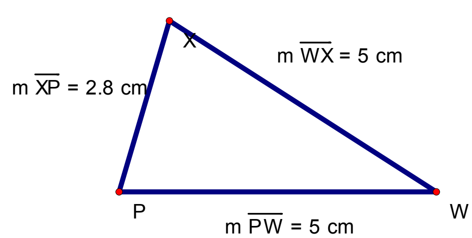
Instruction Set #2: 7 feet, 4.4 feet, 8 feet
Yes, this will create a triangular path.



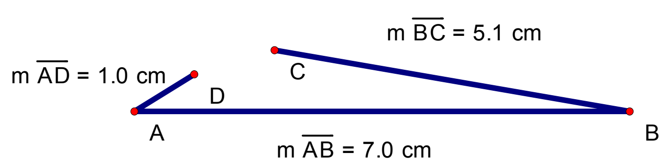
Instruction Set #3: 10 feet, 2 feet, 8 feet
No, this will not create a triangular path.
Since Captain Robot will not retrace his steps he will not follow this path.



Instruction Set #4: 5 feet, 5 feet, 2.8 feet
Yes, this will create a triangular path.



Instruction Set #5: 7 feet, 5.1 feet, 1 foot
No, this will not create a triangular path. Captain Robot will not reach his starting point.



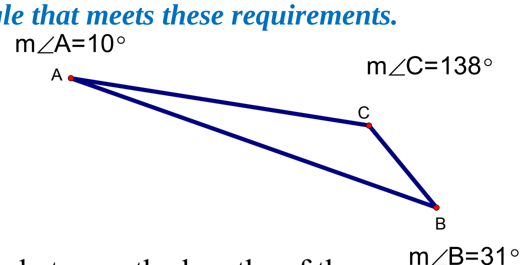
2. Determine a method that will always predict whether or not Captain Robot will move.

Comments: Make sure the students develop the triangle inequality here.

Sample Solution:
In order for the robot to move, the lengths must always be able to create a triangle. If one side is too short it will not create a triangle. For example, on Set #5, the 1.0 and 5.1 lengths together were not long enough to meet using a 7 cm side too. So, the two smaller lengths must add together to be longer than the longest side. Set #3 shows us the sum of the two smaller lengths cannot be equal to the longest side because the triangle collapses.

3. Captain Robot traveled from point A to point B to point C. His largest turn occurred at point C and his smallest turn occurred at point A. Order the sides of the triangle from largest to smallest.

Comment: It will be easiest to solve this problem if the students will sketch or use geometry software to create a triangle that meets these requirements.

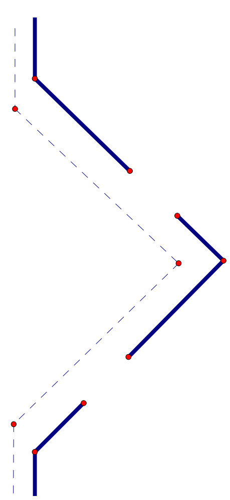


4. Is there a relationship between the lengths of the side and the measures of the angles of a triangle? Explain why or why not.

Comment: Make sure to help students develop the side-angle inequality here. Some students may need to draw and measure many triangles to develop this idea.

Sample Solution:
The largest side of the triangle is opposite the largest angle. The smallest side is opposite the smallest angle.

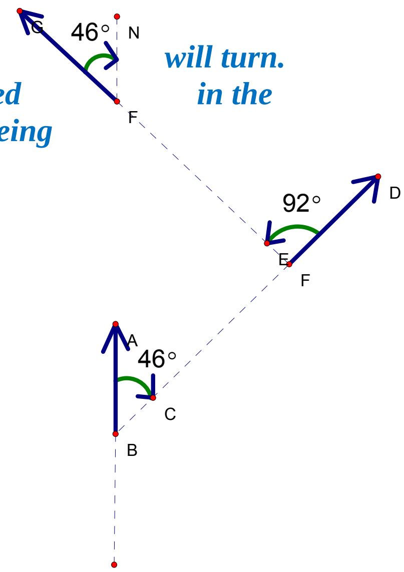
The museum has decided Captain Robot needs to patrol two access doors off the side of the museum. Captain Robot's addition to his route is shown below.



5. Determine the angles Captain Robot will need to turn.

Comment: Note the different directions in which Captain Robot will turn. Many students will need to draw the paths and angles as indicated in the solution below. Make sure they are very clear on what angle is being measured. This is important for later questions.

Solution is given in the picture to the right.

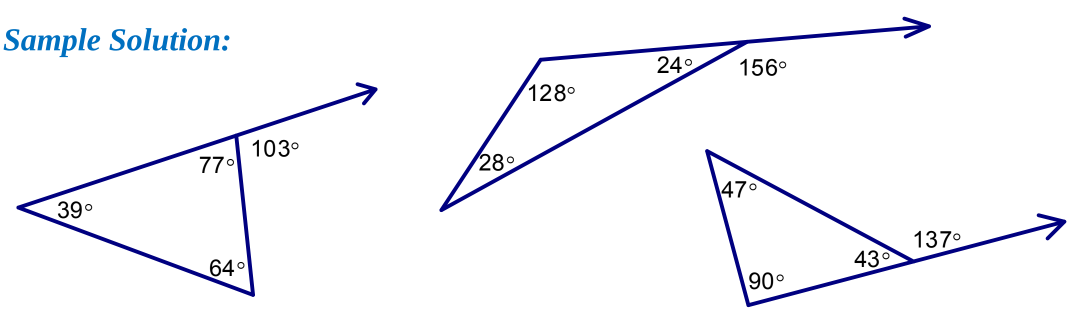


6. Look carefully at the angles you chose for the robot's route. What do you notice?

Answers may vary.
Sample Solutions:
One of the angles is equal to the sum of the other angles.
You can create a triangle by connecting point B and F. Then you have an interior angle and an exterior angle and an angle that is a vertical angle to one the interior angles. In fact, we know two interior angles' and one exterior angle's measure.

7. Draw several triangles. Measure the interior angles and one exterior angle. What pattern do you notice? Make a generalization relating the interior and exterior angles.
Comments:
Students need to learn some new vocabulary as they discuss this problem. They will most likely not know the term 'remote interior' angles. This term makes the generalization much easier to write. However, it's not necessary for the students to know this term in order to make a generalization. The discussion of the term can easily occur as students share the solutions with the class.

Sample Solution:



The two interior angles that don't touch the exterior angles always add to the measure of the exterior angle.

Triangles Learning Task

Notes:
This task provides a guided discovery for the following:
- ***SSS, SAS, ASA, AAS, and HL Congruence Postulates and Theorems can be used to prove triangles are congruent.***
- ***SSA and AAA are not valid methods to prove triangles are congruent***
- ***corresponding parts of congruent triangles are congruent***
- ***congruent triangles can be used to solve problems involving indirect measurement***

Supplies Needed:
There are many ways students can approach this task and the supplies needed will depend upon the method you choose for your students.
- ***Hands-on manipulatives like spaghetti noodles, straws, pipe cleaners, d-stix, etc. can used to represent the lengths of the sides. Protractors will be needed to create the indicated angles between the sides and clay or play dough can be used to hold the sides together.***

- ***Students can use compasses, straightedges and protractors to construct the triangles.***
- ***Geometer's Sketchpad, or similar software, is a good tool to use in these investigations.***

Solutions:

Triangles Learning Task

The students at Hometown High School decided to make large pennants for all 8 high schools in their district. The picture above shows typical team pennants. The Hometown High students wanted their pennants to be shaped differently than the typical isosceles triangle used for pennants and decided each pennant should be a scalene triangle. They plan to hang the final products in the gym as a welcome to all the schools who visit Hometown High.

Jamie wanted to know how they could make sure that all of the pennants are congruent to each other. The students wondered if they would have to measure all six parts of every triangle to determine if they were congruent. They decided there had to be a shortcut for determining triangle congruence, but they did not know the minimum requirements needed. They decided to find the minimum requirements needed before they started making the pennants.

Comment:

As students construct triangles to meet the given restrictions, it may become necessary for them to use a protractor and/or ruler to determine the measures of angles and/or sides not given to convince themselves the triangles are not congruent.

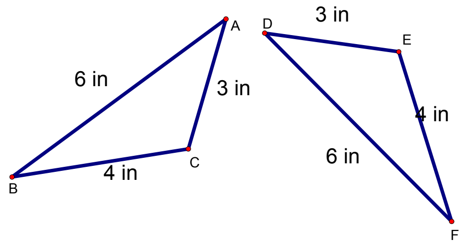
As soon as students begin discussing their triangles, encourage them to use correct geometric terms. They will need to use words like corresponding sides, corresponding angles, included sides and included angles. This common vocabulary will make sharing information about the triangles much easier.

1. Every triangle has __6__ parts, __3___ sides and __3___ angles.

2. First they picked out 3 sides and each person constructed a triangle using these three sides. Construct a triangle with sides of 3 inches, 4 inches, and 6 inches. Compare

your triangle to other students' triangles. Are any of the triangles congruent? Are three sides enough to guarantee congruent triangles? Explain.

Sample Answers:
All the triangles are congruent to each other. Even though some triangles might be rotated in a different direction or reflected, they are all congruent.

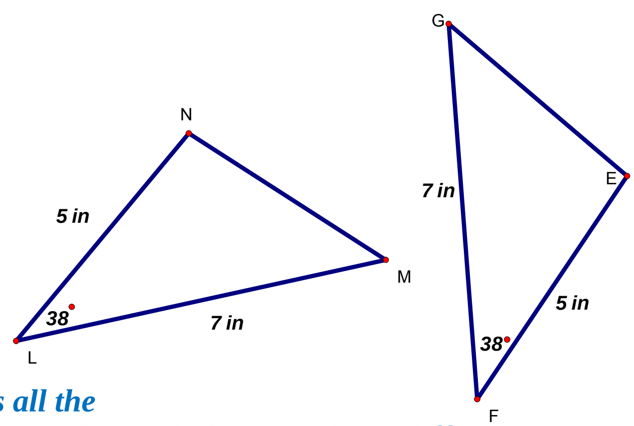


Three sides are enough to guarantee the triangles are congruent.
Nothing about the triangle can be changed when all the sides have to be a specific length.

3. Next the class decided to use only 2 sides and one angle. They choose sides of 5 inches and 7 inches with an angle of 38°. Using these measures, construct a triangle and compare it to other students' triangles. Are any of the triangles congruent?

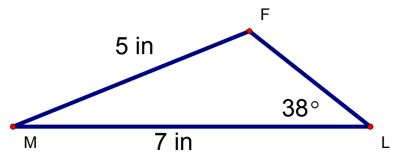
Comments:
Note that the students are not told <u>where</u> to put the given angle. There are three possibilities. They can place the angle between the given side lengths, opposite the 5 inch side, or opposite the 7 inch side. Groups may approach this problem differently. Some groups may only choose to investigate the case where the angle between the two given sides. If so, that is fine as the next problem will push them to look at other possibilities. However, some groups may want to immediately investigate all three options.

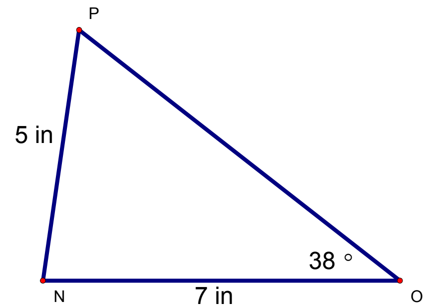
Solutions:
The solutions below include all three cases.
Case 1: Angle between the two given sides



If you put the angle between the two given sides all the triangles will be congruent. Even though some triangles might be rotated in a different direction or reflected, they are all congruent.

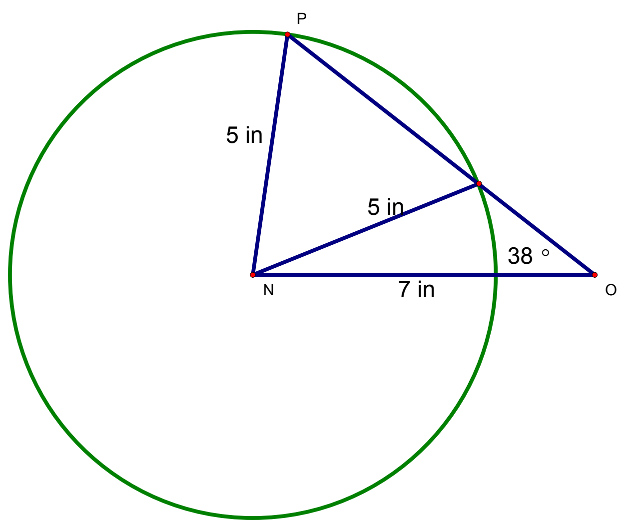
Case 2: Angle opposite the 5 in. side



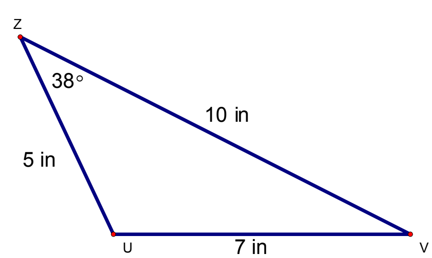


As is the example above, this does not always create two congruent triangles.

This can also be illustrated in the following manner. Draw the 7 inch side and the 38° angle. Since the given angle is at O, the 5 inch side must use N as one of its endpoints. There are an infinite number of segments having a length of 5 inches with N as one endpoint. These can all be represented by constructing a circle using N as the center with a radius of 5 inches. We are interested in where the side of unknown length intersects the circle. Those points of intersection meet all three conditions: the angle is 38°, the side adjacent to the angle is 7 inches and the opposite side is 5 inches. In this case there are two different triangles which satisfy those conditions.



Case 3: Angle opposite the 7 in. side
Comments:
This is the triangle everyone will get when the angle is opposite the side of 7 inches.



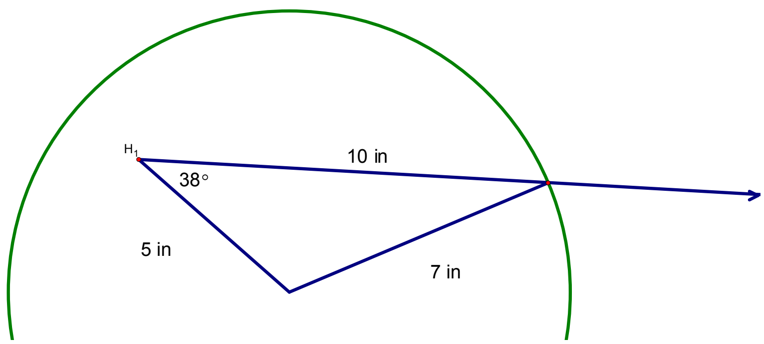
It is important to note that this case will give you congruent triangles.
Ask students to notice the similarities between this case and case 2. In both cases the angle is not included between the two given sides. So, the same pieces of information

are given: side of 5 inches, side of 7 inches and an angle of 38°. But since we did not get always get congruent triangle in case 2, and those triangles are not congruent to the triangles here, this information does not give us a guarantee of congruency.

This may confuse some students but it is critical for them to gain an understanding of this concept. If the constraint sometimes creates congruent triangles and sometimes doesn't, we cannot use it to prove two triangles are congruent.

Ask students: What is the key difference between case 2 and case 3? (e.g. The angle is opposite the larger given side in case 3.) Why do we only get one triangle now? (See drawing below.) If students have time, they might want to investigate the third case a little more to see if anything special is happening there.

The sketch below, using a circle with a radius of 7 inches shows why only one triangle can be created when the 7 inch side is opposite the given angle.



4. Joel and Cory ended up with different triangles. Joel argued that Cory put her angle in the wrong place. Joel constructed his triangle with the angle between the two sides. Cory constructed her sides first then constructed her angle at the end of the 7 in. side not touching the 5 in. side. Everybody quickly agreed that these two triangles were different. They all tried Cory's method, what happened? Which method, Joel's or Cory's will always produce the same triangle?

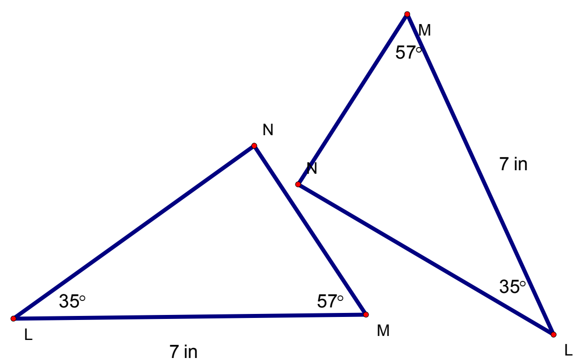
Comments:
If a group investigated all three cases for #3, this will look very familiar.

Sample Answers:
Joel's method will always work. Cory's method sometime works and sometimes doesn't. (For further explanation, see solutions for Case 2 and Case 3 in #3).

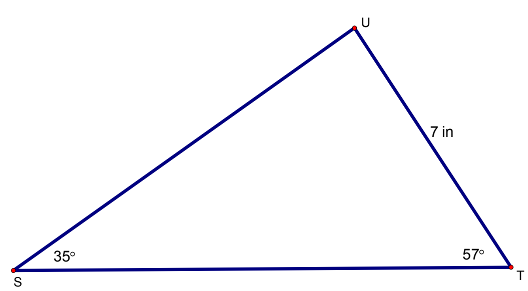
5. Now the class decided to try only 1 side and two angles. They chose a side of 7 in. and angles of 35° and 57°. Construct and compare triangles. What generalization can be made?

Comments:
This problem is similar to #3. Note that students are not told where to put the given side. They have three options. They can place the side between the given angles, opposite the 35° angle, or opposite the 57° angle. . Groups may approach this problem differently. Some groups may only choose to investigate the case where the side between the two given angles. If so, that is fine as the next problem will push them to look at other possibilities. However, some groups may want to immediately investigate all three options.

Solutions:
The solutions below include all three cases.
Case 1: Side between the two given angles
If you put the side between the two given angles all the triangles will be congruent. Even though some triangles might be rotated in a different direction or reflected, they are all congruent.

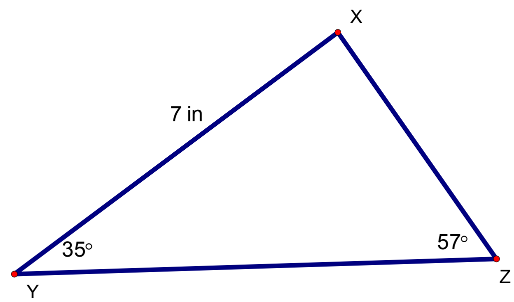


Case 2: 7 in side opposite the 35° angle
The triangle constructed here is not congruent to the triangle constructed in case 1. However, all the triangles constructed <u>in this manner</u> are congruent to each other. (AAS)



Case 3: 7 in side opposite the 57° angle

The triangle constructed here is not congruent to the triangle constructed in case 1. However, all the triangles constructed <u>in this manner</u> are congruent to each other. (AAS)



Comments:
Even though the triangles in cases 2 and 3 may look very similar they cannot be congruent to each other. The missing angle is 88°. That means the 7 inch side, in case 2, has to be the smallest side because it is opposite the smallest angle. In case 3 there is a side that is smaller than the 7inch side. Therefore, the three sides cannot be congruent to each other.

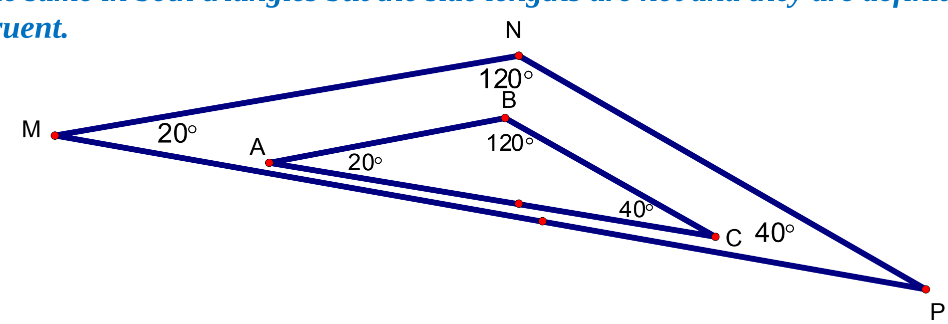
6. Jim noticed that Sasha drew her conclusion given two angles and the included side. He wondered if the results would be the same if you were given any two angles and one side. What do you think?

See the 3 cases discussed in question #5. It is important for students to understand that AAS does work but the order in which the angles and side are listed is very important.

This would be a good time to discuss the connection between AAS and ASA. For example, in case 2, it can be proven that the third angle of the triangle is 88° (using the Triangle Sum Theorem). So all triangles constructed with angles of 35° and 57° will also have a third angle of 88°. This means the side of 7 inches will always be included between the 35° angle and the 88° angle which leads us to ASA found in case 1.

7. The last situation the class decided to try was to use three angles. They chose angles of 20°, 40°, and 120°. How do you think that worked out? Construct a triangle using these three angles and compare with others. Can they prove two triangles are congruent using the three corresponding angles? Explain why or why not.

Solution:
Three angles are not enough information to prove congruence because the side lengths can vary even while the angle measures stay the same. In the picture below the angles are the same in both triangles but the side lengths are not and they are definitely not congruent.



8. Summarize the results using the chart below. Discuss what is meant by the common abbreviations and how they would help to remember the triangle situations you have just explored.

Comments:
The significance of the abbreviations has been written with more formality than the students may initially write on their own. This would be a good time to stop and help students formalize the triangle congruence postulates and theorems. At this point, the students have dealt with specific cases only. They need to expand this to deal with general cases.

This is point at which a discussion of postulates and theorems would be important. Generally, SSS, SAS, and ASA are considered postulates. From these we derive the ASA Theorem.

Solutions:

Common Abbreviation	**Explanation Abbreviation**	**Significance**
SSS	***3 sides lengths***	***If all sides of one triangle are congruent to all sides of another triangle, the triangles will be congruent.***
SAS	***2 sides and the included angle (the angle created by the two sides)***	***If two sides and the included angle in one triangle are congruent to the corresponding sides and included angle in another triangle, the triangles will be congruent.***
SSA	***2 sides and the non-included angle whose vertex lies on the side listed second***	***This does <u>not</u> guarantee that the triangles are congruent to each other.***
ASA	***2 angles and the included side (the side between the two angles)***	***If two angles and the included side in one triangle are congruent to the corresponding angles and included side in another triangle, the triangles will be congruent.***
AAS	***2 angles and the non-included side that is part of the second angle***	***If two angles and the non-included side in one triangle are congruent to the corresponding angles and non-included side in another triangle, the triangles will be congruent.***
AAA	***three congruent angles***	***Creates similar triangles, but <u>not</u> necessarily congruent triangles.***

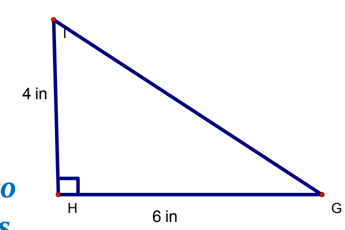
Comments:
The next few problems offer good opportunities to emphasize the importance of proofs. Emphasize the importance of a logical argument and the justification of every statement in a proof. These problems give students a chance to see how the methods discovered above can be applied to determine ways to prove two right triangles are congruent. The solutions below are not formally written proofs, they are logical through processes students might employ to prove the methods are valid.

9. The methods listed in the table, which can be used for proving two triangles congruent, require three parts of one triangle to be congruent to three corresponding parts of another triangle. Nakita thought she could summarize the results but she wanted to try one more experiment. She wondered if the methods might be a bit shorter for right triangles since it always has one angle of 90^{o}. She said: "I remember the Pythagorean Theorem for finding the length of a side of a right triangle. Could this help? My father is a carpenter and he always tells me that he can determine if a corner is square if it makes a 3 – 4 – 5 triangle." Nakita chose to create a triangle with a hypotenuse of 6 inches and a leg of 4 inches. Does her conjecture work? Why or why not?

Solution:
Yes. This method works.

Given $\overline{AC}$ 4 in. and $\overline{AC}$ is 6in.and m∠ACB = 90°you can use the Pythagorean Theorem to determine the length of the other leg. $4^2 + a^2 = 6^2$ Solving this shows the length of a is $\sqrt{20}$ inches. There is no other possible length for this leg. So, any two right triangles with a leg of 4 in. and a hypotenuse of 6 in. will have another leg of $\sqrt{20}$ in. This means all three sides of the triangles will be congruent. Therefore, the triangles are congruent by SSS.
Or-
Using the Pythagorean Theorem we can show the second leg will always be $\sqrt{20}$ inches and the angle between the legs is always 90°. Using the right angle the two legs, you could use SAS to show the triangles will always be congruent.

10. What if Nakita had chosen 6 inches and 4 inches to be the length of the legs. Does her conjecture work? Why or why not?



Solution:
Yes. This conjecture is also true.

Because this is a right triangle, the measure of the angle between the two legs will always be 90°. Therefore, all right triangles with congruent legs will be congruent using SAS.

Or-
Using the Pythagorean Theorem we can find the length of the hypotenuse. Since there is only one possible length for the hypotenuse the three sides of the triangles will always be congruent. According to SSS the triangles will always be congruent.

11. What are the minimum parts needed to justify two right triangles are congruent? Using the list that you already made, consider whether these could be shortened if you knew one angle was a right angle. Create a list of ways to prove congruence for right triangles only.
Comments:
The students should develop the following theorems here. Make sure they investigate cases that are not specifically mentioned above. Also, make sure they connect their theorems to the congruence postulates and theorems they have already discovered.

Solutions:
If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent. (HL) This can be proven using SSS or SAS.

If two legs of one right triangle are congruent to the corresponding legs of another right triangle, the triangles are congruent. (LL) This can be proven using SAS or SSS.

If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and corresponding angle of another right triangle, then the triangles are congruent. (HA) This can be proven using ASA.

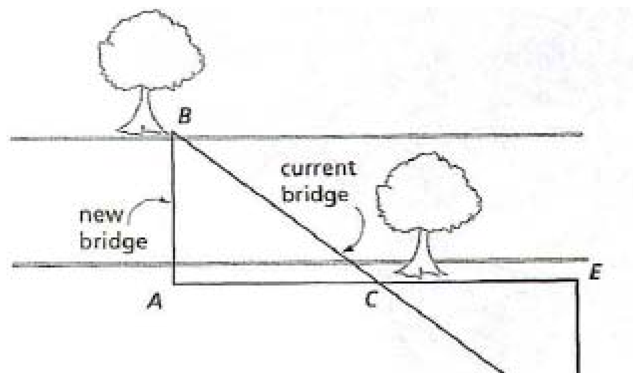
If one leg and an acute angle of a right triangle are congruent to the corresponding leg and angle of another right triangle, then the triangles are congruent. (LA) This can be proven using ASA.

12. Once it is known that two triangles are congruent, what can be said about the parts of the triangles? Write a statement relating the parts of congruent triangles.
Comments:
Students need to develop a clear understanding of CPCTC (corresponding parts of congruent triangles are congruent) here. They will use this in the problems that follow. Students need to use correct mathematical terms as they discuss "corresponding" parts of the triangles.

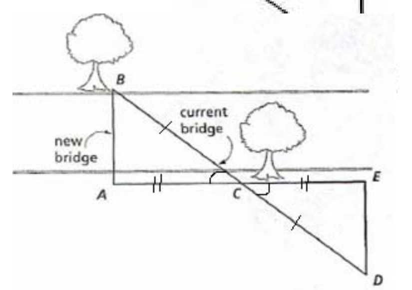
Solutions:
If two triangles are congruent the corresponding angles and sides must also be congruent.

Congruent triangles can be used to solve problems encountered in everyday life. The next two situations are examples of these types of problems.

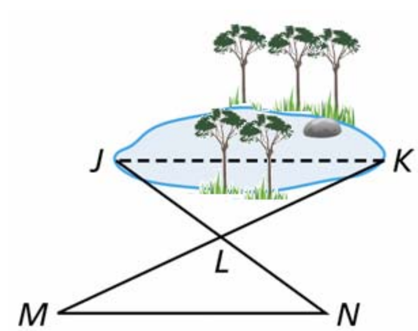
13. In order to construct a new bridge, to replace the current bridge, an engineer needed to determine the distance across a river, without swimming to the other side. The engineer noticed a tree on the other side of the river and suddenly had an idea. She drew a quick sketch and was able to use this to determine the distance. Her sketch is to the right. How was she able to use this to determine the length of the new bridge? You do not have to find the distance; just explain what she had to do to find the distance.



Solution:
The engineer needs to measure the distance across the current bridge and then make sure $\overline{DC}$ is the same length. She can measure the distance from point A to C so she can make sure C is the midpoint of $\overline{AE}$. ∠ACB is congruent to ∠DCE because they are vertical angles. That means ΔACB is congruent to ΔECD by SAS. So, $\overline{AB}$ is congruent to $\overline{ED}$ because they are corresponding parts of congruent triangles. By measuring the length of $\overline{ED}$ the engineer will have found the length of the new bridge.



14. A landscape architect needed to determine the distance across a pond. Why can't he measure this directly? He drew the following sketch as an indirect method of measuring the distance. He stretched a string from point J to point N and found the midpoint of this string, point L. He then made stretched a string from M to K making sure it had same center. He found the length of MN was 43 feet and the length of segment LK is 19 feet. Find the distance across the pond. Justify your answer.



Solution:
Answers may vary about why he can't measure the distance directly.

The distance across the pond is 43 feet.
This can be shown using congruent triangles. The engineer made sure L was the midpoint of $\overline{MK}$ and $\overline{JN}$. $\overline{ML}$ is congruent to $\overline{LK}$ and $\overline{JL}$ is congruent to $\overline{LN}$ by the definition of midpoint. ∠MLN is congruent to ∠KLJ because they are vertical angles.

So ΔMNL is congruent to ΔKJL by SAS. $\overline{MN}$ and $\overline{KJ}$ are congruent by CPCTC. Since the measure of $\overline{MN}$ is 43 feet the measure of $\overline{JK}$ is 43 feet.

<u>Middles and Halves Learning Task</u>

<u>Notes:</u>

This task provides a guided discovery for the following:

- ***a review of the classification of triangles by sides and angles***
- ***a review of basic constructions: angle bisectors, perpendicular lines, and perpendicular bisectors***
- ***review of the definitions of angle bisectors, perpendicular bisectors, and altitudes***
- ***introduction of a new term: median***
- ***special properties of isosceles and equilateral triangles***

<u>Supplies Needed:</u>

- ***miras (or a similar reflective tool)***
- ***patty paper***
- ***compasses***
- ***straightedges***
- ***Geometer's Sketchpad or similar geometry software, if possible***

Middles and Halves Learning Task

Let's take a minute to review the ways we classify triangles using sides and angles.

Comments:

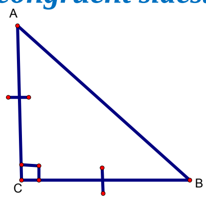
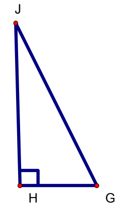
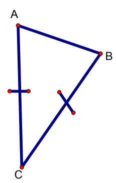
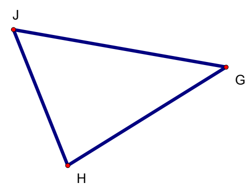
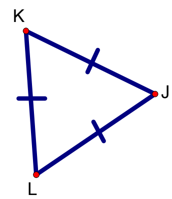
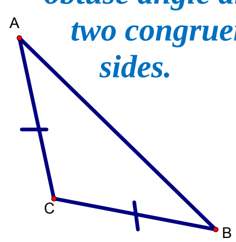
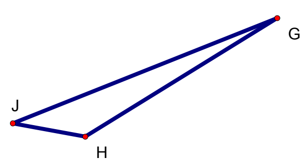
An excellent applet for exploring triangles can be found at:
http://illuminations.nctm.org/ActivityDetail.aspx?ID=142

1. Are the following triangles possible? Why or why not? If possible, draw an example.

Comments:

The classification of triangles by side lengths or angle measures is an important geometric concept. This question challenges students to determine if a specific triangle, described by combining the two types of classification, exists.

Solutions:

Right Isosceles ***Yes. The triangle must have one right angle and two congruent sides.*** 	Right Scalene ***Yes. The triangle must have one right angle and no congruent sides.*** 	Right Equilateral ***No. Equilateral triangles have 3 angles of 60° each. So, you can't have a right angle in an equilateral triangle.***
Acute Isosceles ***Yes. The triangle must have all acute angles and two congruent sides.*** 	Acute Scalene ***Yes. The triangle must have all acute angles and no congruent sides.*** 	Acute Equilateral ***Yes. The triangle must have all acute angles and three congruent sides.*** 
Obtuse Isosceles ***Yes. The triangle must have one obtuse angle and two congruent sides.*** 	Obtuse Scalene ***Yes. The triangle must have one obtuse angle and no congruent sides.*** 	Obtuse Equilateral ***No. Equilateral triangles have 3 angles of 60° each. So, you can't have an obtuse angle in an equilateral triangle.***

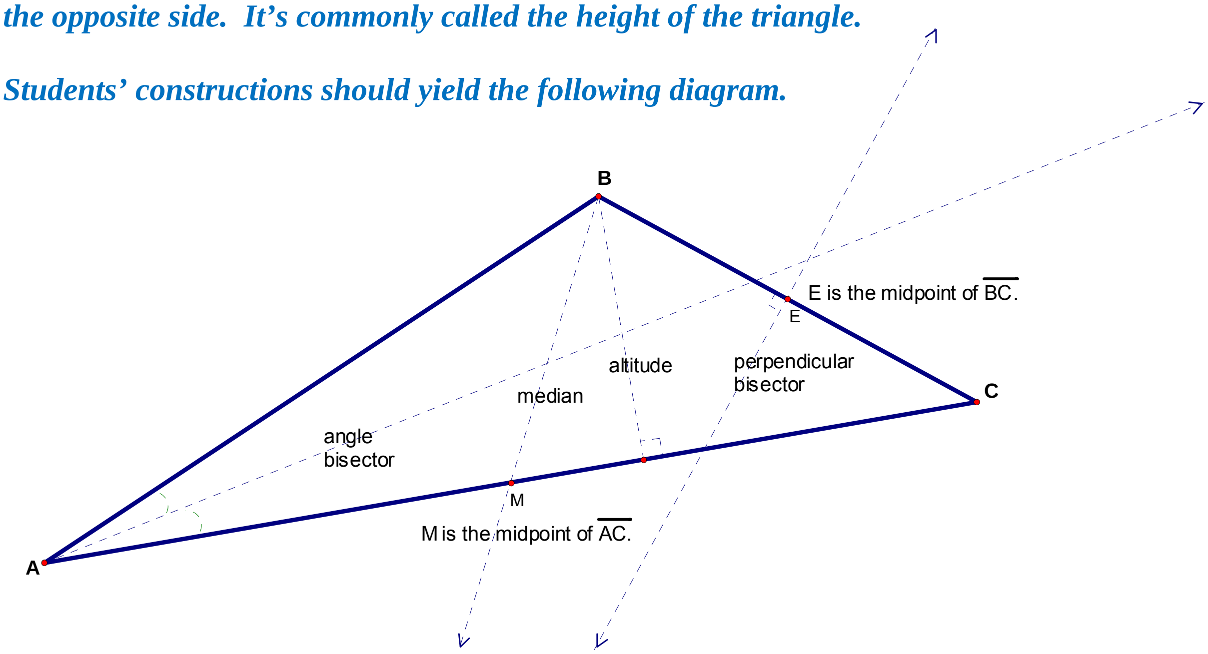
2. Copy the scalene triangle below onto a piece of blank paper. Using patty paper, a MIRA, and formal construction with compass and straight edge, at least once, construct the following for triangle ABC: the altitude from B to AC, the median from B to AC, the perpendicular bisector of BC, and the angle bisector of $\angle A$. (Note: If technology is available, make it your 4th option.) Once this is completed, write a definition in your own words for angle bisector, perpendicular bisector, median, and altitude.

Comments:
Students learned formal construction methods in the seventh grade.

Make sure students discuss their definitions and can identify the critical pieces of information in each definition. This terminology is prerequisite knowledge but may need to be reviewed to ensure students will be successful in the next tasks.

Solutions:
Answers may vary.
Angle bisector: ***a ray that divides an angle into two congruent angles.***
Perpendicular bisector: ***a segment, ray or line perpendicular to a segment at the midpoint.***
Median: ***a segment from the vertex to the midpoint of the opposite side in a triangle.***
Altitude: ***a segment from one vertex of a triangle perpendicular to the line containing the opposite side. It's commonly called the height of the triangle.***

Students' constructions should yield the following diagram.



3. What are some special properties of the angle bisectors and perpendicular bisectors?

Comments:
Students should have explored this previously. These properties are very important for students to remember as they begin to explore triangle centers.

Solution:

- ***If a point lies on the bisector of an angle, then the point is equidistant from the sides of the angle.***
- ***If a point is equidistant from the sides of an angle, then the point lies on the bisector of the angle.***

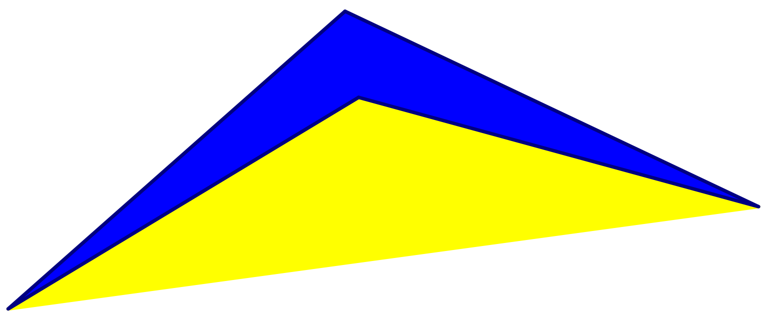
4. Draw an isosceles triangle and an equilateral triangle. What special properties are true of these two types of triangles?

Solution:

There are many special properties of isosceles triangles and equilateral triangles. A few of these are listed below.

- ***If two sides of a triangle are equal, then the angles opposite those sides are equal.***
- ***The bisector of the vertex angle of an isosceles triangle is perpendicular to the base at its midpoint.***
- ***If two angles of a triangle are equal, then the sides opposite those angles are equal.***
- ***An equilateral triangle is also equiangular.***
- ***An equilateral triangle has three 60^o angles.***
- ***An equiangular triangle is also equilateral.***

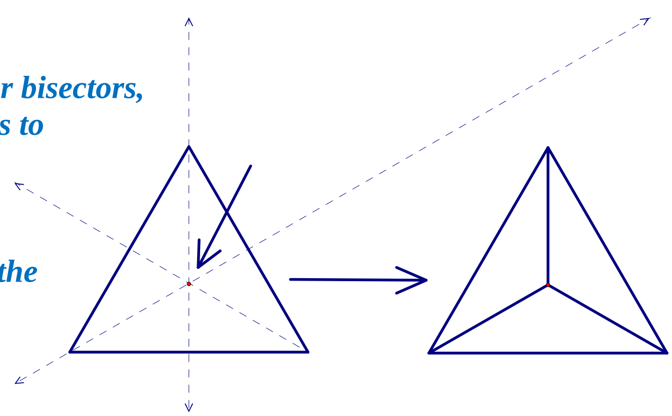
5. The logo below is similar to several in our everyday life (the triangles are isosceles). What one line could you draw to divide the logo exactly in half? Explain.



Solution:
When drawn from the vertex angle, the perpendicular bisector, angle bisector, median, and altitude will all divide the triangle into halves. Students should notice these are all the same line when drawn from the vertex angle of an isosceles triangle.

6. Construct an equilateral triangle. Find the point that when connected to the vertices will divide it into 3 congruent isosceles triangles.

Solution:
The point can be found using the perpendicular bisectors, angle bisectors, medians or altitudes. The key is to connect the point of concurrency to the vertices of the triangle.



Using their knowledge of congruent triangles, the students should be able to <u>prove</u> the three triangles are congruent.

Centers of Triangles Learning Task

Notes:

This task provides a guided discovery and investigation of the points of concurrency in triangles. Students will construct and use the following points:

- ***incenter***
- ***orthocenter***
- ***circumcenter***
- ***centroid***

Supplies Needed:

- ***miras (or a similar reflective tool)***
- ***patty paper***
- ***compasses***
- ***straightedges***
- ***Geometer's Sketchpad or similar geometry software, if possible***

Comments:

In this task students will determine the location for an amusement park by finding the centers of a triangle. The centers will be new to the students but not the constructions. Make sure students remember the significance of points on the perpendicular bisector of a segment (equidistant from the endpoints of the segment) and the points on an angle bisector (equidistant from the sides of the angle).

As students work through the tasks and present their solutions to the class make sure they emphasize the name of the center found, how it was found, and its significance.

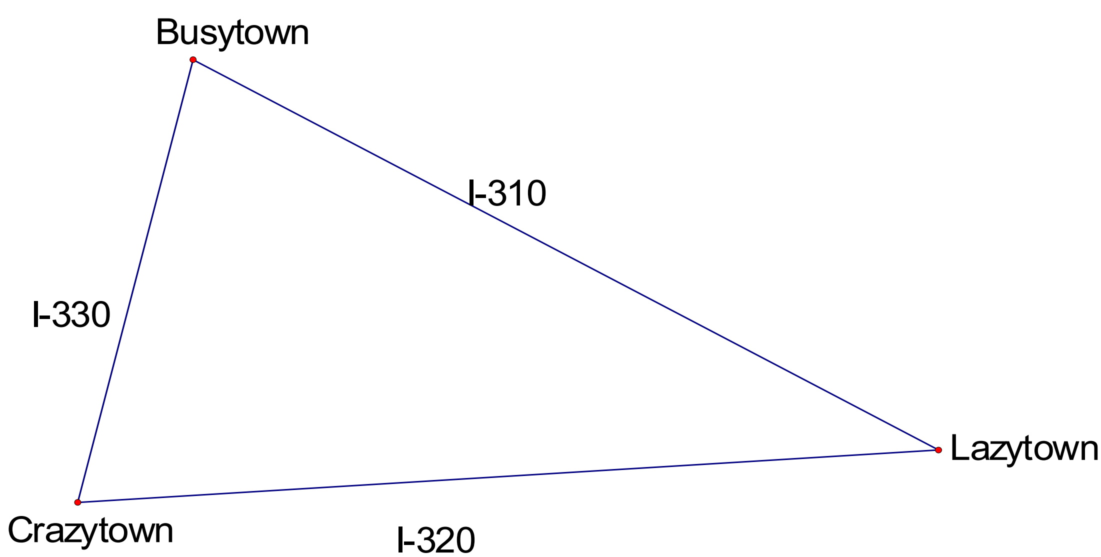
The significance of the circumcenter and incenter can be determined through measurement. If needed, encourage students to measure the distances from the triangle centers to the sides and vertices of the triangle. Students can use properties of the perpendicular bisectors and angle bisectors to justify their conjectures about the significance of the circumcenter and the incenter. Students may require help in determining the significance of the centroid (center of gravity). Students can determine one of the significant features of the centroid through measurement (the centroid is twice the distance from the vertex to the opposite side). Other than being the point of concurrency of the altitudes the orthocenter has no additional significance in this task.

Triangle Center:	*Point of Concurrency of:*	*Significance of:*
Incenter	**Angle bisectors**	**Center of inscribed circle Equidistant from the sides of the triangle**
Circumcenter	**Perpendicular bisectors**	**Center of the circumscribing circle**

		Equidistant from the vertices of the
Orthocenter	**Altitudes**	-
Centroid	*Medians*	*Center of balance or gravity* *The distance from a vertex to the centroid is half the distance from the centroid to the opposite side.*

Centers of Triangles Learning Task

A developer plans to build an amusement park but wants to locate it within easy access of the three largest towns in the area as shown on the map below. The developer has to decide on the best location and is working with the ABC Construction Company to minimize costs wherever possible. No matter where the amusement park is located, roads will have to be built for access directly to the towns or to the existing highways.



1. Just by looking at the map, choose the location that you think will be best for building the amusement park. Explain your thinking.

Comments:
"Just by looking" is important here. Students need to take a moment to look at the towns and make a decision.

Solution:
Answers will vary.

2. Now you will use some mathematical concepts to help you choose a location for the tower.

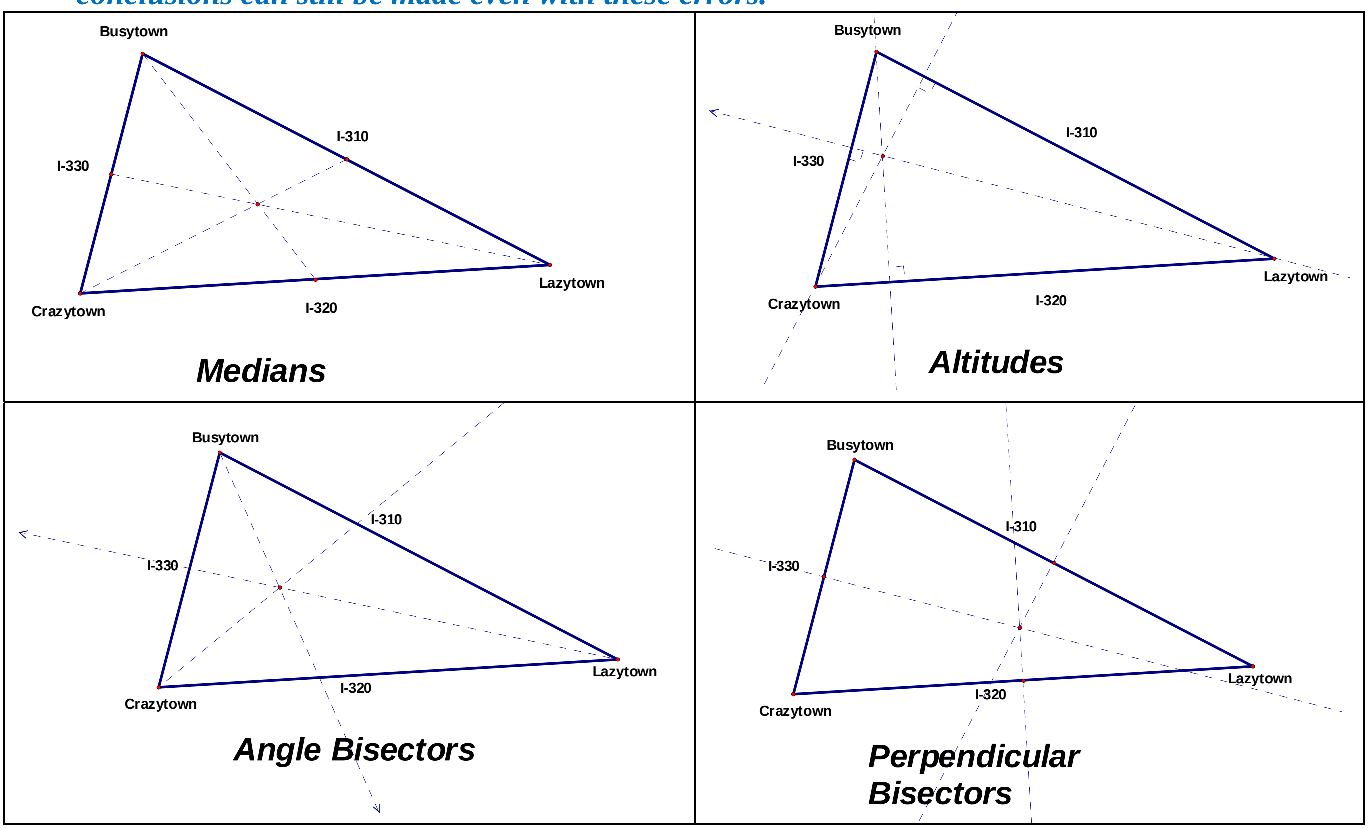
In the previous lesson, you learned how to construct medians and altitudes of triangles. In 7th grade, you learned how to construct angle bisectors and perpendicular bisectors. Investigate the problem above by constructing the following:

a) all 3 medians of the triangle
b) all 3 altitudes of the triangle
c) all 3 angle bisectors of the triangle
d) all 3 perpendicular bisectors of the triangle

You have four different kinds of tools at your disposal- patty paper, MIRA, compass and straight edge, and Geometer's Sketch Pad. Use a different tool for each of your constructions.

Not drawn to scale:

The constructions, regardless of which tool is used, should result in the following. It is very important for students to realize the three lines always intersect in a single point. Because some of the tools they are using are not very precise the students will have some errors in measurement. These errors should be discussed as a group but some conclusions can still be made even with these errors.



3. Choose a location for the amusement park based on the work you did in part 2. Explain why you chose this point.

Solution:

Answers will vary but it is critical for students to have a mathematical justification for their decision. For example, they may choose the circumcenter because it is equidistant

from all three cities. Or they may choose the incenter because it is equidistant from each of the roads. They could choose the centroid instead of the circumcenter because it is closer to two of the cities while not being that much further away from Lazytown.

4. How close is the point you chose in part 3, based on mathematics, to the point you chose by observation?
   Solution:
   Answers will vary.

You have now discovered that each set of segments resulting from the constructions above always has a point of intersection. These four points of intersection are called the ***points of concurrency*** of a triangle.

The intersection point of the medians is called the ***centroid*** of the triangle.
The intersection point of the angle bisectors is called the ***incenter*** of the triangle.
The intersection point of the perpendicular bisectors is called the ***circumcenter*** of the triangle.
The intersection point of the altitudes is called the ***orthocenter*** of the triangle.

Comments:
Students struggle with using the terms <u>point of concurrency</u> and <u>concurrent lines</u> correctly. Make sure they understand what they mean and how to use them. To help them understand the significance of a point of concurrency, ask them to draw three lines on their paper, without looking at anyone else's paper. Then, ask whose lines are drawn in such a way that all three intersect at the same point. Have them compare their drawing and determine the different way three lines can be related to each other.

5. Can you give a reasonable guess as to why the specific names were given to each point of concurrency?

Comments:
Students will need to have an idea of the significance of the triangle centers in order to answer this question. If they have not already done so, they need to go back to their constructions and explore the properties of the triangle centers they found in #2. Some groups may have discovered these properties already.

Make sure the name of the center, how it was found, and its significance are emphasized as students present their solutions.

Solution:
Answers will vary.

6. Which triangle center did you recommend for the location of the amusement park?
   Solution:

The students are only being asked to name their point. They need to decide if their chosen point is the centroid, incenter, circumcenter, or orthocenter based on the definitions above.

7. The president of the company building the park is concerned about the cost of building roads from the towns to the park. What recommendation would you give him? Write a memo to the president explaining your recommendation.
   Comments:
   Since the president is concerned about the cost of the roads, students need to take that into account in their memo. In consideration of the cost of building the roads, some groups may want to change their earlier decision. For example, if a group chose to use the circumcenter because it was an equal distance from each of the cities they may want to choose the incenter or median to reduce the cost of building the roads. Or, students could measure the total distance from a point of concurrency to each of the cities and choose the center that gives the shortest total distance.

   Solution:
   Answers may vary. Mathematical justification of the answer is the most important aspect of this activity.

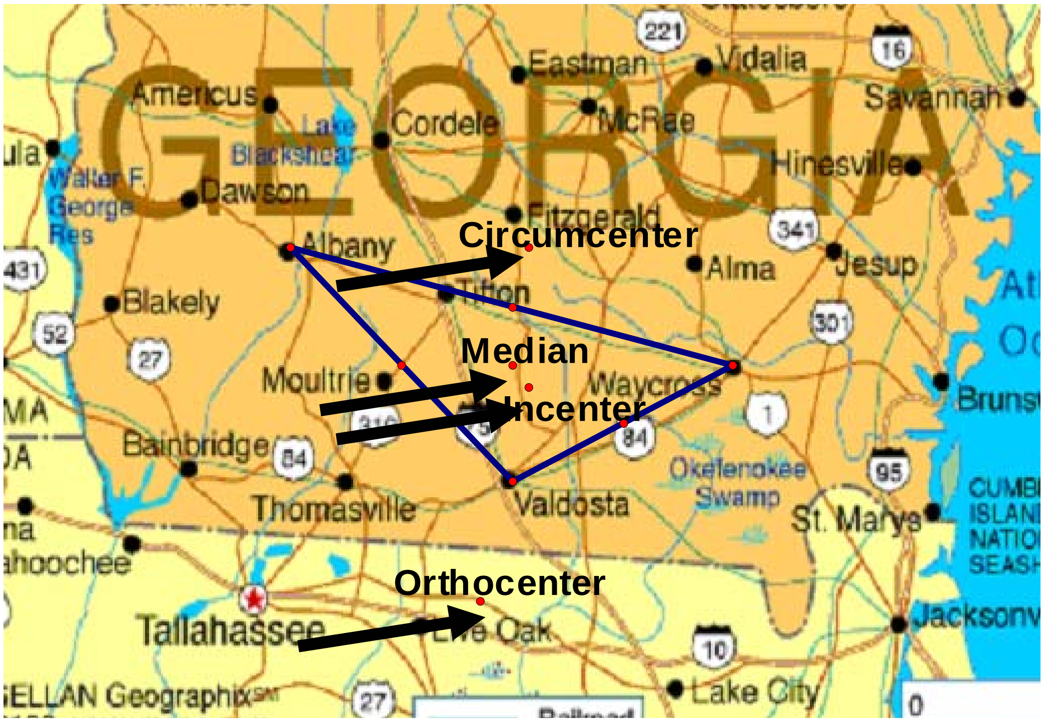
Location Learning Tasks

Comments:
These tasks present an opportunity for students to apply the knowledge gained in the Amusement Park Task to two new scenarios. These are very open ended tasks and could be assessed using a rubric valuing the mathematical content, reasoning, and communication skills demonstrated by students in their solutions. A rubric should be based on the Content Standard MM1G3e and the Process Standards, MM1P1 through MM1P5.

Cell Tower

A cell service operator plans to build an additional tower so that more of the southern part of Georgia has stronger service. People have complained that they are losing service, so the operator wants to remedy the situation before they lose customers. The service provider looked at the map of Georgia below and decided that the three cities: Albany, Valdosta, or Waycross, were good candidates for the tower. However, some of the planners argued that the cell tower would provide a more powerful signal within the entire area if it were placed somewhere between those three cities. Help the service operator decide on the best location for the cell tower.

Solutions:
The triangle centers are indicated below. The students may choose different points as the solution to the problem. They must be able to justify mathematically why they chose that point. For example, they might choose the circumcenter because it is equidistant from the three cities. They might choose the incenter because it is equidistant from the three highways. Or, they might choose the median because it is closer to being in the center of the three cities.



Compose a memo to the president of the cell company justifying your final choice for the location of the tower. Use appropriate mathematical vocabulary and reasoning in your justification.

Solution:
Answers will vary. Students should address their mathematical reasoning behind their choice. See comments above about using a rubric for grading purposes.

Burn Center

Now, you can apply your new found knowledge to a more important situation:

Because of the possible closing of Grady Hospital, several groups have decided to fund an acute trauma hospital located where it would give the most people of Georgia the most access. Using one of the sets of cities indentified below, or your choices of three cities, and the map from the earlier question, decide where the new trauma hospital should be located. Be sure to mathematically justify your answer.

City set one: Bainbridge, Savannah, and Dalton

City set two: Thomasville, Sylvania, and Calhoun

City set three: Valdosta, Augusta, and Marietta

Solution:

Answers will vary. Students should address their mathematical reasoning behind their choice. See comments above about using a rubric for grading purposes.

Constructing with Diagonals Learning Task

Notes:
This task provides a guided discovery and investigation of the properties of quadrilaterals. Students will determine which quadrilateral(s) can be constructed based on specific information about the diagonals of the quadrilateral(s).

Supplies Needed:
There are many ways students can approach this task and the supplies needed will depend upon the method you choose for your students.

- ***Hands-on manipulatives like spaghetti noodles, straws, pipe cleaners, d-stix, etc. can used to represent the lengths of the sides. Protractors will be needed to create the indicated angles between the sides and clay or play dough can be used to hold the sides together.***
- ***Students can use compasses, straightedges and protractors to construct the triangles.***
- ***Geometer's Sketchpad, or similar software, is a good tool to use in these investigations.***

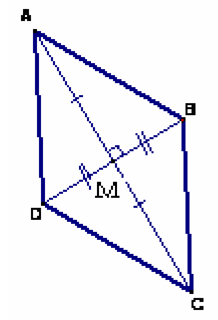
Comments:
Sample proofs are given for each problem. The sample provided are not the only correct way these proofs can be written. Students should realize that proofs can be

logically organized in with differing orders of steps. They should also be given the opportunity to decide which type of proof the prefer writing.

1. Construct two segments of different length that are perpendicular bisectors of each other. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints?

Comment:
Make sure the segments the students draw are of different lengths and <u>both</u> segments are bisected.

Solution:
The quadrilateral is a parallelogram and a rhombus.



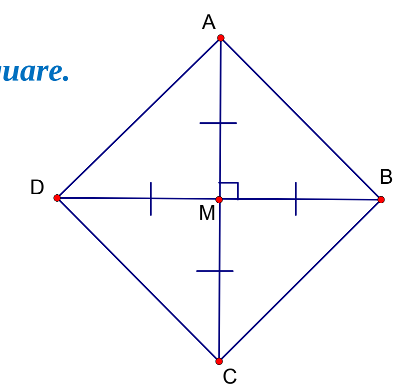
Sample proof:
Given: $\overline{AC} \perp \overline{BD}, \overline{AC}$ bisects $\overline{BD}$, and $\overline{BD}$ bisects $\overline{AC}$

Prove: ABDC is a parallelogram and a rhombus.

Proof: $\overline{AC} \perp \overline{BD}$ is given, so, by definition $\angle AMB$, $\angle BMC$, $\angle CMD$ and $\angle DMA$ are right angles. All right angles are congruent to each other so $\angle AMB \cong \angle BMC \cong \angle CMD \cong \angle DMA$. By the definition of bisect, $\overline{AM} \cong \overline{MC}$ and $\overline{BM} \cong \overline{MD}$. So, ΔADM, ΔABM, ΔCDM, and ΔCBM are congruent by SAS. Using CPCTC, $\angle DAM \cong \angle BCM$ so $\overline{AD} \parallel \overline{CB}$ by the converse of the alternate interior angle theorem. Similarly, using CPCTC, $\angle DCM \cong \angle BAM$ so $\overline{CD} \parallel \overline{BA}$ by the converse of the alternate interior angle theorem. Therefore, by definition ABCD is a parallelogram. Also, $\overline{AD}$, $\overline{AB}$, $\overline{CD}$, and $\overline{CB}$ are congruent by CPCTC. Therefore, ABCD is a rhombus by the definition of a rhombus.

2. Repeat #1 with two congruent segments. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints?

Solution:
The quadrilateral is a parallelogram, rhombus, rectangle and a square.



Sample proof:
Given: $\overline{AC} \perp \overline{BD}, \overline{AC}$ bisects $\overline{BD}$, and $\overline{BD}$ bisects $\overline{AC}$

Prove: ABDC is a parallelogram and a rhombus.

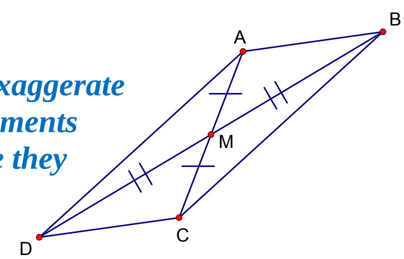
Proof:

Statement	*Reason*
1. $\overline{AC} \perp \overline{BD}$	*1. given*

$\overline{AC} \cong \overline{BD}$ ***$\overline{BD}$ bisects $\overline{AC}$*** ***$\overline{AC}$ bisects $\overline{BD}$***	
2. $\angle AMB$, $\angle BMC$, $\angle CMD$ and $\angle DMA$ are right angles	***2. Definition of $\perp$***
3. $\angle AMB \cong \angle BMC \cong \angle CMD \cong \angle DMA$	***3. All right angles are congruent.***
4. $\overline{AM} \cong \overline{MC}$ and $\overline{BM} \cong \overline{MC}$	***4. Definition of bisect***
5. $\overline{AM} \cong \overline{MC} \cong \overline{BM} \cong \overline{MC}$	***5. $\overline{AC} \cong \overline{BD}$***
6. $\triangle ADM \cong \triangle ABM \cong \triangle CDM \cong \triangle CBM$	***6. SAS***
7. $\angle DAM \cong \angle BCM$ ***$\angle DCM \cong \angle BAM$***	***7. CPCTC***
8. $\overline{AD} \parallel \overline{CB}$ ***$\overline{CD} \parallel \overline{BA}$***	***8. Converse of the Alternate Interior Angles Theorem***
9. <u>ABCD is a parallelogram</u>	***9. Definition of parallelogram***
10. $\overline{AD} \cong \overline{AB} \cong \overline{CD} \cong \overline{CB}$	***10. CPCTC***
11. <u>ABCD is a rhombus</u>	***11. Definition of a rhombus***
12. <u>ABCD is a rectangle</u>	***12. The diagonals of the parallelogram are congruent.***
13. <u>ABCD is a square</u>	***13. The parallelogram is a rectangle with 4 congruent sides.***

3. Construct two segments that bisect each other but are not perpendicular. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints?

Comment:
Students can keep from making incorrect conjectures if they will exaggerate the shape of the figure they are exploring. They should not use segments that seem to be congruent and they should not make them look like they are perpendicular. If they are not careful in drawing this, it would be easy to for them to think this figure would always be a rhombus or even a rectangle.

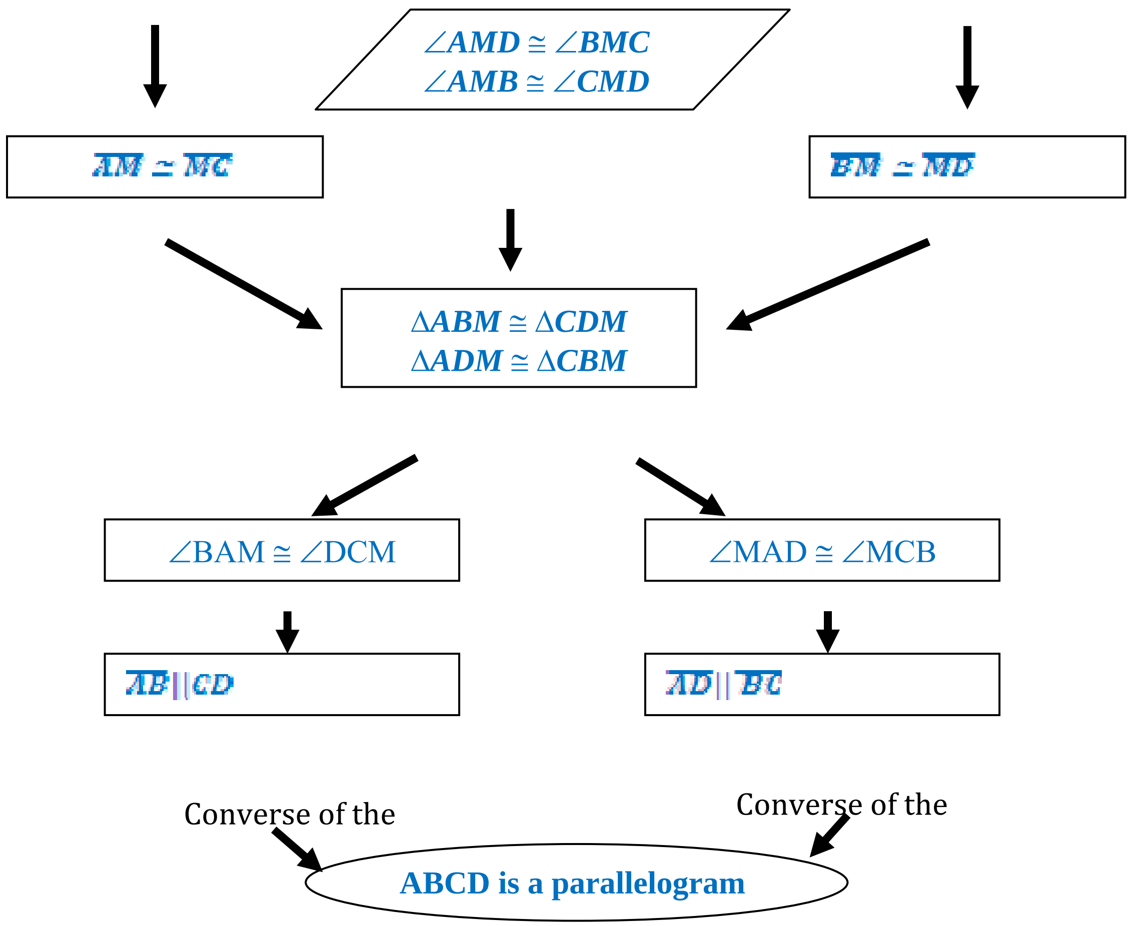


Solution:
The quadrilateral is a parallelogram.

Sample proof:
Given: $\overline{AC}$ bisects $\overline{BD}$, and $\overline{BD}$ bisects $\overline{AC}$
Prove: ABDC is a parallelogram.
Proof:

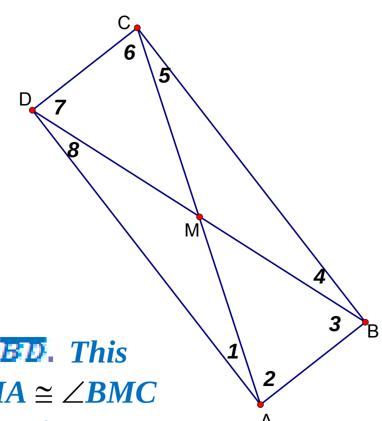
$\overline{AC}$ bisects $\overline{BD}$

$\overline{BD}$ bisects $\overline{AC}$



Definition of

4. What if the two segments in #3 above are congruent in length? What type of quadrilateral is formed? What names can be used to describe the quadrilaterals formed using these constraints?



Solution:

The quadrilateral is a parallelogram and a rectangle.

Sample proof:

Given: *$\overline{AC}$ bisects $\overline{BD}$, $\overline{BD}$ bisects $\overline{AC}$, and $\overline{AC} \cong \overline{BD}$*

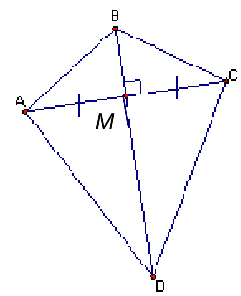
Prove: *ABDC is a parallelogram and a rectangle.*

Proof: *We are given that $\overline{AC}$ bisects $\overline{BD}$, $\overline{BD}$ bisects $\overline{AC}$, and $\overline{AC} \cong \overline{BD}$. This means $\overline{AM} \cong \overline{MC} \cong \overline{BM} \cong \overline{MC}$. Since vertical angles are congruent ∠DMA ≅ ∠BMC and ∠DMC ≅ ∠BMA. So, ΔDAM ≅ ΔBCM and ΔCDM ≅ ΔBAM by SAS. Using CPCTC, ∠6 ≅ ∠2 and ∠4 ≅ ∠2 which leads to $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$ by the Converse of the Interior Angle Theorem. Since the opposite sides are parallel, by definition, ABCD is a parallelogram. Using the Interior Angle Theorem the following pairs of angles are*

congruent: $\angle 7$ and $\angle 3$, $\angle 5$ and $\angle 1$. Since $\overline{AM} \cong \overline{MC} \cong \overline{BM} \cong \overline{MC}$, ΔDAM, ΔBCM, ΔCDM, and ΔBAM are isosceles triangles. This means $\angle 1 \cong \angle 8$, $\angle 2 \cong \angle 3$, $\angle 4 \cong \angle 5$ and $\angle 6 \cong \angle 7$ by the Isosceles Base Angles Theorem. So, $\angle ABC \cong \angle BCD \cong \angle CDA \cong \angle DAB$ by the angle addition postulate. Since the sum of the angles in any quadrilateral is 360° and the 4 angles are congruent, each angle in the quadrilateral measures 90°, giving us four right angles. Therefore, ABCD is a rectangle, since it is a parallelogram with 4 right angles and 2 pairs of congruent opposite sides.

5. Draw a segment and mark the midpoint. Now construct a segment that is perpendicular to the first segment at the midpoint but is not bisected by the original segment. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints?

Comment:
Make sure only one segment is bisected in the students drawings. Encourage students to exaggerate the part of the drawing that is supposed to be different. If they draw two segments that appear to be congruent it's easy to draw incorrect conclusions. Students may be unfamiliar with the mathematical definition of a kite. It may be necessary to have them look up information about this figure.



Solution:
The quadrilateral is a kite.

Sample proof:
Given: $\overline{AC}$ bisects $\overline{BD}$, $\overline{BD}$ bisects $\overline{AC}$, and $\overline{AC} \cong \overline{BD}$
Prove: ABDC is a parallelogram and a rectangle.

Statement	***Reason***
1. $\overline{AC} \perp \overline{BD}$ $\overline{BD}$ bisects $\overline{AC}$	1. given
2. $\overline{AM} \cong \overline{MC}$	2. Definition of bisect
3. $\angle AMB$, $\angle BMC$, $\angle CMD$ and $\angle DMA$ are right angles	3. Definition of $\perp$
4. $\angle AMB \cong \angle BMC \cong \angle CMD \cong \angle DMA$	4. All right angles are congruent
5. $\overline{BM} \cong \overline{BM}$, $\overline{MD} \cong \overline{MD}$	5. Reflexive property
6. $\Delta AMB \cong \Delta CMB$ and $\Delta AMD \cong \Delta CMD$	6. SAS
7. $\overline{AB} \cong \overline{BC}$ and $\overline{AD} \cong \overline{DC}$	7. CPCTC
8. ABCD is a kite	8. Definition of kite

6. In the above constructions you have been discovering the properties of the diagonals of each member of the quadrilateral family. Stop and look at each construction. Summarize any observations you can make about the special quadrilaterals you constructed. If there are any quadrilaterals that have not been constructed yet, investigate any special properties of their diagonals.

Comment:
Quadrilaterals not investigated include trapezoids and isosceles trapezoids.

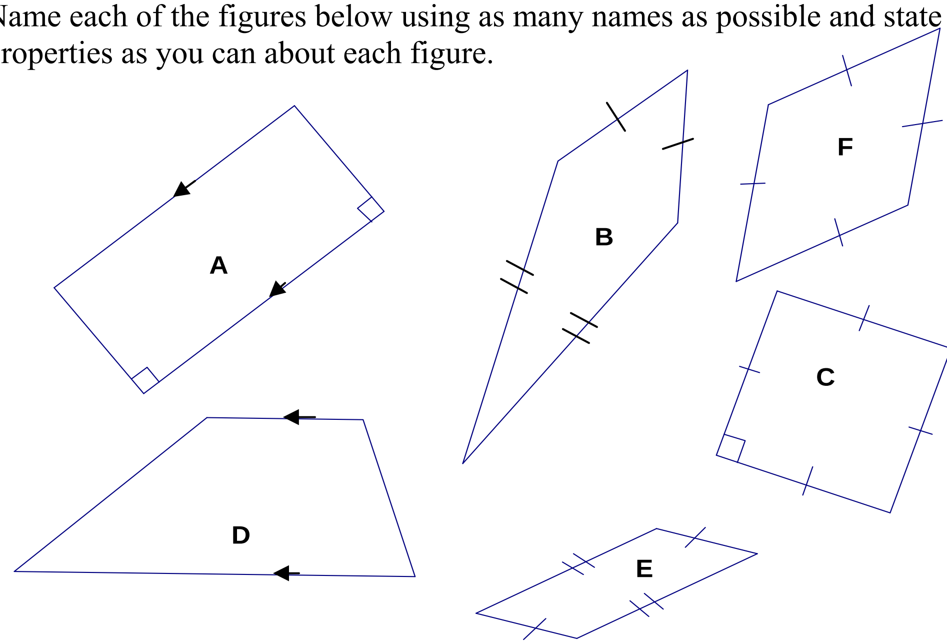
7. Complete the chart below by identifying the quadrilateral(s) for which the given condition is necessary.

Solution:
Look closely for students' thoughts when they explain their reasoning. This should be written in their own words and doesn't have to be a proof. The explanations below are written in a manner similar to the way a student explain their reasoning.
All students should agree and the middle column but explanations may vary.

Conditions	Quadrilateral(s)	Explain your reasoning
Diagonals are perpendicular.	***Rhombus*** ***Square*** ***Kite***	***This is always true of a rhombus, square and kite. But there are some parallelograms and rectangles that do not have perpendicular diagonals.***
Diagonals are perpendicular and only one diagonal is bisected.	***Kite***	***All parallelograms have diagonals that bisect each other so this can't be true for any of them. Both diagonals are bisected in an isosceles trapezoid so this would have to be a kite.***
Diagonals are congruent and intersect but are not perpendicular.	***Rectangle*** ***Isosceles Trapezoid***	***This is true for a rectangle but not a square. The square's diagonals are perpendicular. The isosceles trapezoid is special and this is true of that type of trapezoid.***
Diagonals bisect each other.	***Parallelogram*** ***Rectangle*** ***Rhombus*** ***Square***	***This is true of all parallelograms. This is not true of kites and isosceles trapezoids.***
Diagonals are perpendicular and bisect each other.	***Rhombus***	***This is true of rhombuses and is also true of squares. But unless we know the diagonals are congruent we don't know if it is a square.***
Diagonals are congruent and bisect each other.	***Rectangle***	***This is always true of a rectangle. But unless we know the diagonals are perpendicular we don't know if it is a square.***
Diagonals are congruent, perpendicular and bisect each other	***Square***	***Only a square has all three of these properties. The rectangle's diagonals are not always perpendicular and the diagonals of rhombuses are not always congruent.***

8. As you add more conditions to describe the diagonals, how does it change the types of quadrilaterals possible? Why does this make sense?
   Solution:
   As more restrictions are placed on the quadrilateral, fewer types of quadrilaterals meet all the restrictions.

9. Name each of the figures below using as many names as possible and state as many properties as you can about each figure.



Solution:
Answers may vary but they should include the properties listed below. Students may discover more properties than listed here. As long as they can prove it to always be true they should list as many properties as possible.

Figure	Names	Properties
A	***Parallelogram*** ***Rectangle***	***Opposite sides are congruent.*** ***Diagonals are congruent and bisect each other.***
B	***Kite***	***Diagonals are perpendicular.*** ***Angles between non-congruent sides are congruent to each other.*** ***Diagonal between congruent angles is bisected by the other diagonal.***
C	***Parallelogram*** ***Rectangle*** ***Rhombus*** ***Square***	***Diagonals bisect each other and are perpendicular.***
D	***Trapezoid***	***No specific properties.***

E	***Parallelogram***	***Diagonals bisect each other.***
F	***Parallelogram Rhombus***	***Diagonals bisect each other and are perpendicular.***

10. Identify the properties that are always true for the given quadrilateral by placing an X in the appropriate box.

Solutions:

Property	Parallelogram	Rectangle	Rhombus	Square	Isosceles Trapezoid	Kite
Opposite sides are parallel	***X***	***X***	***X***	***X***		
Only one pair of opposite sides is parallel					***X***	
Opposite sides are congruent	***X***	***X***	***X***	***X***		
Only one pair of opposite sides is congruent					***X***	
Opposite angles are congruent	***X***	***X***	***X***	***X***		
Only one pair of opposite angles is congruent						***X***
Each diagonal forms 2 $\cong$ triangles	***X***	***X***	***X***	***X***	***X***	
Diagonals bisect each other	***X***	***X***	***X***	***X***		
Diagonals are perpendicular			***X***	***X***		***X***
Diagonals are congruent		***X***	***X***	***X***	***X***	
Diagonals bisect vertex angles			***X***	***X***		
All $\angle$s are right $\angle$s		***X***	***X***	***X***		
All sides are congruent			***X***	***X***		

Two pairs of consecutive sides are congruent						***X***

11. Using the properties in the table above, list the **minimum** conditions necessary to prove that a quadrilateral is:

Solution:

A few solutions are listed. Students may think of more. As long as they can prove it to always be true they should list it.

a. a parallelogram
   opposite sides are parallel
   -or
   opposite sides are congruent
   -or
   diagonals bisect each other
b. a rectangle
   diagonals are congruent and bisect each other
   -or
   opposite sides are parallel and congruent
   -or
   Opposite angles are congruent and diagonals are congruent
c. a rhombus
   diagonals are perpendicular and bisect each other
   -or
   opposite sides are congruent and diagonals bisect each other
d. a square
   diagonals are congruent, perpendicular and bisect each other
   -or
   opposite sides are congruent and diagonals are congruent and perpendicular
e. a kite
   diagonals are perpendicular and one diagonal is bisected
   or
   diagonals are perpendicular and two pairs of consecutive sides are congruent
f. an isosceles trapezoid
   only one pair of sides is parallel and diagonals are congruent

Landscaping Culminating Task:

Notes:
This task asks students to use the geometric knowledge they have gained in this unit to create a park. This is a very open-ended task and may be especially appealing to artistic students. Additional constraints can be added to this task as desired.

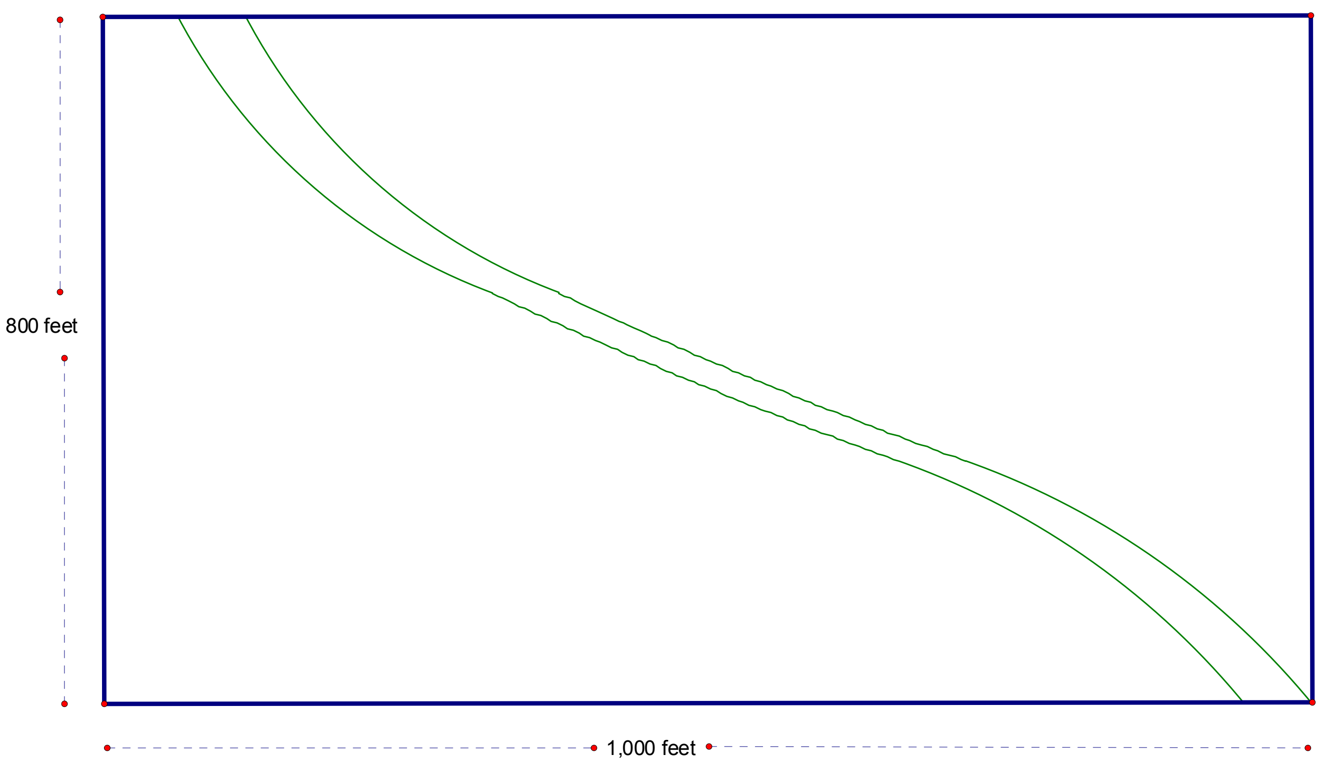
This task could be assessed using a rubric valuing the mathematical content, reasoning, and communication skills demonstrated by students in their solutions. A rubric should be based on the Content Standard MM1G3 and the Process Standards, MM1P1 through MM1P5.

Supplies Needed:
Students may want to draw their blueprint on a larger piece of paper or poster board. You can allow the students to use as much creativity, in the design and presentation of their design, as desired. Some students may be very comfortable using auto-cad programs and might choose to create their design on the computer.

Landscaping Culminating Task:

Your landscape architect company has been commissioned to design a city park. The area you will be using is sketched below. A creek that runs through the park, averaging a width of 10 feet, begins 20 feet from top left corner of the park and continues as indicated below.

Note: this is not drawn to scale.



The city has given you the following requirements that must be met. The park must include, but is not limited to:

- 3 special quadrilaterals
- 1 convex and 1 concave polygon
- 2 different pairs of congruent triangles
- an inscribed or circumscribed circle
- a central meeting point

Think about the things that make a park enjoyable: trees, flower beds, walkways, bridges, gazebos, etc.

Draw a set of blueprints outlining your plan for the park. Make sure you indicate your scale and all special features of your design. Write a brief description of your design which includes:

- the names of the 3 special quadrilaterals
- identification of the convex and concave polygons
- proof of the congruency of the triangles
- how you constructed the inscribed and circumscribed circles
- how you chose the central meeting point.

Solution:
Designs will vary. Make sure students address all the mathematical content of the task. Students need to draw the blueprint to scale and include the scale on the blueprint. Remind students to include lengths and angle measures on their blueprint.

Acquisition Lesson
Concept: Polygons
Mathematics 1 Unit 3
Time allotted for this Lesson: 1 hours

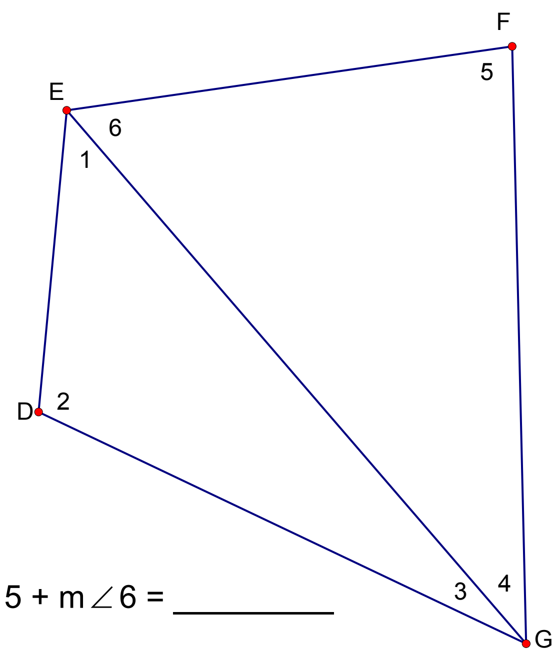
Essential Question: How can I find the sum of the interior angles of a polygon?
Activating Strategies: (Learners Mentally Active) **A set of cards will be made for each of the following polygons: quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon, decagon and dodecagon. Each set will include 4 cards: the name of the polygon, a sketch of the polygon, the number of sides for the polygon and the number of interior angles for the polygon. As the students enter the class they are handed a card. To begin class, the students are instructed to find the other students who have the cards that correspond to their polygon.** **Acceleration/Previewing: (Key Vocabulary) : convex polygon, convex polygon**
Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers) **Graphic Organizer: What is a Polygon? As an Anticipation Guide for the unit, have students complete the first 4 rows of the graphic organizer with guidance from the teacher.** **Task: Working in the cooperative group formed in the Activating Strategy, students complete the Interior Angle Investigation, problems 1-7. Discuss the answers as a class.** **Direct Instruction: teacher leads students through the solutions to problems 8 and 9.** **Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)** **See Problems for Practice at the end of the Interior Angle Investigation.**
Summarizing Strategies: Learners Summarize & Answer Essential Question **Fill in row 5 of the Graphic Organizer (What is the sum of the interior angles of a polygon?)**

Polygons Graphic Organizer

What is a polygon?	
Examples	Non-examples
What is a regular polygon?	
What is a concave polygon?	What is a convex polygon?
What is the sum of the interior angles of a polygon? Interior Angle Sum Theorem	
What is the measure of an interior angle of a regular polygon?	
What is the sum of the exterior angles of a convex polygon? Exterior Angle Sum Theorem	
What is the measure of an exterior angle of a regular polygon?	

Interior Angle Investigation:

In this investigation you are going to discover an easier way to find the sum of the interior angles of a polygon, by dividing a polygon into triangles.



1. Consider the quadrilateral to the right. Diagonal $\overline{EG}$ is drawn. A diagonal is a segment connecting a vertex with a nonadjacent vertex.

The quadrilateral is now divided into two triangles, Triangle DEG and Triangle FEG.

Angles 1, 2, and 3 represent the interior angles of Triangle DEG and Angles 4, 5, and 6 represent the interior angles of Triangle FEG.

$m\angle 1 + m\angle 2 + m\angle 3 =$ __________ $m\angle 4 + m\angle 5 + m\angle 6 =$ __________

$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 =$ __________

2. What is the relationship between the sum of the angles in the quadrilateral and the sum of the angles in the two triangles?

This procedure can be used to find the sum of the interior angles of any polygon.

1. Sketch the polygon.
2. Select one vertex.
3. Draw all possible diagonals from that vertex.
4. Determine the number of triangles formed.

4. Draw a sketch of each polygon and use this same procedure to determine the sum of the angles for each polygon in the table.

Polygon	**Sketch**	**Number of sides**	**Number of triangles formed**	**Number of degrees in a triangle**	**Interior angle sum**
Triangle					
Quadrilateral					
Pentagon					
Hexagon					
Heptagon					
Octagon					
Decagon					
Dodecagon					
n-gon					

5. What patterns do you notice in the table? When you know the number of triangles formed, how do you find the sum of the interior angles of the triangle?

6. In the last row of the table you should have developed a formula for finding the sum of the interior angles of a polygon. Use this formula to find the sum of the interior angles of a 20-gon.

7. Write a sentence explaining how to find the sum of the interior angles of a polygon.

8. The measures of the angles in a convex quadrilateral are 2x, 2(2+1), x-5, and 3(x-2).

a. Sketch and label the figure.

b. What is the sum of the interior angles of a convex quadrilateral?

c. Find x

d. Find each angle measure.

9. The measures of the angles in a convex pentagon are 120^{o}, 2y-5, 3y-25, 2y, and 2y.

a. Sketch and label the figure.

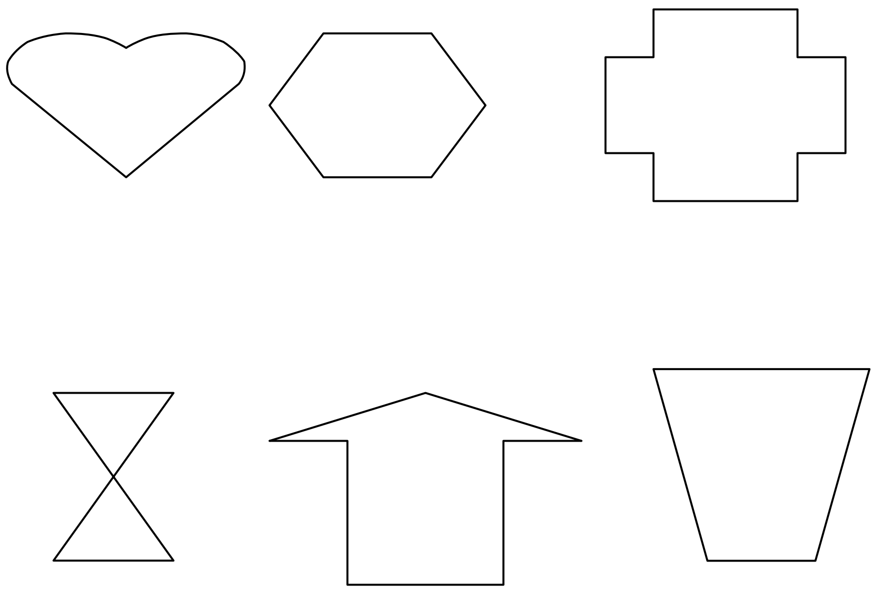
b. What is the sum of the interior angles of a convex quadrilateral?

c. Find y

d. Find each angle measure.

Problems for Practice:

1. Classify the following as either a polygon or not a polygon. If it is a polygon, further classify is as convex or concave.



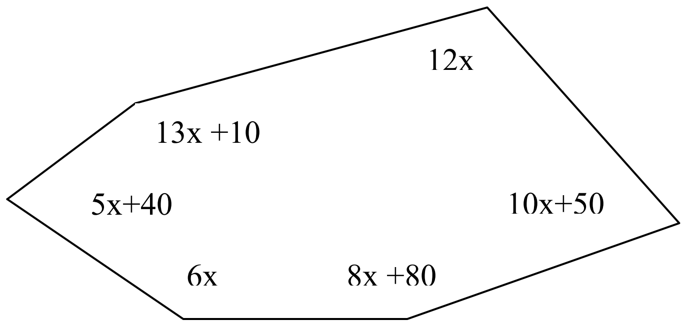
2. Find the sum of the interior angles of the following convex polygons:

24-gon:

13-gon:

3. The four interior angles of a quadrilateral measure x-5, 3(x+8), 3x+6, and 5x-1. Find the measures of the four angles.

4. Find each angle measure in the figure below.


12x
13x +10
5x+40
10x+50
6x
8x +80

Acquisition Lesson
Concept: Polygons
Mathematics 1 Unit 3
Time allotted for this Lesson: 1 hour

Essential Question: How can I find the measure of one interior angle of a regular polygon?

Activating Strategies: (Learners Mentally Active): Use Think-Pair-Share to answer question 1 from the Interior Angle Investigation Part 2.

Acceleration/Previewing: (Key Vocabulary): regular polygon, non-regular polygon will need to be reviewed

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)
Collaborative Pairs: Complete the Interior Angle Investigation Part 2 which includes a graphic organizer (table), and share findings in whole class discussion.

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

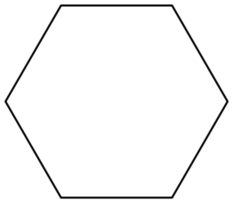
1. **What is another name for a regular triangle?**
2. **What is the interior angle sum of a 60-gon?**
3. **What is the measure of one interior angle of a regular 60-gon?**
4. **Three angles of quadrilateral measure 98^o, 75^o, 108^o. Find the measure of the fourth angle.**
5. **Can a triangle be equiangular, but not equilateral? Draw a picture to justify your answer.**

Summarizing Strategies: Learners Summarize & Answer Essential Question
Writing Prompt: Write a statement explaining how to find each of the following:

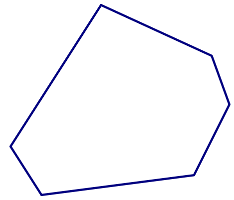
1. **The sum of the interior angles of a polygon.**
2. **The measure of one interior angle of a regular polygon.**
3. **As the number of sides of a regular polygon increases, what**

happens to the sum of the interior angles? What happens to the measure of each interior angle?

Interior Angel Investigation Part 2



Regular Polygon



Nonregular Polygon

1. Compare the two polygons shown above. How would you define a regular polygon and a nonregular polygon?

2. What is the sum of the interior angles of a hexagon?

3. What is the measure of one angle of a regular hexagon?

4. If you know the sum of the angles of a regular polygon, how can you find the measure of one of the congruent angles?

5. Complete the table below:

Regular Polygon	**Interior angle sum**	**Measure of one angle**
Triangle	180^o	
Quadrilateral	360^o	
Pentagon		
Hexagon		
Heptagon		
Octagon		
Decagon		
Dodecagon		
n-gon		

6. Find the measure of one angle of a regular 18-gon.
7. Find the measure of one angle of a regular 14-gon.

Acquisition Lesson
Concept: Polygons
Mathematics 1 Unit 3
Time allotted for this Lesson: 1 hour

Essential Question: How do I find the sum of the exterior angles of a polygon? How do I find the measure of one exterior angle of a regular polygon?

Activating Strategies: (Learners Mentally Active) In Collaborative pairs, students complete the Exterior Angles of Polygons Launch activity.

Acceleration/Previewing: (Key Vocabulary): exterior angle

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)
In Collaborative pairs: Pairs will receive 2 copies of one of the polygon shapes. Each student will label the exterior angles on their sheet, measure them with a protractor, and find the sum. On one of the sheets, students will cut the exterior angles out and arrange and tape them on a blank sheet of paper to discover the sum of the exterior angles. The papers will be displayed on the wall. Students will discuss their findings with the whole class.

Teacher Demonstration: Teacher will demonstrate another approach to finding the sum of the exterior angles of a polygon (see Teacher Demonstration sheet).

Graphic Organizer – Angles of Polygons – Teacher can lead students through the first 2 rows and then have them finish.

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Finish the Graphic Organizer
In Class Practice:

1. An exterior angle of a regular polygon measures 72^{o}. What is the measure of its corresponding interior angle?

2. What is the measure of one interior angle of a regular 40-gon?

3. What is the measure of one exterior angle of a regular 40-gon?

4. If the exterior angles of a quadrilateral measure 93^{o}, 78^{o}, 104^{o}, and x^{o}, find the value of x.

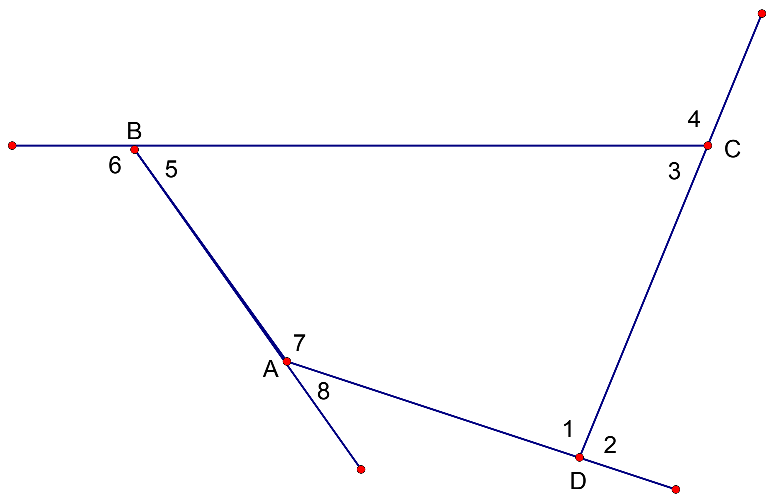
5. If an exterior angle of a regular polygon measures 45^{o}, how many sides does the polygon have?

6. If an interior angle of a regular polygon measures 160^{o}, how many sides does the polygon have?

Summarizing Strategies: Learners Summarize & Answer Essential Question
Have students finish filling in the Polygon Graphic Organizer (from first lesson).

Exterior Angles of Polygons Launch

An <u>exterior angle</u> of a polygon is formed by extending a side of the polygon outside the figure. An exterior angle forms a linear pair with an interior angle. In the drawing below $\angle 2$ is an exterior angle of polygon ABCD.



1. How many exterior angles does quadrilateral ABCD have? Name them.

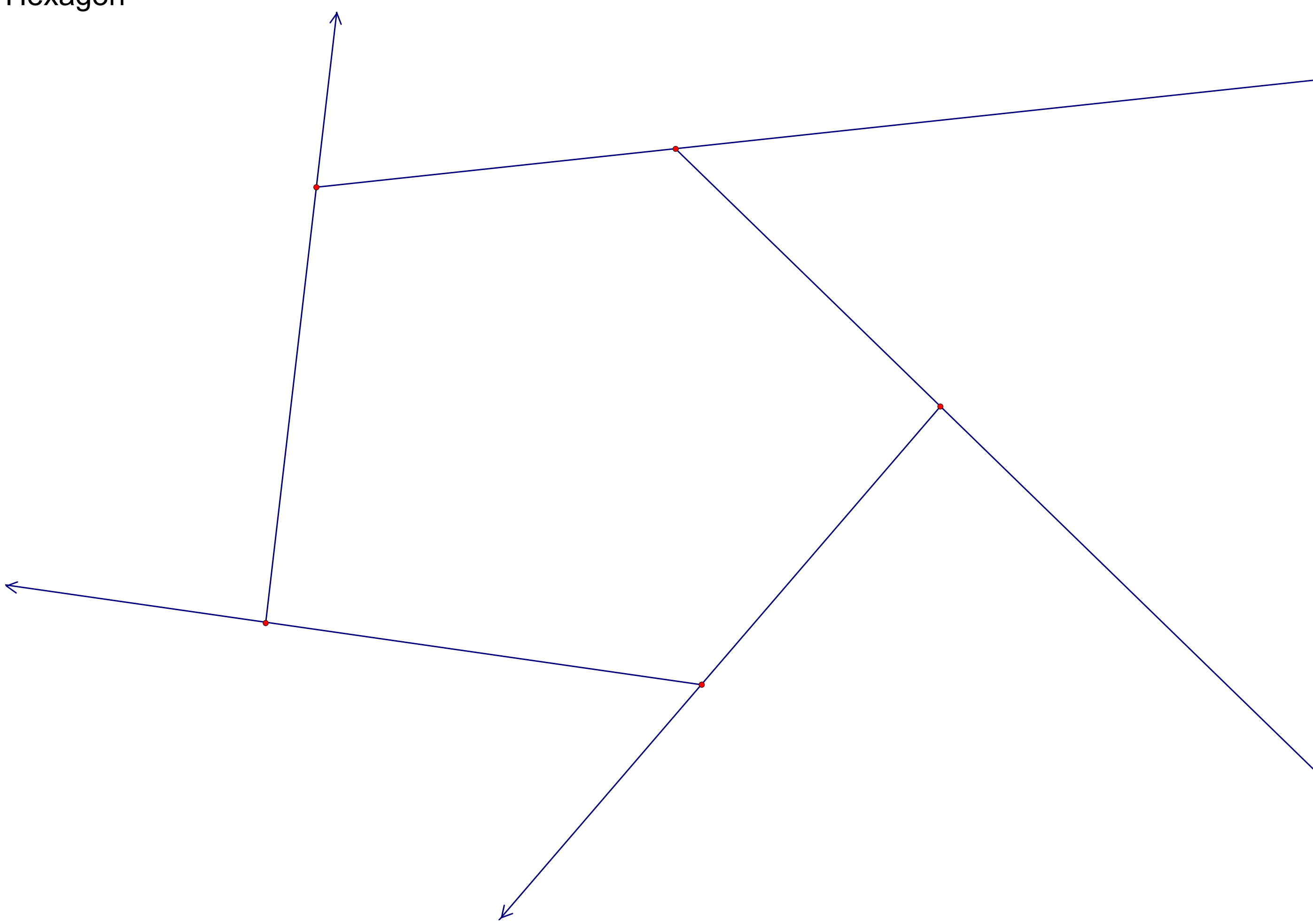
2. Find the measure of $\angle 1$.

3. What kind of angle do interior $\angle 1$ and exterior $\angle 2$ form?

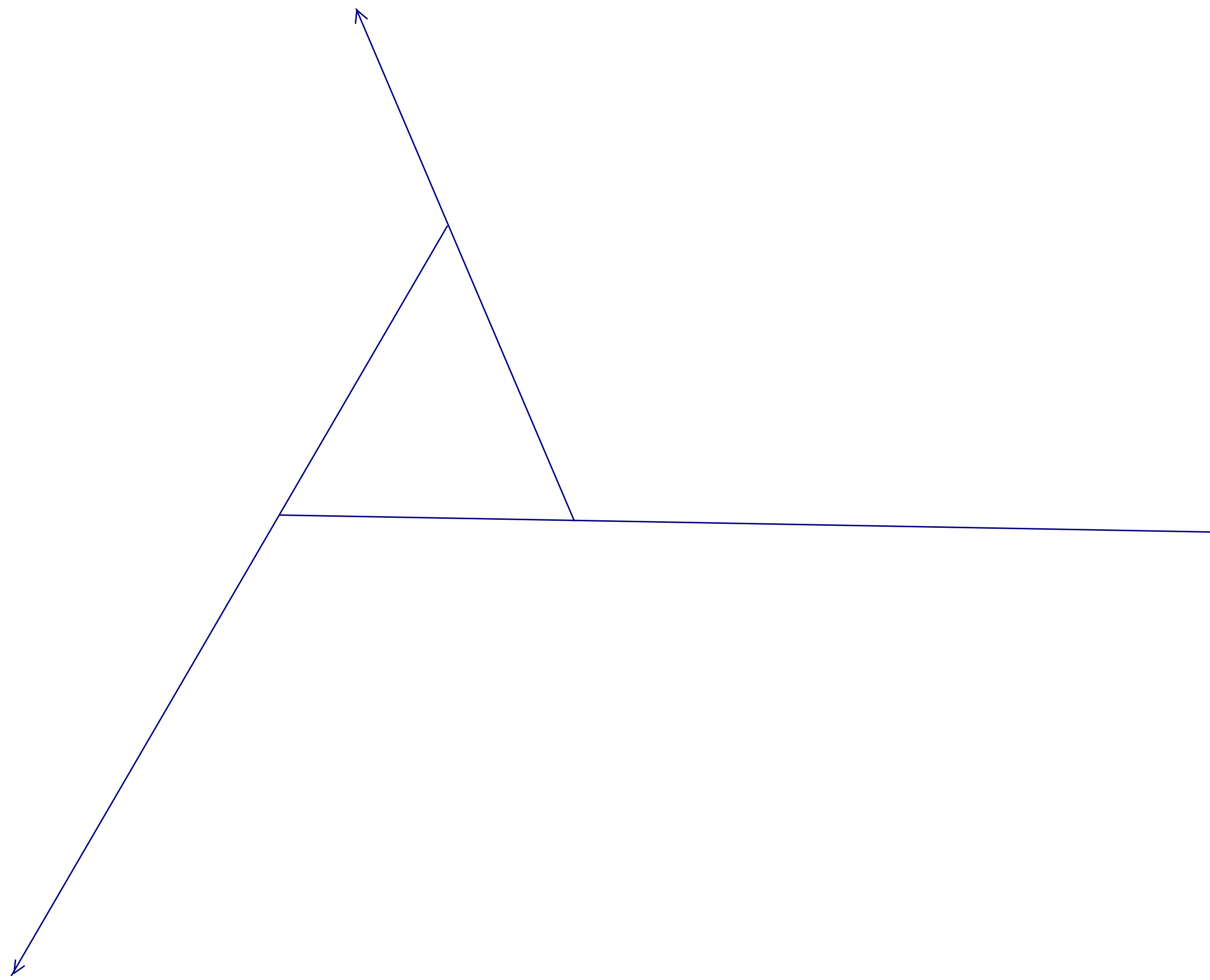
4. Find the measure of $\angle 2$.

5. Find the measure of each of the remaining exterior angles.

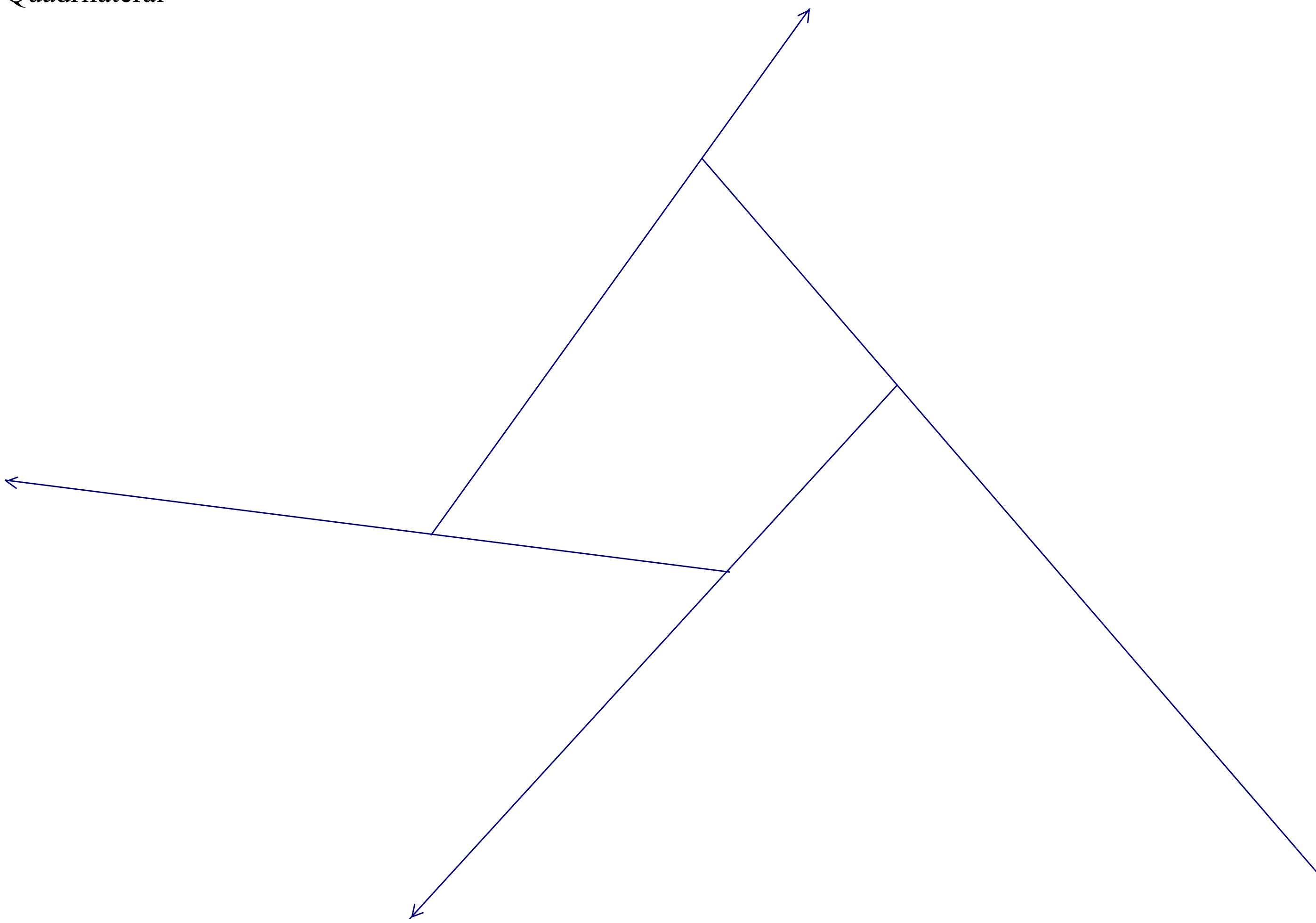
Hexagon



Triangle

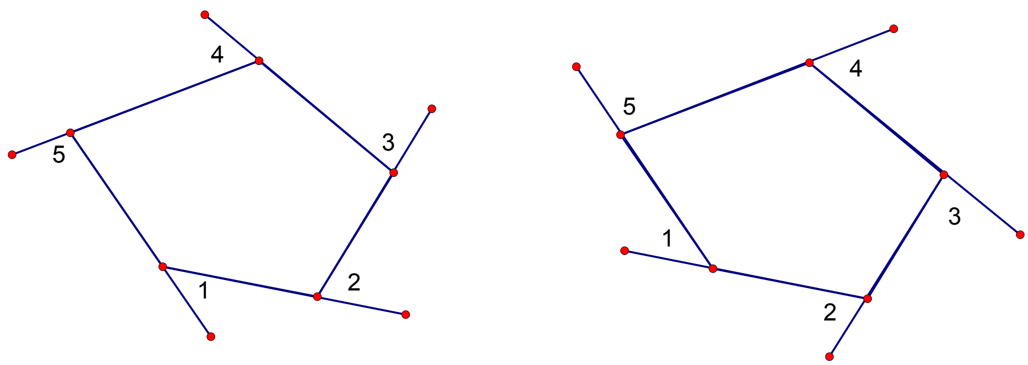


Quadrilateral



Teacher Demonstration – Exterior Angles

Part 1—Sum of exterior angles of any polygon



In the Launch you found that an <u>exterior angle</u> of a polygon is formed by extending a side of the polygon outside the figure. Every polygon has two sets of exterior angles depending on whether you choose to draw the exterior angles progressing in a clockwise or counterclockwise direction. Notice in the figures above that exterior $\angle 1$ is the same measure in each polygon. Exterior $\angle 2$ is the same measure in each polygon, and so on. Also you observed that each interior angle forms a straight angle with an exterior angle.

The sum of the exterior angles of a polygon can be found by visualizing what would happen if you walked around the figure. Imagine starting out at one vertex on a walk around one of the pentagons above. At each vertex you will turn and continue walking until you return to the starting point facing in your original direction. When completed you should have turned around one time.

You can simulate this with your pencil. Place your pencil on a side of the polygon with the eraser on one vertex and the pencil tip pointing along a side. Slide the pencil until the eraser reaches the next vertex, then turn so that it is pointing along the next side. Notice that the angle through which the pencil turns is an exterior angle of the polygon. Continue until the eraser is back too its starting point.

1. Through how many degrees does the pencil rotate from start to finish?

2. Draw a hexagon and sketch in one set of exterior angles. Use your pencil to take you on a journey around the figure.

3. Would the sum of the angles be different if you walked around an octagon? A decagon? A 50-gon?

4. Would the sum of the angles be different if the lengths of the sides of the polygon changed?

5. Write a statement about how to find the sum of the exterior angles of a polygon.

Name of Polygon	# of Sides	Sum of Interior Angles	Measure of an Interior Angle in a Regular Convex Polygon	Sum of exterior angles	Measure of an Exterior Angle in a Regular Convex Polygon
	3				
	4				
	5				
	6				
	7				
	8				
	9				
	10				

	n				

Conclusions:

1. What do you notice about the measures of the interior and exterior angles of a regular convex polygon?

2. Given the measure of an exterior angle of a regular polygon, how would you determine the number of sides in the polygon?

3. Given the measure of an interior angle of a regular polygon, how would you determine the number of sides in the polygon?

Acquisition Lesson Planning Form
Concept: Polygons
Mathematics 1 Unit 3
Time allotted for this Lesson: 1 hour

Essential Question:
What is the relationship between an exterior angle of a triangle and its 2 remote interior angles?

Activating Strategies: (Learners Mentally Active)
Wordsplash: Write down all the things that come to mind when you hear the word "remote".

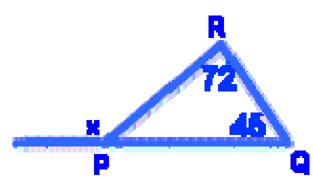
Acceleration/Previewing: (Key Vocabulary): remote interior angles of a triangle

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)
The Exterior Angle Theorem Activity - Distribute the Activity Sheet. Each student will receive either the acute or obtuse triangle. Students will label the exterior angle 4, the two remote interior angles as 1 and 2. Students will tear off angles 1 and 2, and try to arrange them to fit on angle 4. Direct students to find someone else with the same triangle to compare results. Student volunteers can present their findings to the class. Write an equation involving those three angles.

Students will be given the following problems without the answers. Groups of three will work on finding the solutions to these problems. Groups will present their findings to the class.

In $\triangle PQR$, $m\angle Q = 45^\circ$, and $m\angle R = 72^\circ$. Find the measure of an exterior angle at P.

It is always helpful to draw a diagram and label it with the given information.



Then, using the theorem above, set the exterior angle (x) equal to the sum of the two non-adjacent interior angles (45° and 72°.)

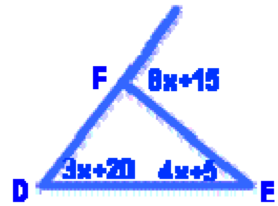
$x = 45 + 72$
$x = 117$

So, an exterior angle

at P measures 117°.

In △DEF, an exterior angle at F is represented by 8x + 15. If the two non-adjacent interior angles are represented by 4x + 5, and 3x + 20, find the value of x.

First, draw and label a diagram.



Next, use the theorem to set up an equation.

Then solve the equation for x.

$8x+15=(4x+5)+(3x+20)$
$8x+15 = 7x + 25$
$8x = 7x + 10$
$x = 10$

So, $x = 10$

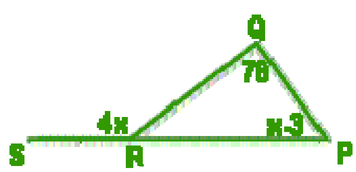
Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

1. Find x

2. Find the measure of angle SRQ

3. Find the measure of angle QPR

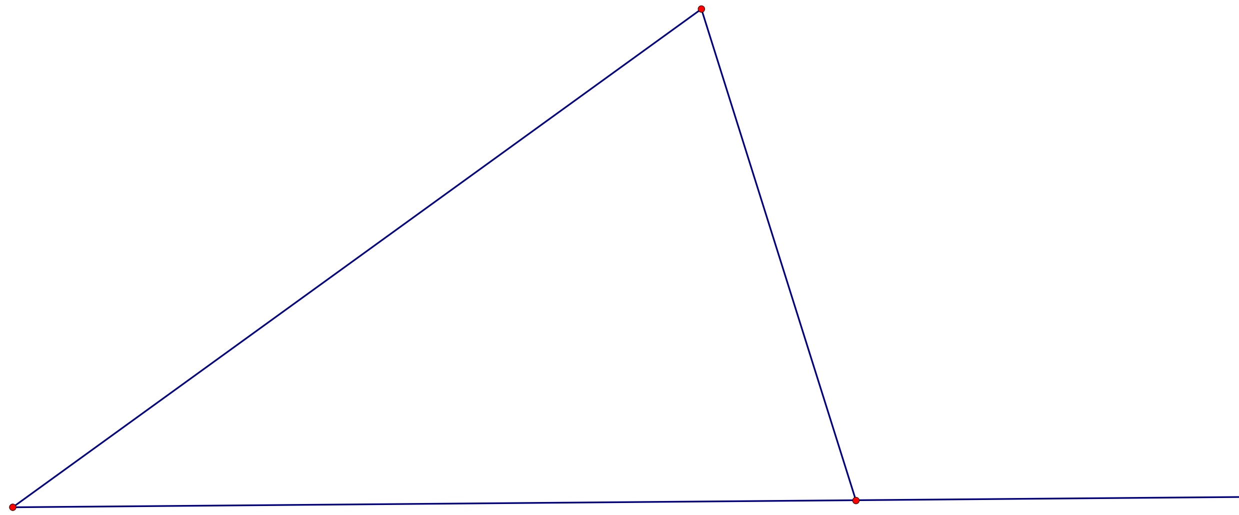
4. Find the measure of angle QRP

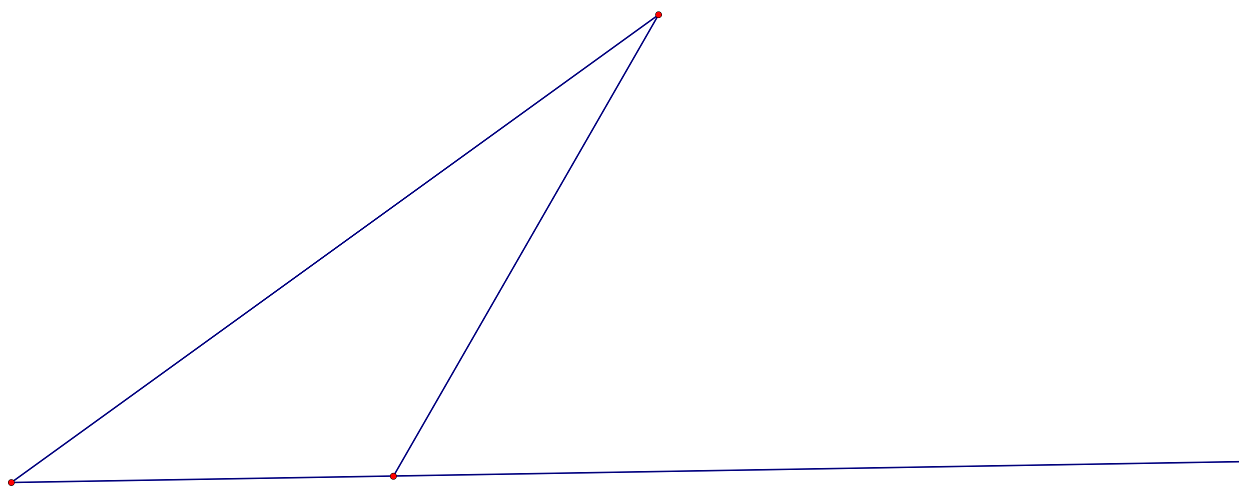


Summarizing Strategies: Learners Summarize & Answer Essential Question

Write the exterior angle theorem in your own words as a ticket out the door.

Exterior Angle Theorem Activity Sheet





Mathematics 1
Unit 3
Acquisition Lesson—Triangular Inequalities
Session 1

Essential Question:

What does the sum of two sides of a triangle tell me about the third side?

Activating Strategies: (Learners Mentally Active)
Activation strategy to be used to begin the concept of triangle inequalities.

Math 1 and Math 1 Support:

- KWL: Students list what they know about triangles. Using collaborative pairs, have the pairs list at least one thing they want to know about triangles. (This KWL is for the entire concept of triangular inequalities.)

OR

- Anticipation Guide: Students will answer the anticipation guide based on their prior knowledge of triangles. (This anticipation guide is for the entire concept of triangular inequalities.)

Acceleration/Previewing: (Key Vocabulary)

Math 1 Support: Have students complete measuring triangle sides worksheet. Also, have students complete the inequality for sides of a triangle worksheet.

Vocabulary: Triangle Inequality

Maintain Vocabulary: Scalene, Isosceles, Equilateral

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

Task:

- Have one student perform a demonstration. Give him/her three segments (use straws or linguini). Have that student use those segments to construct a triangle.

- Next, have another student try to make a triangle with different segments (this time, the given segments should not form a triangle).

- Pose the question: Why does this not work?

- In small groups, students will complete The Triangle Inequality Learning Task (adapted from Illuminations).

- Teacher will have large chart paper on the board with a diagram similar to the student task sheet.

- Each group will post one of his/her results from step 1 and step 2. After each group is posted, the class will discuss what works and what did not work and why.

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Students will be given a Frayer Model (attached) and will summarize their conclusions on the Triangle Inequality Theorem. This graphic organizer will be used throughout the entire concept of triangular inequalities.

Summarizing Strategies: Learners Summarize & Answer Essential Question

Ticket-out-the-door: Each student is given an index card with three segment measures. Have the students write their names on their card. Some of the segments will make a triangle and some will not. As students leave the classroom, they will post their card on the board under the columns “Does Make a Triangle” or “Does NOT Make a Triangle”.

This is a form of formative assessment.

Needed Materials:

- KWL or Anticipation Guide
- Linguini or Straws
- The Triangle Inequality Task Sheet
- Chart Paper
- Chart Markers
- Rulers
- Frayer Diagram
- Index Cards with segment measurements
- Columns on Board: “Does Make a Triangle” or “Does NOT

Name: ____ Date: __________________

Math 1 – Unit 3 – Session 1
Triangle Inequality Learning Task

During this activity, you will compare the sum of the measures of any two sides of a triangle with the measure of the third side.

1. Break a piece of linguini into three pieces or cut straws into three pieces, and use the pieces to form a triangle. Measure each side length to the nearest tenth of a centimeter. In the table below, record the measures of each side of the triangle from smallest to largest; then, find the sum of the measures of the small and medium sides. Repeat this activity twice, with two other triangles, to complete the chart.

Small	Medium	Large	Small + Medium

2. Break a piece of linguini or cut straws into three pieces so that it is impossible to form a triangle. Measure each side of the non-triangle to the nearest tenth of a centimeter. In the table below, record the measures of each side of the non-triangle from smallest to largest; then, find the sum of the measures of the small and medium sides. Repeat this activity twice, with two other non-triangles, to complete the chart.

Small	Medium	Large	Small + Medium

3. Compare the sum of the measures of the small and medium sides to the measure of the large side for each triangle you created. Describe what you notice.

4. Compare the sum of the measures of the small and medium sides to the measure of the large side for each non-triangle you created. Describe what you notice.

5. Based on your observations, write a conjecture about the relationship between the sum of the measures of the small and medium sides of a triangle and the measure of the large side of the triangle. Provide a reason for your conjecture.

6. Using a partner's measurements, test your conjecture. If your conjecture holds for your partner's measurements, provide a convincing reason why your conjecture would hold for any triangle. If your conjecture does not hold for your partner's measurements, revise your conjecture.

7. Is it possible to have a triangle such that the sum of the measures of the small and medium sides is equal to the measure of the large side? Provide a convincing reason for your answer. (You may use linguini, if you like.)

8. If the sum of the measure of the small and medium sides of the triangle is greater than the measure of the large side of the triangle, we can conclude that the sum of the measures of any other pair of sides of the triangle will be greater than the measure of the remaining side. Explain why this conclusion is possible.

9. In the box below, write three inequalities that are always true for a triangle with side lengths s, m, and l. (These inequalities should be based on your conclusion from Question 8.)

Triangle Inequality Theorem

In a triangle with side lengths s, m, and l

______ +______ > ______

______ +______ > ______

______ +______ > ______

10. Consider the situation where you know that two sides of a triangle are 4 and 7. Would each of the following values be possibilities for the third side of the triangle?

Possible length of 3rd side	**Yes or No?**
3	
3.5	
4	
5	
6	
7	
8	
9	
10	
10.5	
11	

11. Using the information from number 10, what are all the possible values for the third side? Write your answer as an inequality.

Triangular

Triangle Inequality

Definition

Characteristics

It Works!

It Does NOT Work!

Side-Angle Inequality

Definition

Characteristics

It Works!

It Does NOT Work!

Side-Side-Side Inequality

Definition

Characteristics

It Works!

It Does NOT Work!

Exterior Angle Inequality

Definition

Characteristics

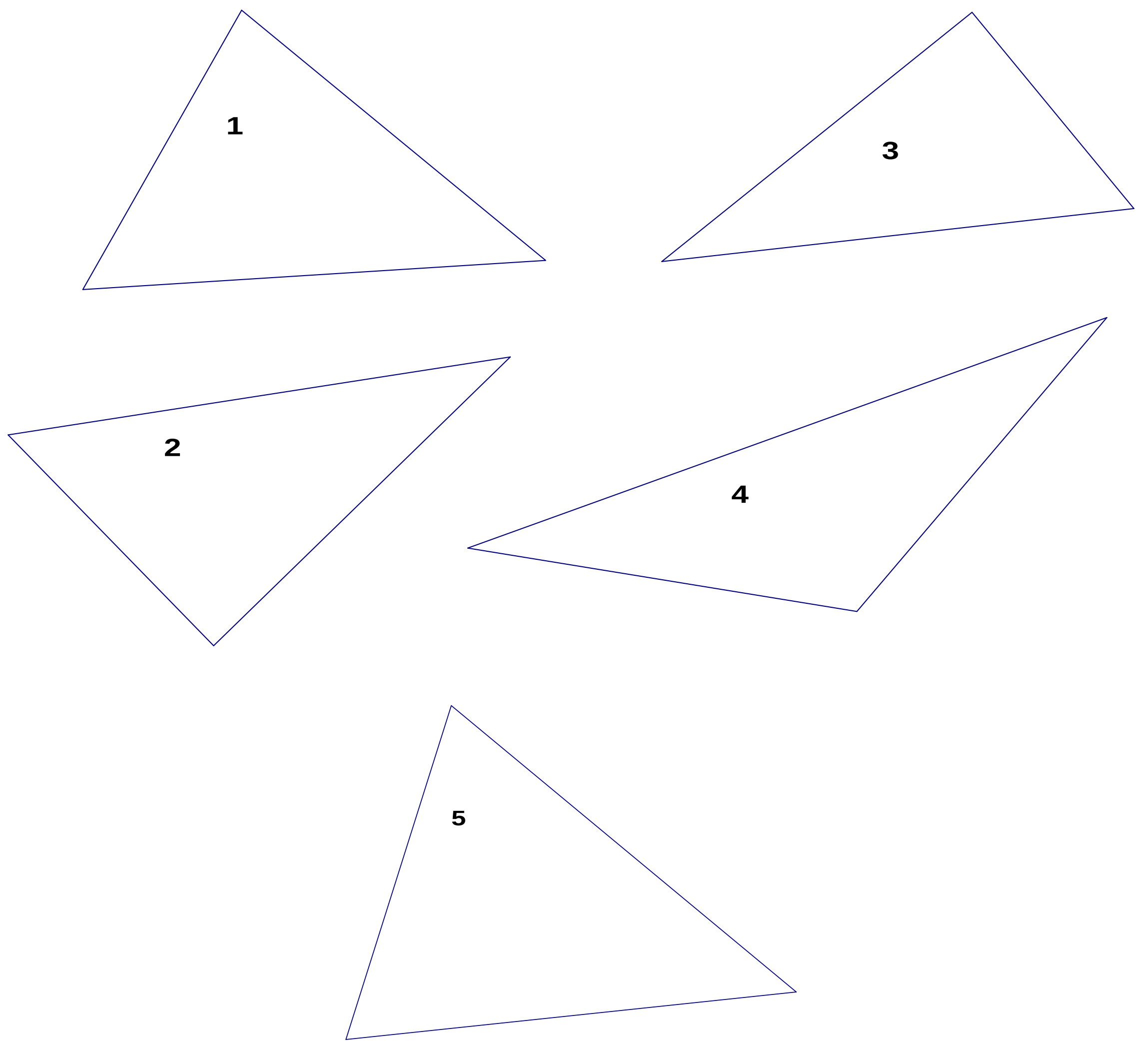
It Works!

It Does NOT Work!

Name: ______________________________________ Date: _______________________

Math 1 – Unit 3 – Session 1
Math Support – Measuring Triangle Sides

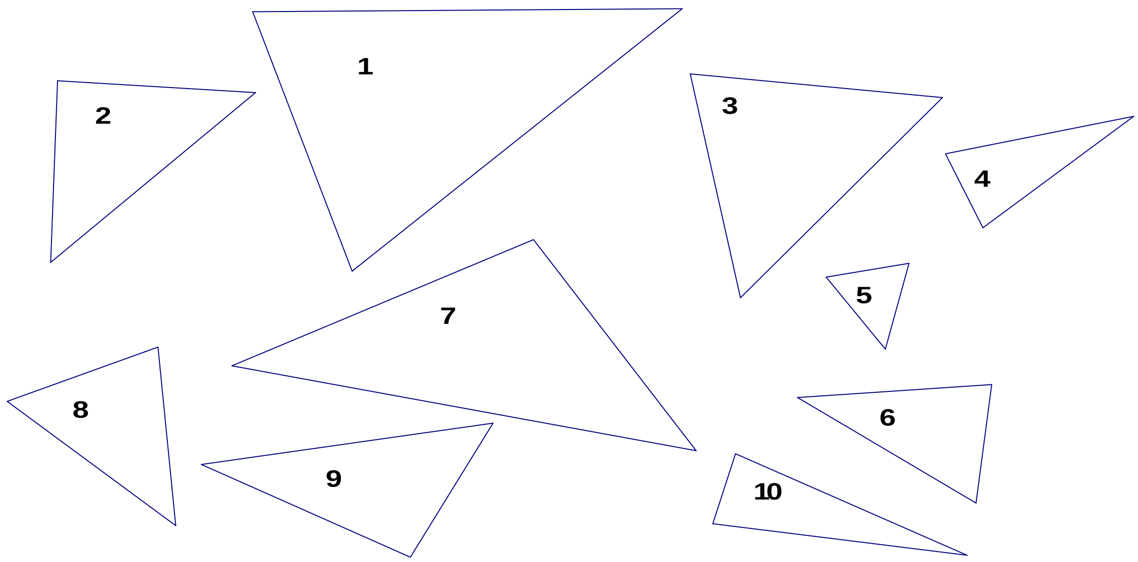
Measure the sides in inches and centimeters of each of the following triangles. Write in those measures on each triangle.



Name: ____ Date: ___________________

Math 1 – Unit 3 – Session 1
Math Support - Inequalities for Sides

For each triangle, measure each of the sides. Then, fill in the chart with the needed information.



Triangle #	Shortest Side	Middle Side	Short + Middle	Inequality < or >	Longest Side
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

Mathematics 1
Unit 3
Acquisition Lesson—Triangular Inequalities
Session 2

Essential Question:

In a triangle, what is the relationship between an angle and its opposite side?

Activating Strategies: (Learners Mentally Active)

Teacher poses the following question (This is a review from Session 1.)

If I know that two sides of my triangle are 11cm and 17cm, what are the possibilities of the third side?

Acceleration/Previewing: (Key Vocabulary)

Math 1 Support: Students will complete classifying triangles worksheet.
Note: Three different angle measures indicate three different side measures.

Vocabulary: Side-Angle Inequality

Maintain Vocabulary: Acute, Obtuse, Scalene, Right, Isosceles, Equilateral, Equiangular, opposite

Preview: Distribute "Are the triangles possible?" half sheet.

- Think-pair-share: Individual students will determine if the given triangles are possible before pairing with another student to compare conclusions. Then the class will discuss the possible triangles and why.

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

Task:

- In small groups, students complete Inequalities for Sides and Angles Learning Task worksheet (adapted from Illuminations.)
- The groups will be given a piece of chart paper to present their answers.
- Each group will present their discoveries to the class and the teacher will lead the class to the correct vocabulary.
- The teacher will use the Triangle Inequalities Frayer from Session 1 to define Side-Angle Inequality.

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

What does the side-angle inequality tell us about a triangle?
What are three real-life applications to the side-angle inequality?

Summarizing Strategies: Learners Summarize & Answer Essential Question

Answer the essential question: In a triangle, what is the relationship of angle of a triangle and its opposite side?

Needed Materials:
- Protractor
- Rulers
- Preview Session 2 Worksheet – Review of Classifying
- Sides and Angles Learning Task
- Chart Paper
- Chart Markers
- Triangle Inequality Frayer Diagram (from Session 1)

Name: ______________________________________ Date: _____________________

Math 1 – Unit 3 – Session 2
Inequalities for Sides and Angles Learning Task

During this activity, you will examine the relationship between the locations of sides and angles in a triangle.

1. **Use a ruler to draw scalene ΔABC, such that segment $\overline{AB}$ is the longest side and segment $\overline{AC}$ is the shortest side.**

2. **Measure each side to the nearest tenth of a centimeter, and measure each angle to the nearest degree. Indicate these measurements on your drawing above.**

3. **Describe the relationship between the location of the longest side and the location of the largest angle of scalene ΔABC.**

4. **Describe the relationship between the location of the shortest side and the location of the smallest angle of scalene ΔABC.**

5. **Using your answers from numbers 3 and 4, what conclusions can you draw about the relationship between a triangle's sides and angles?**

6. **Explain why it would be impossible to draw a triangle where the longest side was not opposite the largest angle.**

Name: ______________________________________ Date: ____________________

Math 1 – Unit 3 – Session 2
Preview - Review of Classifying Triangles

Are each of the following triangle possible – yes or no? Why or why not?

Right Isosceles

Right Scalene

Right Equilateral

Acute Isosceles

Acute Scalene

Acute Equilateral

Obtuse Isosceles

Obtuse Scalene

Obtuse Equilateral

Name: ______________________________________ Date: ____________________

Math 1 – Unit 3 – Session 2
Preview – Review of Classifying Triangles

Are each of the following triangle possible – yes or no? Why or why not?

Right Isosceles

Acute Isosceles

Obtuse Isosceles

Right Scalene

Acute Scalene

Obtuse Scalene

Right Equilateral

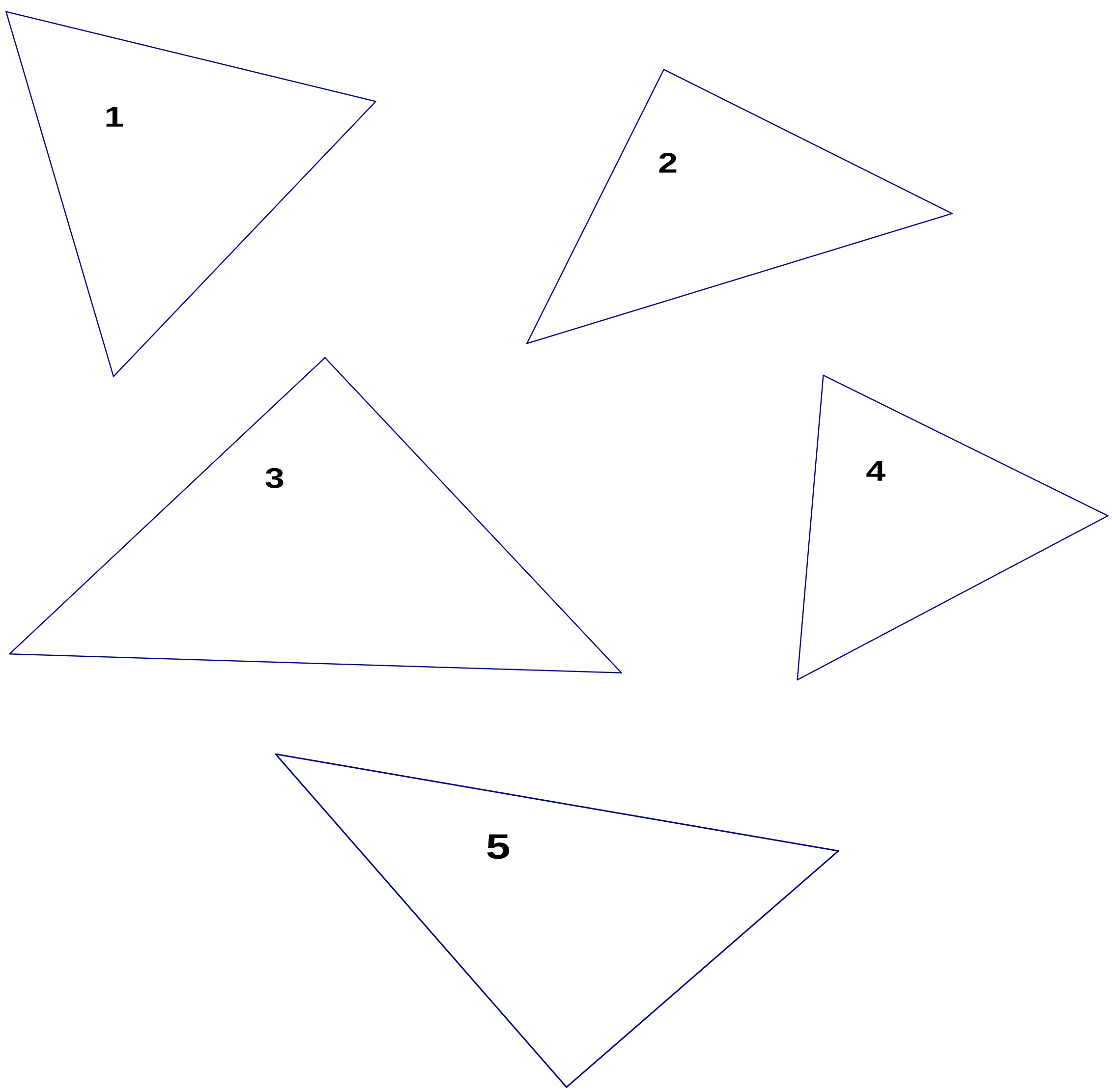
Acute Equilateral

Obtuse Equilateral

Name: ______________________________________ Date: ________________________

Math 1 – Unit 3 – Session 2
Math Support – Classifying Triangles

Measure the sides in and angles of each of the following triangles. Write in those measures on each triangle. Then, determine which of the following can be used to describe each triangle: acute, right, obtuse, scalene, isosceles, equilateral, and equiangular.



Mathematics 1
Unit 3
Acquisition Lesson—Triangular Inequalities
Session 3

Essential Question:

What is the relationship between the exterior angle of a triangle and either of its remote interior angles?

Activating Strategies: (Learners Mentally Active)

Students will be given half sheet on finding angle measures in a triangle and will find the missing measures of an interior and exterior angles (see attached half sheet).

Acceleration/Previewing: (Key Vocabulary)

Math 1 Support: Students will complete exterior and interior angle sheet.

Vocabulary: Exterior Angle Inequality

Maintain Vocabulary: Exterior angle, remote interior angles

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

Task:

- Students will work in collaborative pairs to complete Exterior Angles and Remote Interior Angles learning task sheet.
- With the task sheet enlarged and displayed on the board, the teacher will collect the information from each partner group.
- Teacher will lead discussion on students' conclusions from the task.
- As the students draw their own conclusions, they will fill in the definition for the Exterior Angle Inequality on the Triangle Inequalities Frayer Diagram from session 1.

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to

Initiate Periodic Practice or Summarizing) Students will work independently to find all missing angles on the "Angles of Triangles" sheet (attached).
Summarizing Strategies: Learners Summarize & Answer Essential Question **Ticket-out-the-door:** Answer the essential question. What is the relationship between the exterior angle of a triangle and either of its remote interior angles?

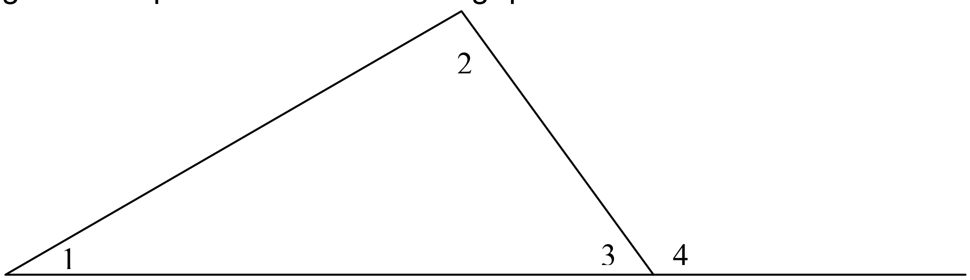
Needed Materials:

- Finding Angle Measures half sheet
- Exterior and Interior Angles Sheet Activator
- Exterior Angles and Remoter Interior Angles Learning Task
- Triangle Inequality Frayer Diagram from Session 1
- Angles of Triangles Sheet
- Chart paper for enlarged task sheet

Name: ______________________________________ Date: ___________

Math 1 – Unit 3 – Session 3
Activator – Exterior and Remote Interior Angles

Use this figure to help visualize the following questions.

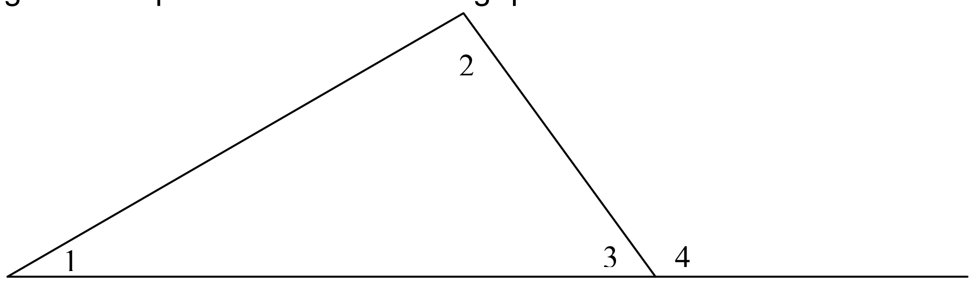


1. Given m<1 = 30 and m<2 = 65, find the m<4. ______________________
2. Given m<1 = 43 and m<2 = 87, find the m<4. ______________________
3. Given m<1 = 56 and m<2 = 64, find the m<4. ______________________
4. Given m<1 = 35 and m<2 = 68, find the m<4. ______________________
5. Given m<1 = 48 and m<2 = 73, find the m<4. ______________________

Name: ____ Date: ____________

Math 1 – Unit 3
Activator – Exterior and Remote Interior Angles

Use this figure to help visualize the following questions.



1. Given m<1 = 30 and m<2 = 65, find the m<4. ______________________
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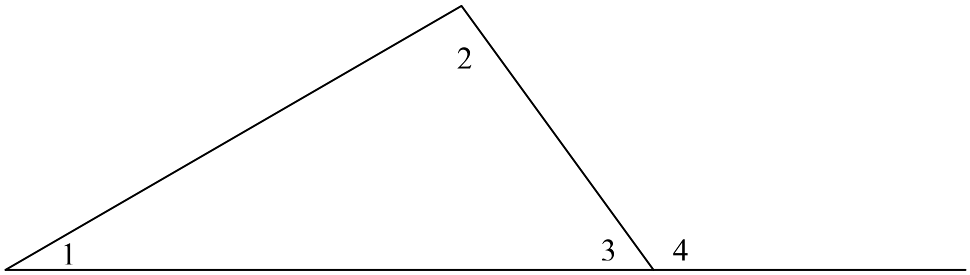
3. Given m<1 = 56 and m<2 = 64, find the m<4. ______________________
4. Given m<1 = 35 and m<2 = 68, find the m<4. ______________________
5. Given m<1 = 48 and m<2 = 73, find the m<4. ______________________

Math 1 – Unit 3
Activator – Exterior and Remote Interior Angles
Answer Key

Use this figure to help visualize the following questions.

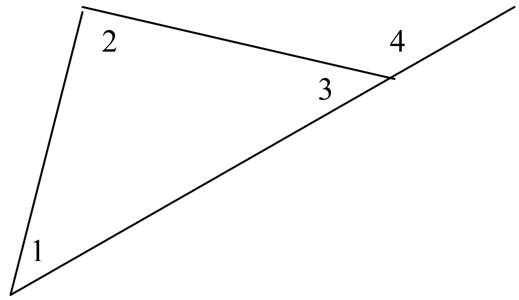


1. Given m<1 = 30 and m<2 = 65, find the m<4. _____95___________________
2. Given m<1 = 43 and m<2 = 87, find the m<4. _____130_________________
3. Given m<1 = 56 and m<2 = 64, find the m<4. ______120_________________
4. Given m<1 = 35 and m<2 = 68, find the m<4. ______103_________________
5. Given m<1 = 48 and m<2 = 73, find the m<4. ______121__________________

Name: ______________________________ Date: ______________

Math 1 - Unit 3 – Session 3
Math Support – Exterior and Interior Angles

Triangle 1



Materials:
Straightedge
Protractor

Step 1: Name the 3 interior angles of the above triangle.

Step 3: From Step 1, which of these angles are remote interior angles with reference to angle 4?

Step 2: Name the exterior angle of the above triangle.

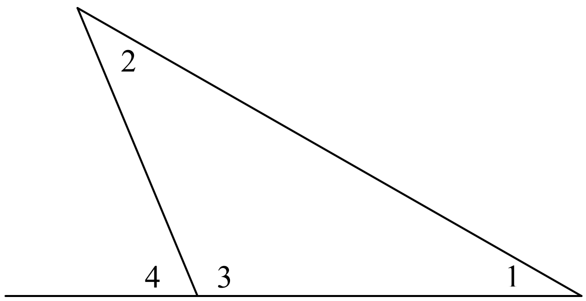
Step 3: Using a protractor, measure the angles of the triangle and the exterior angle.

Step 4: Record the needed information in the table below.

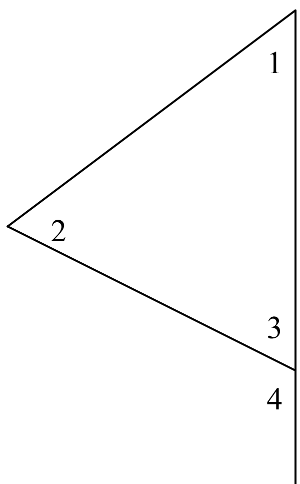
m < 1	m < 2	m < 1 + m < 2	m < 4

Now, follow the same directions as above for the next 3 triangles. Fill in your information on the chart above.

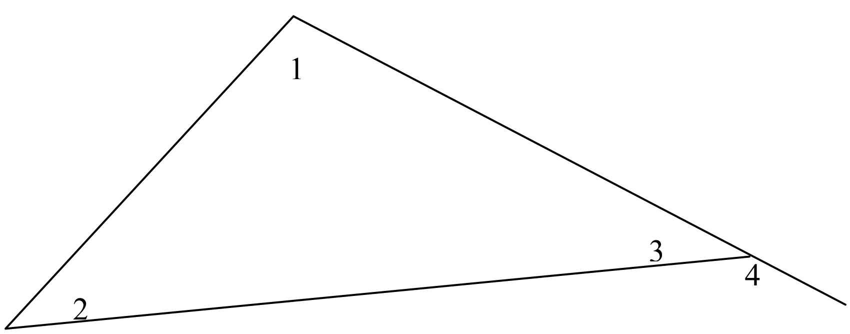
Triangle 2



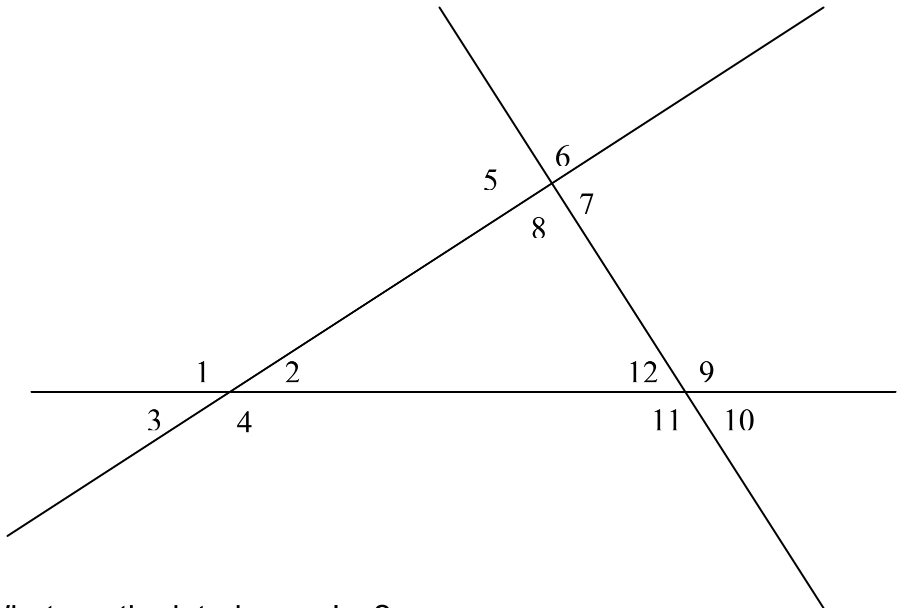
Triangle 3



Triangle 4



Answer the following questions using the following figure.



1. What are the interior angles?

2. What are the exterior angles?

3. With reference to angle 9, what are the two remote interior angles?

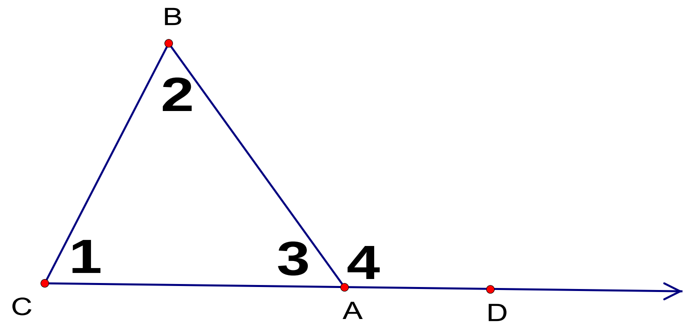
4. Given remote interior angles 8 and 12, what is the exterior angle?

5. With reference to angle 4, what are the two remote interior angles?

6. Given remote interior angles 2 and 12, what is the exterior angle?

Name: ____ **Date:** ___________________

Math 1 – Unit 3 – Session 3
Learning Task – Exterior Angles & Remote Interior Angles



Materials:
- Straightedge
- Protractor

1. Measure the angles of the triangle and the exterior angle.

2. Record the needed information in the table below.

m < 1	m < 2	m < 1 + m < 2	m < 4

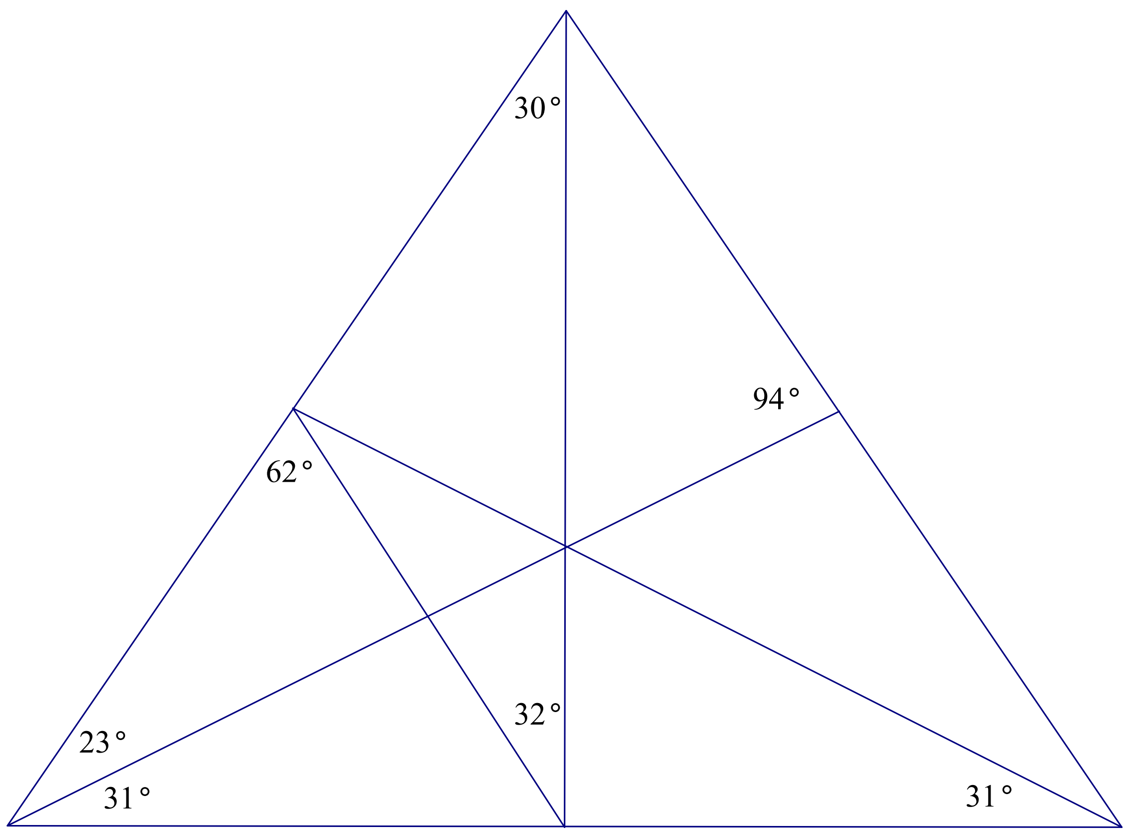
3. Draw (3) other triangles and collect same data. Record the data in the table above.

4. Make a conjecture that describes the pattern you observe when comparing exterior angle and either remote interior angle?

Name: ____ Date: ________________

Math 1 – Unit 3 – Session 3
Angles of Triangles

Knowing some facts about specific triangles can help you to deduce other facts. Study the triangles below. Decide which measures of the angles you can find, then calculate these measures. Record your answer near the vertex. Follow the same procedure until you have found the measure of each angle. (Do not use your protractor.)



Mathematics 1
Unit 3
Acquisition Lesson—Triangular Inequalities
Session 4(1.5 Sessions)

Essential Question:

For two triangles with two pairs of congruent corresponding sides, what is the relationship between the included angles and the third side?

Activating Strategies: (Learners Mentally Active)

Students will do Review of Corresponding Sides and Included Angles worksheet.

Acceleration/Previewing: (Key Vocabulary)

Math 1 Support: Preview activity sheet reviewing included angles and corresponding parts. Teacher should lead a discussion on what happens when you open and close a door.

Vocabulary: Side-side-side Inequality

Maintain Vocabulary: Corresponding parts, included angle, opposite side, opposite angle, inequality, triangle

NOTE: Essential Question commonly known as Hinge Theorem.

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

Task:

- Students will work in collaborative pairs using geometers sketch pad or Cabri geometry following the directions on the Exploring the Hinge Theorem Learning Task sheet.

- After students have completed the activity, students will summarize findings.

- Teacher can further demonstrate using attached Power Point which includes an animated demo.

POSSIBLE EXTENSIONS:

Students can continue exploring other cases of the Hinge Theorem using Geometers Sketchpad.

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Students will write down their conclusions about the Side-side-side inequality in the Triangle Inequality Frayer Diagram (from Session 1).

Summarizing Strategies: Learners Summarize & Answer Essential Question

Ticket-out-the-door: Answer the essential question.

For two triangles with two pairs of congruent corresponding sides, what is the relationship between the included angles and the third side?

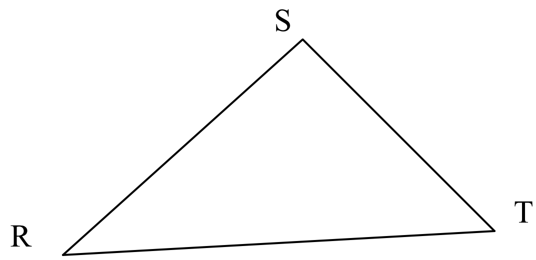
<u>Needed Materials:</u>

- Activator of Corresponding Sides
- Preview Activity Sheet for Math Support
- Exploring Hinge Theorem Lab Sheet
- Power Point
- Frayer Diagram
- Geometers Sketch Pad or Cabri

Name: ___________________________________ Date: ______________

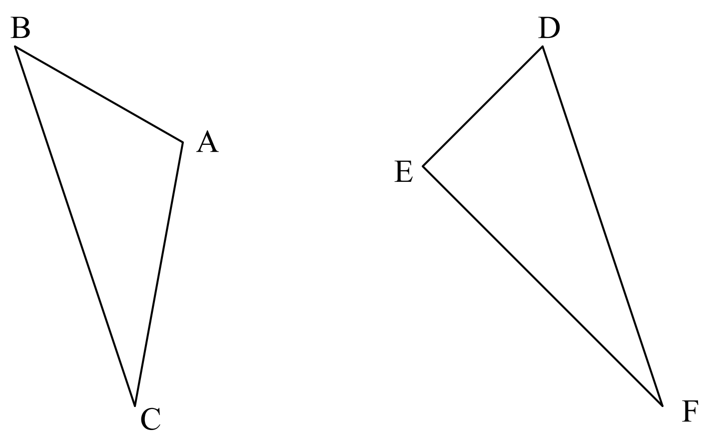
Math 1 – Unit 3 – Session 4
Activator – Review of Corresponding Sides and Included Angles

Part 1 – Review of Included Angles
Answer the following questions, using this figure.



1. What is the included angle of $\overline{RS}$ and $\overline{TR}$?

Part 2 – Review of Corresponding Parts
Answer the following questions using these figures.
Triangle ABC $\cong$ Triangle DEF



2. Which angle corresponds to angle A?

3. Which segment corresponds to $\overline{DE}$?

Given Triangle DEF $\cong$ Triangle RST

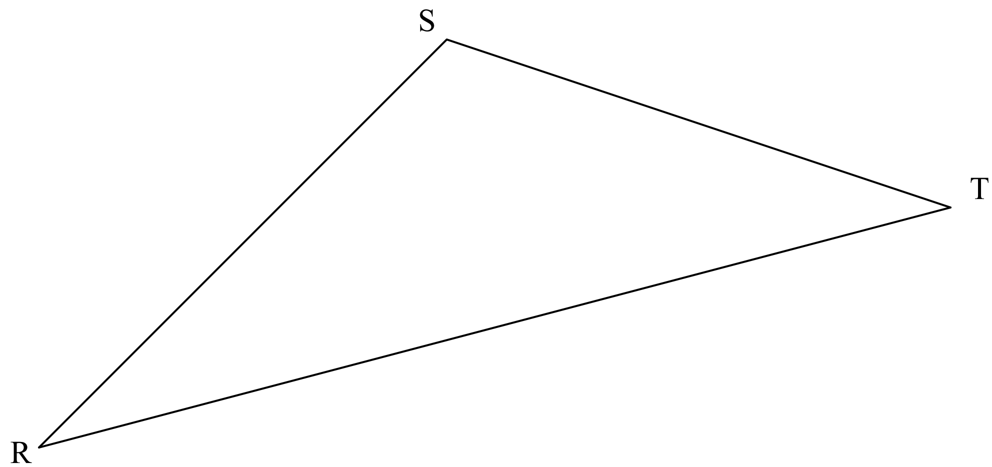
4. Name the 3 corresponding pairs of segments.

5. Name the 3 corresponding pairs of angles.

Name: ____________________________________ Date: _______________

Math 1 – Unit 3 – Session 4
Math Support – Review of Corresponding Parts and Included Angles

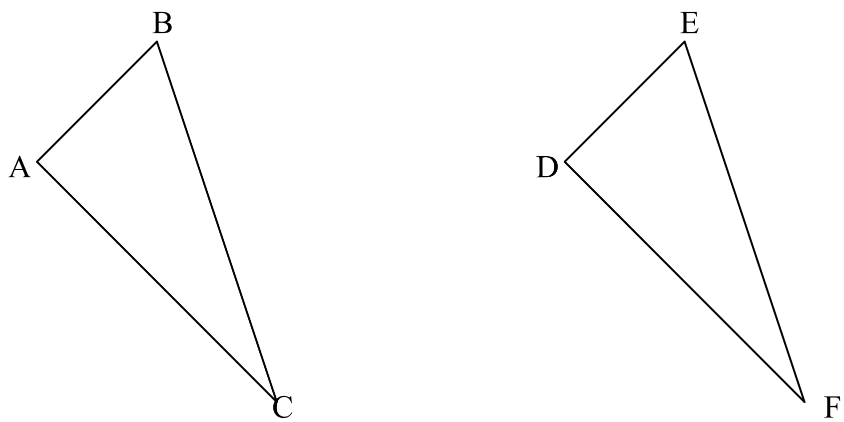
Part 1 – Review of Included Angles
Answer the following questions, using this figure.



1. What is the included angle of $\overline{RS}$ and $\overline{TR}$?
2. What is the included angle of $\overline{ST}$ and $\overline{RS}$?
3. What is the included angle of $\overline{RT}$ and $\overline{TS}$?

Part 2 – Review of Corresponding Parts
Answer the following questions using these figures.

Triangle ABC ≅ Triangle DEF



4. Name the corresponding parts.

Which angle corresponds to angle A?

Which segment corresponds to $\overline{DE}$?

Which angle corresponds to angle F?

Which segment corresponds to $\overline{BC}$?

Which angle corresponds to angle B?

Which segment corresponds to $\overline{AC}$?

5. Name the corresponding parts.

Triangle RST ≅ ZXY

Name the 3 corresponding pairs of segments.

Name the 3 corresponding pairs of angles.

Exploring the Hinge Theorem Learning Task

Lesson Summary:

This guided activity allows students to discover the Hinge Theorem. The Hinge Theorem can be used to write inequalities between two triangles given two pairs of congruent sides. The Hinge Theorem states *If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first triangle is greater than the included angle of the second triangle, then the third side of the first triangle is greater than the third side of the second triangle.*

Key Words:

Hinge theorem, inequality, triangle

Background Knowledge:

Students must be familiar with Cabri geometry, either on the P.C or on the TI-92 calculator. This activity can be modified and used on Geometers Sketchpad. Students must also be able to identify corresponding congruent parts of triangles and understand basic triangle inequalities.

Objectives

1) Students will be able to compare, order, and determine equivalence of real numbers.

2) Students will be able to write inequalities for various triangular relationships.

Learning Objectives

1) Students will be able to use inequalities involving triangle side lengths and angle measures to solve problems.

Materials

Computers or calculators with Cabri geometry installed or Geometer's Sketchpad
Exploring the Hinge Theorem lab worksheet

Procedures

Review the various inequalities that can be written to describe relationships in triangles.
Divide the students into groups of no more than three (two is preferable)
Have students complete the lab worksheet.
Monitor students' progress during the activity.
Use results from the lab activity as assessment.
Review findings prior to the conclusion of class.

Exploring the Hinge Theorem
Lab Worksheet

Team members: ____
File Name: ____

Lab Goals: Students will investigate and discover the Hinge Theorem, which describes inequalities for two triangles. The primary activity examines a special case of the Hinge Theorem and the extension looks at all possible cases of the Hinge Theorem.

Procedures

1) Draw a circle C. (use circle tool)
2) Place points A and B on circle C. (use points on object tool)
3) Draw segments *AC*, *BC*, and *AB* to form triangle V*ABC*. (use segment tool)
4) Grab point A or point B and move it along circle C. Watch what happens to the length of segment *AB* . Describe what happens to segment *AB* as the measure of angle $\angle C$ changes.

____

____

5) Draw another triangle ΔDCE following the same steps above.
6) List the congruent corresponding parts of the two triangles. _______________

____

7) Grab the points and move them such that angle $\angle ACB$ is larger than $\angle DCE$. How do segments *AB* and *DE* compare? _______________________________
Now grab the points and move them such that angle $\angle ACB$ is smaller than angle $\angle DCE$. Now how do segments *AB* and *DE* compare? ___________________
8) The first activity had you examine a special case of the Hinge Theorem where the two triangles were isosceles and had congruent sides. Explain why you know that the triangles are isosceles with congruent sides.

____
____

9) Finish the Hinge Theorem: *If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first triangle is greater than the included angle of the second triangle, then*

____
____

Extension: Exploring other cases of the Hinge Theorem

As an extension, create two non-isosceles triangles using Cabri. Design triangle ΔABC and triangle ΔDCE so that segments AB and AD are congruent and segments AC and AE are congruent. Follow the procedures below.

1) Draw two concentric circles. Concentric circles have the same center and difference radii. (use circle tool)
2) Draw segments AB with one endpoint at A and the second endpoint B on the smaller circle. Then draw segment AC with one endpoint at A and the second endpoint C on the larger circle. Next, draw segment BC to form triangle ΔABC.
3) Repeat step 2 to form a second triangle ΔADE. Make segment AB congruent to segment AD congruent and segment AC congruent to segment AE.
Explain why this construction guarantees that segments AB and AD are congruent and segments AE and AC are congruent.

____

____

(use segment tool)
(use segment tool)

4) Now measure angle $\angle BAC$ and angle $\angle DAE$. Next find the length of segment BC and segment DE. Label the values appropriately. Drag the values to the side.
5) Make angle $\angle BAC$ larger than angle $\angle DAE$. How does the length of segment DE compare to the length of segment BC?

____

____

6) Make a conjecture about the length of segment DE compared to segment BC if angle $\angle DAE$ is smaller than angle $\angle BAC$.

____

Try this. Is your conjecture accurate? ______________________
7) In your own words, write the Hinge Theorem. _________________________

____

____

8) Why is it called the Hinge Theorem? _________________________________

____

(use distance and length tool, use angle tool)

The Hinge Theorem

Math I Unit 3 Concept: Triangular Inequalities

Optional PowerPoint included at the end of the unit.

Mathematics 1
Unit 3
Acquisition Lesson—Triangular Inequalities
Session 5 (1.5 Sessions)

Essential Question:

How do I use the Converse of the Pythagorean Theorem to determine if a triangle is a right triangle?

Activating Strategies: (Learners Mentally Active)

Review of Pythagorean Theorem worksheet.

Acceleration/Previewing: (Key Vocabulary)

Math 1 Support: Do the Converse of the Pythagorean Theorem Worksheet

Maintain Vocabulary: hypotenuse, right angle, legs

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

Task:

- In groups of two or three, students will explore the Converse of the Pythagorean Theorem Learning Task using Geometers Sketchpad following the directions on the learning task worksheet.

- As students make their discoveries, they will apply the information to their learning task worksheet.

- Teacher will lead discussion to summarize conclusions and complete the graphic organizer on the Converse of the Pythagorean Theorem.

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

- Students will complete the square puzzle on the Pythagorean Theorem.

Summarizing Strategies: Learners Summarize & Answer Essential

Question

- Students will refer back to the KWL chart and anticipation guide used as a Session 1 activator and complete the column for what they learned throughout this concept.

Ticket-out-the-door:

- Students will be given an index card with three segments. The student must decide if the three segments form a triangle (using the Triangle Inequality Theorem), decide which side would be the possible hypotenuse, and decide what kind of triangle is formed (using the Converse of the Pythagorean Theorem).

Needed Materials:

- Pythagorean Review Worksheet
- Math 1 Support: Pythagorean Theorem Sheet
- Converse of Pythagorean Lab Sheet
- Converse of Pythagorean Graphic Organizer
- Square Puzzle
- Index Cards of Segments

Name: ____________________________________ Date: _________________________

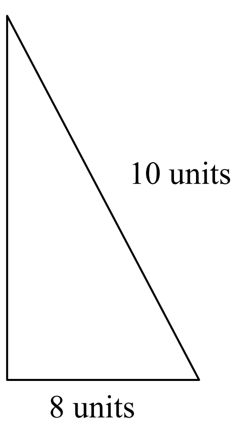
Math 1 – Unit 3
Activator - Review of the Pythagorean Theorem

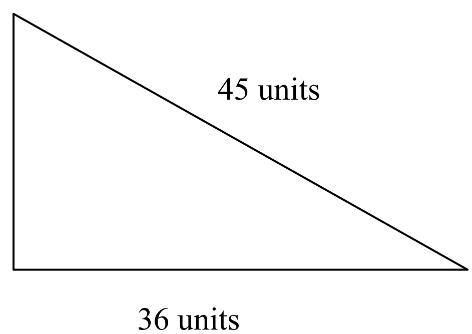
Pythagorean Theorem

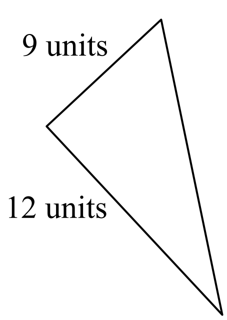
Pythagoras, a Greek philosopher and mathematician who lived in the 6th century B.C., developed the famous theorem that describes the special relationship between the legs and hypotenuse of a right triangle and states that $a^2 + b^2 = c^2$. a and b represent the lengths of the legs of the triangle and c represents the hypotenuse.

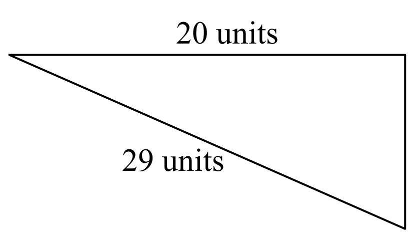
In any right triangle, the hypotenuse is the side opposite the right angle and the longest side. The legs are the other two sides.

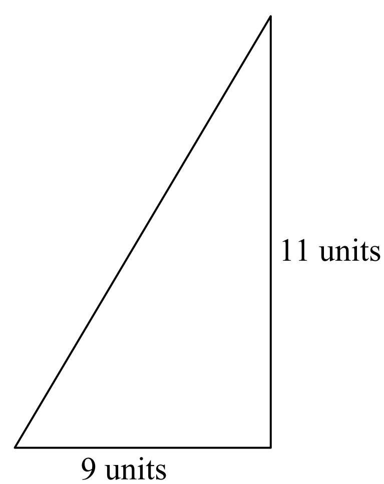
Find the lengths of the missing sides of the following Right triangles.











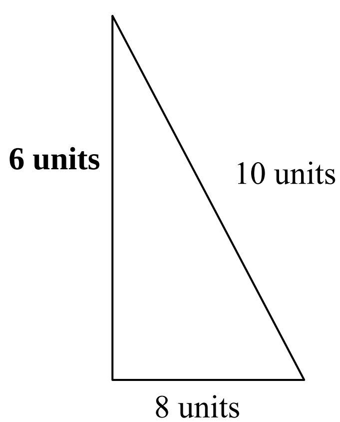
Name: ___________________________________ Date: _________________________

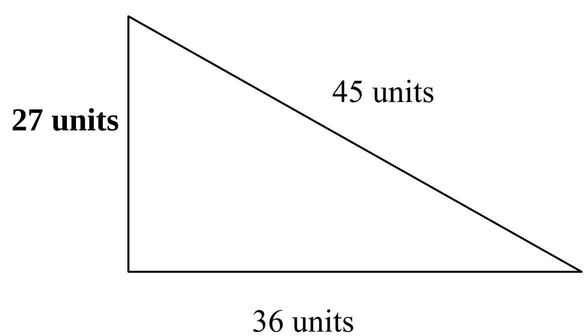
Math 1 – Unit 3
Activator - Review of the Pythagorean Theorem
Answer Key
Pythagorean Theorem

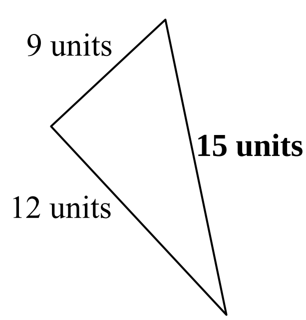
Pythagoras, a Greek philosopher and mathematician who lived in the 6th century B.C., developed the famous theorem that describes the special relationship between the legs and hypotenuse of a right triangle and states that $a^2 + b^2 = c^2$. a and b represent the lengths of the legs of the triangle and c represents the hypotenuse.

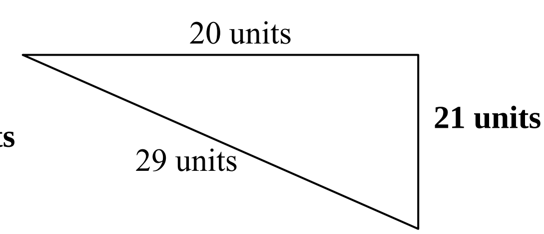
In any right triangle, the hypotenuse is the side opposite the right angle and the longest side. The legs are the other two sides.

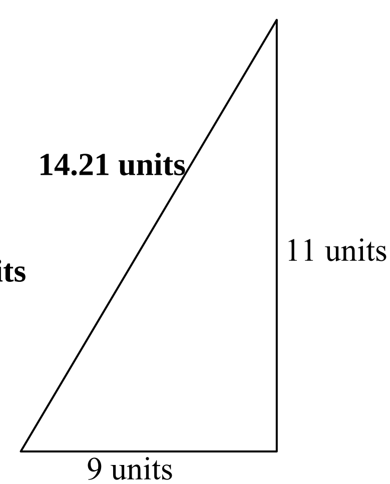
Find the lengths of the missing sides of the following Right triangles.











Name: ____ Date: __________________

Math 1 – Unit 3 – Session 5
Math Support – Converse of the Pythagorean Theorem

Pythagorean Theorem: $a^2 + b^2 = c^2$

Given the following lengths, find the missing length of the following RIGHT triangles.

1. leg a: 65	leg b: 72	hypotenuse:
2. leg a:	leg b: 6	hypotenuse: 10
3. leg a: 39	leg b:	hypotenuse: 89
5. leg a:	leg b: 5	hypotenuse: 14
6. leg a: 12	leg b: 16	hypotenuse:
4. leg a: 9	leg b:	hypotenuse: 1
7. leg a: 9	leg b: 12	hypotenuse:
8. leg a: 18	leg b:	hypotenuse: 30
9. leg a: 20	leg b:	hypotenuse: 101
10. leg a:	leg b: 35	hypotenuse: 37

Draw each of the following triangles and label the legs and hypotenuse.

11. a = 5 cm b = 12 cm c = 13 cm

12. a = 3cm b = 4 cm c = 5 cm

Math 1 – Unit 3 – Session 5
Math Support – Converse of the Pythagorean Theorem
Answer Key

Pythagorean Theorem: $a^2 + b^2 = c^2$

Given the following lengths, find the missing length of the following RIGHT triangles.

1. leg a: 65	leg b: 72	hypotenuse: **97**
2. leg a: **8**	leg b: 6	hypotenuse: 10
3. leg a: 39	leg b: **80**	hypotenuse: 89
4. leg a: 9	leg b: **35.7**	hypotenuse: 12
5. leg a: **32.7**	leg b: 5	hypotenuse: 14
6. leg a: 12	leg b: 16	hypotenuse: **20**
7. leg a: 9	leg b: 12	hypotenuse: **15**
8. leg a: 18	leg b: **24**	hypotenuse: 30
9. leg a: 20	leg b: **99**	hypotenuse: 101
10. leg a: **12**	leg b: 35	hypotenuse: 37

Draw each of the following triangles and label the legs and hypotenuse.

11. a = 5 cm b = 12 cm c = 13 cm

12. a = 3cm b = 4 cm c = 5 cm

Converse of the Pythagorean Theorem Investigation Learning Task

Procedures:

1. Introduce the lesson with a discussion of the Pythagorean Theorem. Have students state the algebraic formula $a^2 + b^2 = c^2$ and its application to a right triangle with legs of length a and b and hypotenuse of length c. Have them name a set of values that satisfy this equation.

2. Students will work in pairs on the computer on Investigation #1.

3. For a standard class, have students complete the "Pythagorean Theorem Investigation" in **Exploring Geometry with *The Geometer's Sketchpad***. This is a "Dissection Proof of the Pythagorean Theorem" Another suggestion is to divide the class into two groups and have each group work on one of the investigations. In this case, have each group explain their investigation.

4. Students will work on investigation #3 to test the Converse of the Pythagorean Theorem.

5. Students will continue with an investigation with ***The Geometer's Sketchpad***.

6. Additional worksheets can be used as either follow up or enrichment activities.

Evaluation:
The teacher will circulate around the classroom to make certain all students are on task. Student work done using *The Geometer's Sketchpad* will be assessed through the worksheets and additional activities.

Extension/Follow Up:

- Have students design a proof of the Pythagorean Theorem using a regular polygon other than a square on each of the sides of the right triangle. Evaluation of the project can be assessed by demonstration and observation or printed and collected.
- Write a conjecture for finding the length in the third dimension. Are there any Pythagorean quadruples in the third dimension?

Investigation I: Does the Pythagorean Theorem Work?

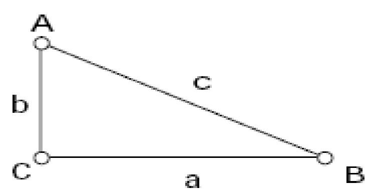
In this investigation you will construct a right triangle and demonstrate that if **a** and **b** are the legs of the triangle and **c** is the hypotenuse, then $a^2 + b^2 = c^2$.
Start with a blank sketch and construct a right triangle. Label the vertices A, B, C, with C being the vertex of the right angle, and the sides opposite the angles a, b, and c respectively.

Investigate

Measure the length of the legs and the hypotenuse.

Calculate $a^2 + b^2$ and c^2.

Does $a^2 + b^2 = c^2$?________________________



Create a table for the values of $a^2 + b^2$ and c^2. Select point A and drag it. Add these measurements to your table by double clicking on the table. Repeat these two steps for points B and C.
What do you notice about the sum of the squares of the legs and the square of the hypotenuse when the measurements change?

____

____

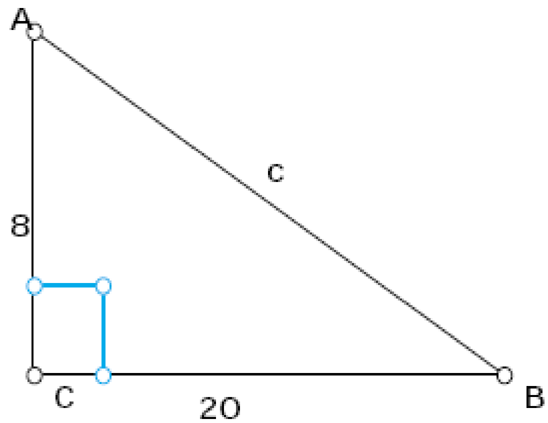
Does the Pythagorean Theorem hold true in each case? If so, why or why not?____

____

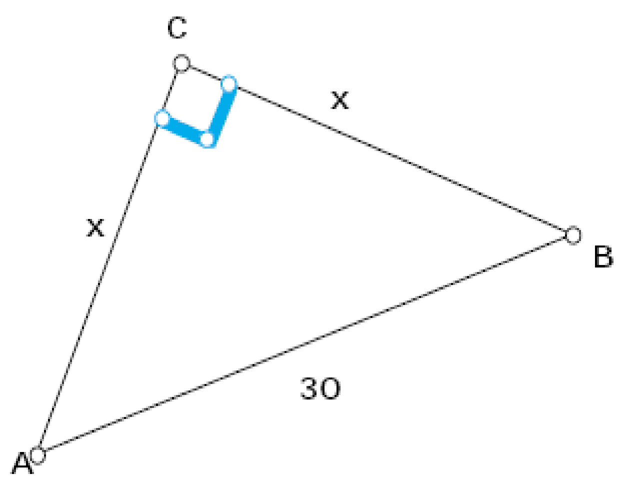
____

WORKSHEET FOR INVESTIGATIONS 1 and 2
I. Find the measures of the missing side. SHOW ALL WORK!!!

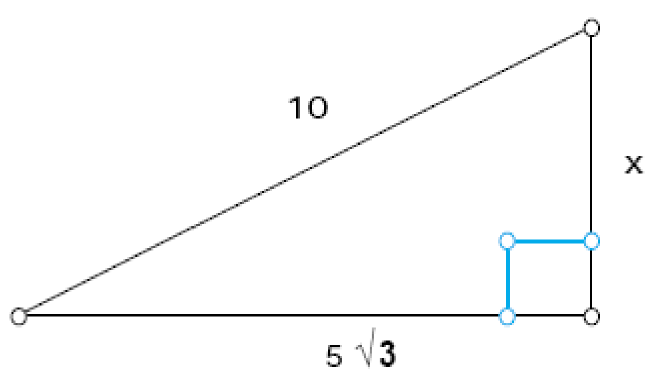
a.



b.



c.



II. Solve the following problems. Show all work!!

1. Given a right triangle with legs 5cm and 12cm, find the length of the hypotenuse.

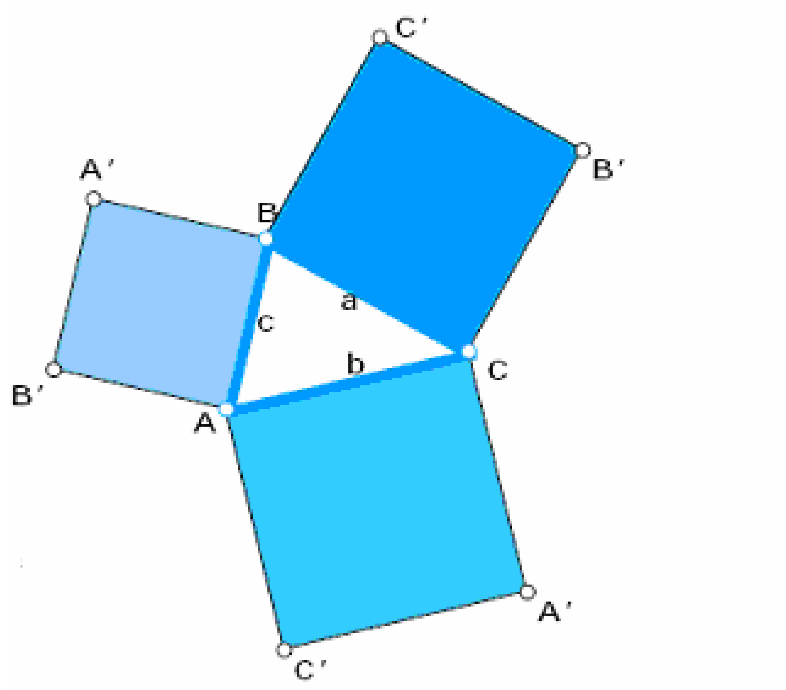
2. Given an isosceles right triangle with side 6m, what is the length of the hypotenuse?

3. If the length of the diagonal of a square is 5yds, find the length of each side of the square. Also find the area of the square.

4. If the length of the hypotenuse is $9\sqrt{3}$cm and one leg measures $8\sqrt{2}$cm, find the length of the other leg. Also, find the area of the right triangle.

5. Investigation 3: Is the Converse True?

In this investigation you'll create a script for constructing a square, then construct squares on the sides of a non-right triangle. You will test whether or not the Pythagorean Theorem applies to other triangles.



Sketch:

Record a script for construction a square:

Step 1: Open new script and click "record".
Step 2: Construct AB.
Step 3: Mark point B as center in Transform Menu, and rotate the segment and point A 90 degrees about point B.
Step 4: Mark A' as center and rotate BA' and B 90 degrees about point A'.
Step 5: Construct AB'. Stop your script.

Start with a blank sketch and construct a non-right triangle (not too large and approximately in the center of your screen). Label the vertices A, B, C and the segments a, b, c respectively.

Investigate

Place your square script on the two endpoints of each side of your triangle. If your script constructs the square to fall into the triangle, undo and select the points in the opposite order. Construct the interiors of the squares using different colors. Measure the areas of the squares and <ACB. Find $a^2 + b^2$. Create a table showing m<ACB, $a^2 + b^2$, and c^2.

1. a) Does $a^2 + b^2 = c^2$?

   b) What is the m<ACB?

Now drag point A to change m<ACB. Add the new measurements to your table by double clicking on your table.

2. a) Does $a^2 + b^2 = c^2$?

   b) What is the m<ACB?

Drag point A in another direction. Add these measurements to your table.

3. a) Does the Pythagorean Theorem hold in this case?

   b) What is the m<ACB?

Drag point A until $a^2 + b^2$ does equal c^2.

4. What is the m<ACB?

5. When $a^2 + b^2 = c^2$, what kind of triangle do you have?

What you have just discovered is the **Converse of the Pythagorean Theorem.** State the converse in your own words below.

Exercise Set

1. Use the Converse of the Pythagorean Theorem to determine whether each triangle is a right triangle.

    15, 8, 17
    12, 36, 35
    □3, □2, □5

2. A triangular plot of land has boundary lines 45 meters, 60 meters, and 70 meters long. The 60 meter boundary line runs north-south. Is there a boundary line for the property that runs due east-west?

PYTHAGOREAN WORKSHEET

1. A rope 17 m long is attached to the top of a flagpole. The rope is able to reach a point on the ground 8 m from the base of the pole. Find the height of the flagpole.

2. The base of an isosceles triangle is 16 cm long. The equal sides are each 22 cm long. Find the altitude of the triangle.

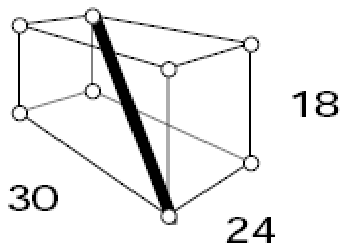
3. The dimensions of a rectangular doorway are 200 cm by 80 cm. Can a circular mirror with a diameter of 220 cm be carried through the doorway?

4. At Martian high noon, Dr. Rhonda Bend leaves the Martian U.S. Research Station traveling due east at 60 km/hr. One hour later Professor I.M. Bryte takes off from the station heading north straight for the polar ice cap at 50 km/hr. How far apart will the doctor and the professor be at 3 P.M. Martian time? Express your answer to the nearest kilometer.

5. A flagpole has cracked 9 feet from the ground and fallen as if hinged. The top of the flagpole hit the ground 12 feet from the base. How tall was the flagpole before it fell?

6. Ellen is standing on a dock 1.5 m above the water. She is pulling in a boat that is attached to the end of a 3.9 m rope. If she pulls in 1.2 m of rope, how far did she move the boat?

7. What is the longest stick that can be placed inside a box with inside dimensions of 24 inches, 30inches, and 18 inches?


18
30
24

Answers for Worksheets

Investigations 1 and 2

I. a. $4\sqrt{29}$ or 21.54
$15\sqrt{2}$ or 21.21
5

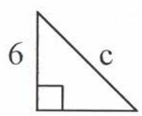
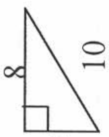
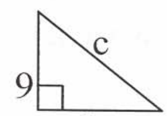
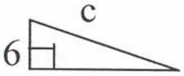
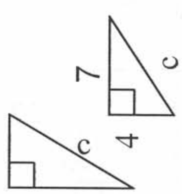
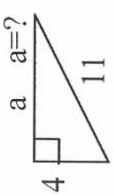
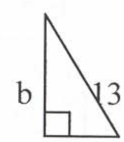
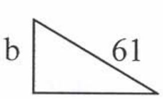
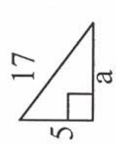
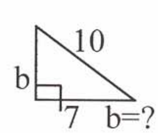
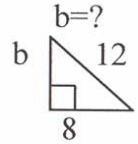
II. a. 13 cm
$6\sqrt{2}$ or 8.49 cm
side = $(5\sqrt{2})/2$ or 3.54 yds; area = 12.5 sq. yds
leg = $\sqrt{115}$ or 10.72 cm; area = $4\sqrt{230}$ or 242.65 sq. cm

Pythagorean Worksheet

1. 15

Identifying Right (or not) Triangles w/the Pythagorean Theorem Square Puzzle

Directions: Cut out the squares below. Rearrange these squares back into a 4x4 grid by working a given problem, finding the answer (simplified fully) on a square, and placing the problem and answer on adjacent edges. When all problems have been completed, if your work is correct, the pieces will fit perfectly to re-form a 4x4 grid. Paste or tape your completed puzzle to paper. Figures are NOT drawn to scale.

 6, c, 9 c=? a=3, b=4, c=5	not a right triangle right triangle  8, 10, b b=? a=5. b=8. c=9	not a right triangle $\sqrt{65}$  9, c, 9 c=? a=15, b=20, c=25	$5\sqrt{5}$ a=16, b=30, c=34 $\sqrt{105}$
right triangle 6 right triangle a=7, b=15, c=17	right triangle  6, c, 4 c=? a=4. b=5. c=6	12 c=?  5, c, 9	11 a=7, b=24, c=25  c, 6, 6 c=?
not a right triangle a=9, b=12, c=15  7, c, 4 c=? 5, c, 10 c=?	$\sqrt{51}$ $4\sqrt{5}$ right triangle	$9\sqrt{2}$  a, a=?, 11, 4 $\sqrt{106}$	$6\sqrt{2}$ right triangle  b, 13, 5 b=?
8  b, 61, 60 b=?	$2\sqrt{13}$  17, a, 15 a=? a=3, b=3, c=$3\sqrt{2}$	$3\sqrt{13}$  10, b, 7 b=? a=10, b=24, c=26	b=?  b, 12, 8 right triangle

Triangular Inequality

Anticipation Guide

Name: ______________________________________ Date: __________ Period: ______

TRUE or FALSE

_____ 1. The lengths of the sides of a triangle affects the size of the angles of the triangle.

_____ 2. Any (3) segments can form a triangle.

_____ 3. A triangle can have (2) right angles.

_____ 4. In a right triangle, the hypotenuse is always the side with the greatest measure.

_____ 5. An exterior angle of a triangle is always greater than either of its remote interior angles.

_____ 6. The sum of (2) sides of a triangle will be less than the length of the third side.

_____ 7. The Pythagorean Theorem can show if a triangle is acute.

_____ 8. The smallest angle in a triangle is opposite the longest side of the triangle.

_____ 9. Obtuse triangles have more degrees in the triangle than acute triangles.

_____ 10. The largest angle in a triangle is opposite the longest side of the triangle.

Triangular Inequality
Answer Key
Anticipation Guide

Name: ___________________________________ Date: _________ Period: ______

TRUE or FALSE

False 1. The lengths of the sides of a triangle affects the size of the angles of the triangle.

False 2. Any (3) segments can form a triangle.

False 3. A triangle can have (2) right angles.

True 4. In a right triangle, the hypotenuse is always the side with the greatest measure.

True 5. An exterior angle of a triangle is always greater than either of its remote interior angles.

False 6. The sum of (2) sides of a triangle will be less than the length of the third side.

True 7. The Pythagorean Theorem can show if a triangle is acute.

False 8. The smallest angle in a triangle is opposite the longest side of the triangle.

False 9. Obtuse triangles have more degrees in the triangle than acute triangles.

True 10. The largest angle in a triangle is opposite the longest side of the triangle.

TRIANGULAR INEQUALITY

ACTIVATOR

KWL

What We Know	What We Want to Find Out	What We Learned	How Can We Learn More

Triangular Inequalities

Definition	Characteristics
Triangle Inequality	
It Works!	It Does Not Work!

Definition	Characteristics
Side-Angle Inequality	
It Works!	It Does Not Work!

Definition	Characteristics
Side-Side-Side	
It Works!	It Does Not Work!

Definition	Characteristics
Exterior Angle Inequality	
It Works!	It Does Not Work!