Student Learning Map
Unit 5

Algebra in Context

Mathematics I
MM1A1h,i MM1A3b,c,d

Key Learning(s):

*The even/odd nature of graphs
*The basic shapes of rational, radical and quadratic functions
*Techniques for solving simple quadratic, rational and radical equations
*The fact that intersections of graphs can represent solutions to problems

Unit Essential Question(s):

1. What can the equation of a function tell you about its graph?
2. What real life situations might be solved using a quadratic equation?
3. What situations require you to look for restrictions on the domain and range?

Optional Instructional Tools: Graphing Calculator, Promethean board

Concept: 1

Functions

Lesson Essential Questions

1. How do you determine if a function has symmetry?

2. How do you determine if a function is even, odd, or neither?

3. How can you interpret the equation f(x) = g(x) and determine its solution and what solutions they have in common?

Vocabulary
1. Even function
2. Odd function
3. Point symmetry

Concept: 2

Quadratic Equations

Lesson Essential Questions

1. How do you model quadratic expressions and equations?

2. How can you factor quadratic expressions and equations with leading coefficient of 1?

3. How do you solve quadratic equations with leading coefficient of 1 using the zero product property?

Vocabulary
1. Leading coefficient
2. Zero Product Property
3. Quadratic
4. Factors

Concept: 3

Equations with radicals

Lesson Essential Questions

1. What restrictions must be considered when solving equations with radicals under the set of real numbers?

2. How would you solve an equation with radicals algebraically?

Vocabulary
1. Restricted Domain
2. Restricted Range
3. Extraneous Solution

Concept: 4

Rational Equations

Lesson Essential Questions

1. How do variables in the denominators impact solutions when solving rational equations?

2. How would you solve a rational equation that can be simplified into a linear or quadratic equation?

Vocabulary:
1. Excluded value

Notes:

Notes:

Notes

Notes:

For your information:

"All Framework Tasks can be accessed at www.georgiastandards.org under Mathematics Frameworks. The Mathematics 1 Units can be downloaded in Word Format so that increased spacing can be included between problems for students. You will be required to log on to access these documents. Create a login username and password. You may need to access these from your Central Office."

Acquisition Lesson Planning Form
Math I, Unit 5, Concept 1, Session I
Key Standards: MM1A1.c,h,i
Time Allotted: 2 hours

Essential Question: Session 1 – Functions

How do you determine if a function has symmetry?
How do you determine if a function is even, odd, or neither?

Activating Strategies: (Learners Mentally Active)

Math 1 and Math 1 Support

1. KWL—Have students list what they know about symmetry. During the class discussion, the teacher will list responses from students on the board. Based upon the responses, the teacher will assess where instruction should begin on symmetry. (See attachment #1)

2. With a partner, have students use graphic organizer to identify and explain the symmetries they see in each logo. (See attachment #2)

3. Anticipation guide—can be used as a review of rotation and line symmetry and their connections to the real world (See attachment #3)

Acceleration/Previewing: (Key Vocabulary)
OPENING

Students fill out Graphic Organizer and Task sheet (See attachment 2 and 3c)
Teacher will guide students through the following task.
Introduce even functions using Task 2 Logo Symmetry, part 4, a-e (from Frameworks)

Even function (define, give examples, and determine symmetry)

- Definition: $f(x)$ is an even function if for each x in the domain of f, $f(-x) = f(x)$
- Prove algebraically: $f(x) = x^2$ is even.
  Given that $f(x) = x^2$ — *Given function*
  then $f(-x) = (-x)^2 = x^2$ *(–x should be read "opposite of x")* — *Substitute x with –x and simplify*
  Since $f(x) = f(-x)$, then the function $f(x) = x^2$ is even. — *Compare f(-x) and f(x)*
- Determine Symmetry: Use the graphing calculator to graph $y = x^2$. Notice that y is representing f(x). Draw this graph on patty paper (wax paper). Show whether the graph has line or rotational symmetry by folding. We say that an even function is symmetric to the y-axis.

Introduce odd functions—Task 2 Logo Symmetry, part 11, a-e (Attachment 3c)

Odd function (define, give examples, and determine symmetry)

- Definition: $f(x)$ is an odd function if for each x in the domain of f, $f(-x) = -f(x)$
- Example: $f(x) = x^3$ (Given function)
  $f(-x) = (-x)^3 = -x^3$ Substitute x with –x
  $-f(x) = -(x^3) = -x^3$ Simplify and compare f(-x) to –f(x)
  Since $f(-x) = -f(x)$, the function is odd.

- Determine Symmetry: Use the graphing calculator to graph $y = x^3$. Notice that y is representing f(x). Draw this graph on patty paper (wax paper). Show whether the graph has line symmetry or rotational symmetry by folding. (Hint: Rotating a figure 180° is the same as reflecting the figure over one axis and then the other axis) We say that the graph of an odd function has rotational symmetry of 180° about the origin, also called symmetry with respect to the origin.

*Point symmetry with respect to origin

- Every part has a matching part which is the same distance from the central point but in the opposite direction (also known as Origin symmetry because the origin is the central point about which the shape is symmetrical)
- Examples: playing cards; Letters X, H, I, S, N, and Z

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)
WORK SESSION

(Use attachment 3c)
GPS Framework Task 2: Logo Symmetry, part 11 f,g,h (relate to transitions of the function families)

- Students will work individually on task.
- Collaborative pairs will compare their answers.
- Use graphic organizer (attachment #2) as a reference.

Students share out their work during work session or closing.

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)
QUESTIONING

Prompts for students:

1. Describe the symmetry of an even function.
   Graph of an even function has symmetry with respect to the y-axis.
2. Describe the symmetry of an odd function.
   Graph of an odd function has symmetry with respect to the origin or has rotational symmetry of 180° about the origin.
3. Describe a situation in which a function would be neither even or odd.
   The graph of $y= \sqrt{x}$ is neither because it is not symmetric to the y-axis and does not have rotational symmetry of 180° about the origin.

Summarizing Strategies: Learners Summarize & Answer Essential Question
CLOSING

How do you determine if a function has symmetry?
How do you determine if a function is even, odd, or neither?

Homework:

Using the Function Family Worksheet (attachment #3b), determine whether the graphs of the equations are even, odd, or neither based on their symmetry.

Symmetry KWL

What We Know	What We Want to Find Out	What We Learned	How Can We Learn More

Attachment 2

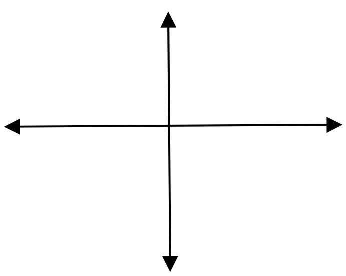
Is a Function odd or even?

Graphically   Algebraically

Even Function

Symmetric to the ______________

Prove Geometrically: $f(x) = x^2 + 1$ is even.



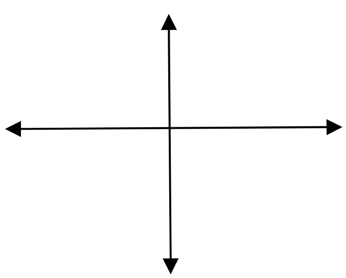
F(x) is an even function if for each x in the domain of f, $f(-x)=f(x)$

Prove Algebraically: $f(x) = x^2 + 1$ is even.

Odd Function

Symmetric with respect to the _____________

Prove Geometrically: $f(x) = x^3$ is odd.



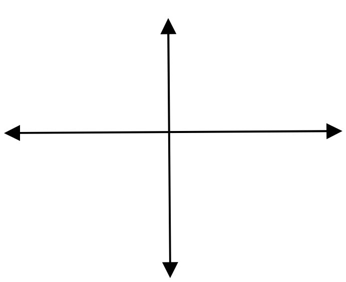
F(x) is an odd function if for each x in the domain of f, $f(-x)=-f(x)$

Prove Algebraically: $f(x) = x^3$ is odd.

Neither

Graph is not symmetric to the ___________
or with respect to the ___________________

Prove Geometrically: $f(x) = \sqrt{x} + 1$ is neither even nor odd.



F(x) is neither even nor odd if it is not symmetrical to the y-axis or is not symmetric with respect to the origin.

Prove Algebraically: $f(x) = \sqrt{x} + 1$ is neither even nor odd.

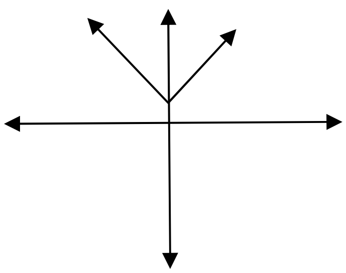
Attachment 2 Answer Key

Is a Function odd or even?

Graphically

Even Function

Symmetric to the y-axis
Prove $f(x) = x^2 + 1$ is even



If you fold this graph along the y-axis, it will lie on top of itself. Therefore this graph is symmetric to the y-axis and an even function

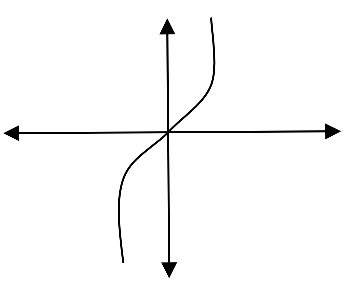
Algebraically

F(x) is an even function if for each x in the domain of f, f(-x)=f(x)
Prove Algebraically: If $f(x) = x^2 + 1$ is even,

Given that $f(x) = x^2 + 1$
then $f(-x) = (-x)^2 + 1 = x^2 + 1$.
Since $f(x) = f(-x)$, then f(x) is an even funtion by definition.

Odd Function

Symmetric with respect to the origin
Prove geometrically $f(x) = x^3$ is odd.



If you fold this graph over the y-axis and then over the x-axis it will lie on top of itself. A reflection over one axis and then over the other axis is the same as rotating the graph 180°. This shows the graph is symmetric with respect to the origin; therefore it is an odd function.

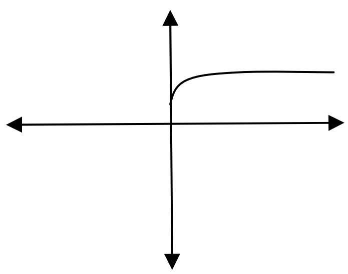
F(x) is an odd function if for each x in the domain of f, f(-x) = -f(x)
Prove algebraically: $f(x) = x^3 + 2x$ is odd.

Given that $f(x) = x^3 + 2x$
then $f(-x) = (-x)^3 + 2(-x) = -x^3 - 2x$
and $-f(x) = -(x^3 + 2x) = -x^3 - 2x$
Since $f(-x) = -f(x)$, then f(x) is odd by definition.

Neither

Graph is not symmetric to the __y-axis____ or with respect to the __origin________

Prove geometrically: $f(x) = \sqrt{x} + 1$ is neither even nor odd.



The graph will not reflect over the y- axis and lie on top of itself, so it is not an even function.
If the graph is reflected over the y-axis and then over the x-axis it will not lie on top of itself and is not symmetric with respect to the origin. Therefore it is not an odd function.
The graph does not represent an even or odd function.

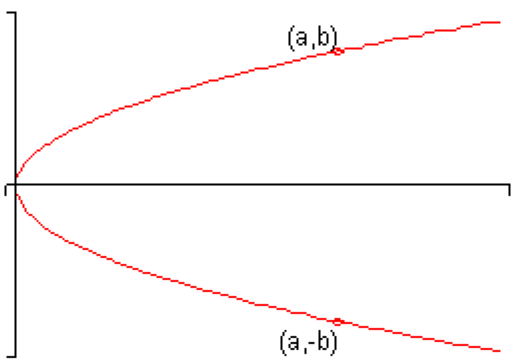
Prove algebraically: $f(x) = \sqrt{x} + 1$ is neither even nor odd.

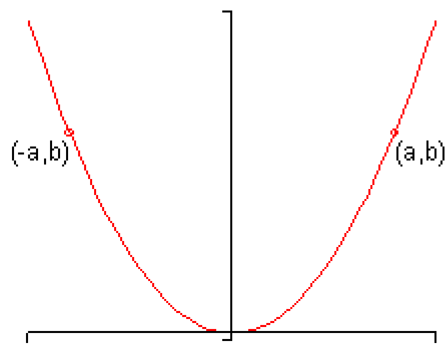
Given that $f(x) = \sqrt{x} + 1$
then $f(-x) = \sqrt{(-x)} + 1$
Also, $-f(x) = -(\sqrt{x} + 1) = -\sqrt{x} - 1$.
Since $f(-x) \neq f(x)$ this is not an even function by definition. Since $-f(x) \neq f(-x)$, this is not an odd function by definition.

Attachment 3

Symmetry

When two halves of a figure are mirror images of one another, the figure is said to have line symmetry. The line that separates the figure into these matching halves is the line of symmetry. A reflection is a transformation that flips a figure over a line called the line of reflection. On the coordinate plane, there are general rules that explain the reflection of points over the x- and y-axes: (x,y) reflected over the x-axis is (x, -y), and
(x,y) reflected over the y-axis is (-x, y).





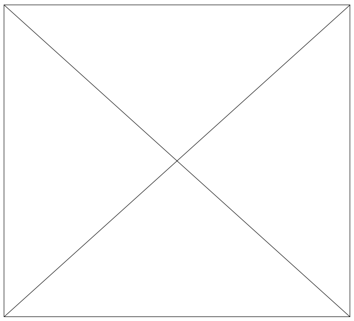
Draw a graph that reflects over the x-axis.
How can you show this is true?

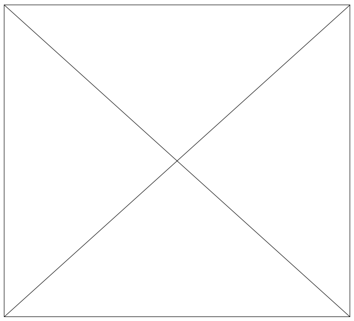
Draw a graph that reflects over the y-axis.
How can you show this is true?

Rotational Symmetry

Rotational Symmetry

With Rotational Symmetry, the shape or image can be rotated and it still looks the same.

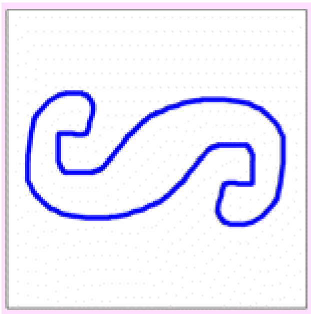
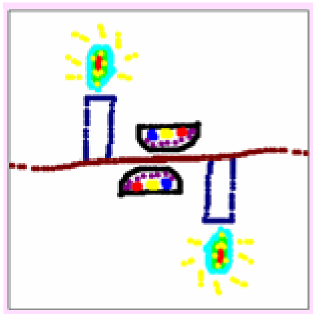
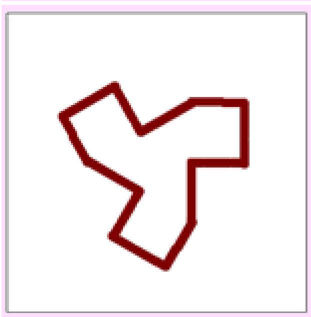
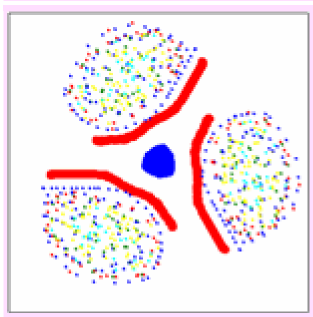




How many **matches** there are as you go **once around** is called the **Order**.

If you think of propeller blades (like below) it makes it easier.

Examples of Different Rotational Symmetry Order

Order	Example Shape	Artwork
		(using Symmetry Artist)
		
		

Draw a graph that has rotational symmetry about the origin.

Attachment 3

Real World Examples

A Dartboard has Rotational Symmetry of Order 10

The US Bronze Star Medal has Order 5

The London Eye has Order … oops, I lost count!

Attachment 3c

LOGO SYMMETRY

Problems 4 and 11
(GPS Unit 5 Framework Task)

4. We call a function f an **<u>even function</u>** if, for any number x in the domain of f, $-x$ is also in the domain and $f(-x) = f(x)$.
   a. Suppose f is an even function and the point (3, 5) is on the graph of f. What other point do you know must be on the graph of f? Explain.

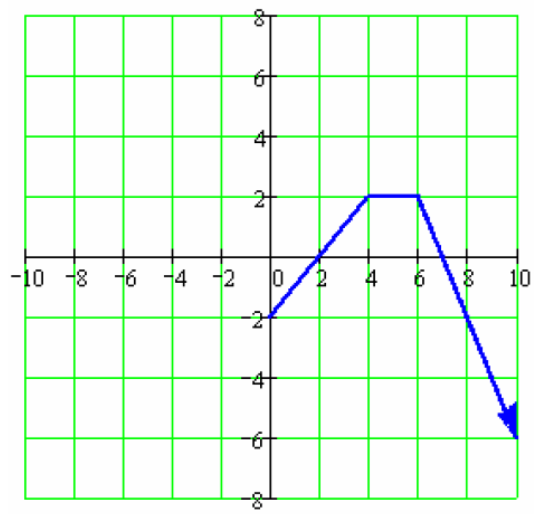
   b. Suppose f is an even function and the point (– 2, 4) is on the graph of f. What other point do you know must be on the graph of f? Explain.

   c. If (a, b) is a point on the graph of an even function f, what other point is also on the graph of f?

   d. What symmetry does the graph of an even function have? Explain why.

Attachment 3c

e. Consider the function *k*, which is an even function. Part of the graph of *k* is shown at the right. Using the information that *k* is an even function, complete the graph for the rest of the domain.



The graph of the function *k* for the nonnegative part of the domain.

11. We call a function *f* an **odd function** if, for any number *x* in the domain of *f*, – *x* is also in the domain and *f*(– *x*) = – *f*(*x*).

a. Suppose *f* is an odd function and the point (3, 5) is on the graph of *f*. What other point do you know must be on the graph of *f*?

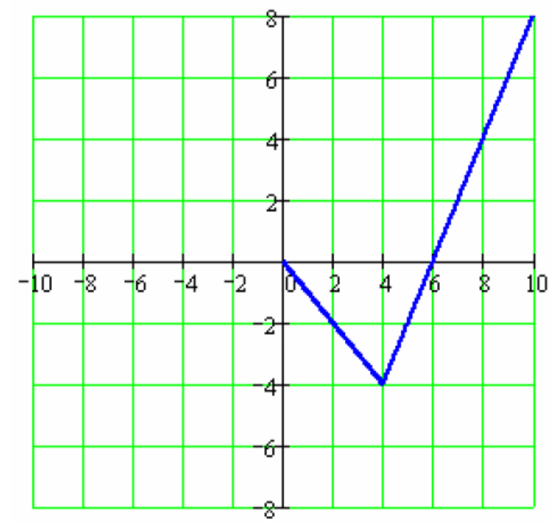
b. Suppose *f* is an odd function and the point (– 2, 4) is on the graph of *f*. What other point do you know must be on the graph of *f*?

c. If (*a*, *b*) is a point on the graph of an odd function *f*, what is *f*(*a*)?

What other point is also on the graph of *f*?

d. What symmetry does the graph of an odd function have? Explain why.

e. Consider the function k, which is an odd function. The part of the graph of k which has nonnegative numbers for the domain is shown at the right below. Using the information that k is an odd function, Complete the graph for the rest of the domain.

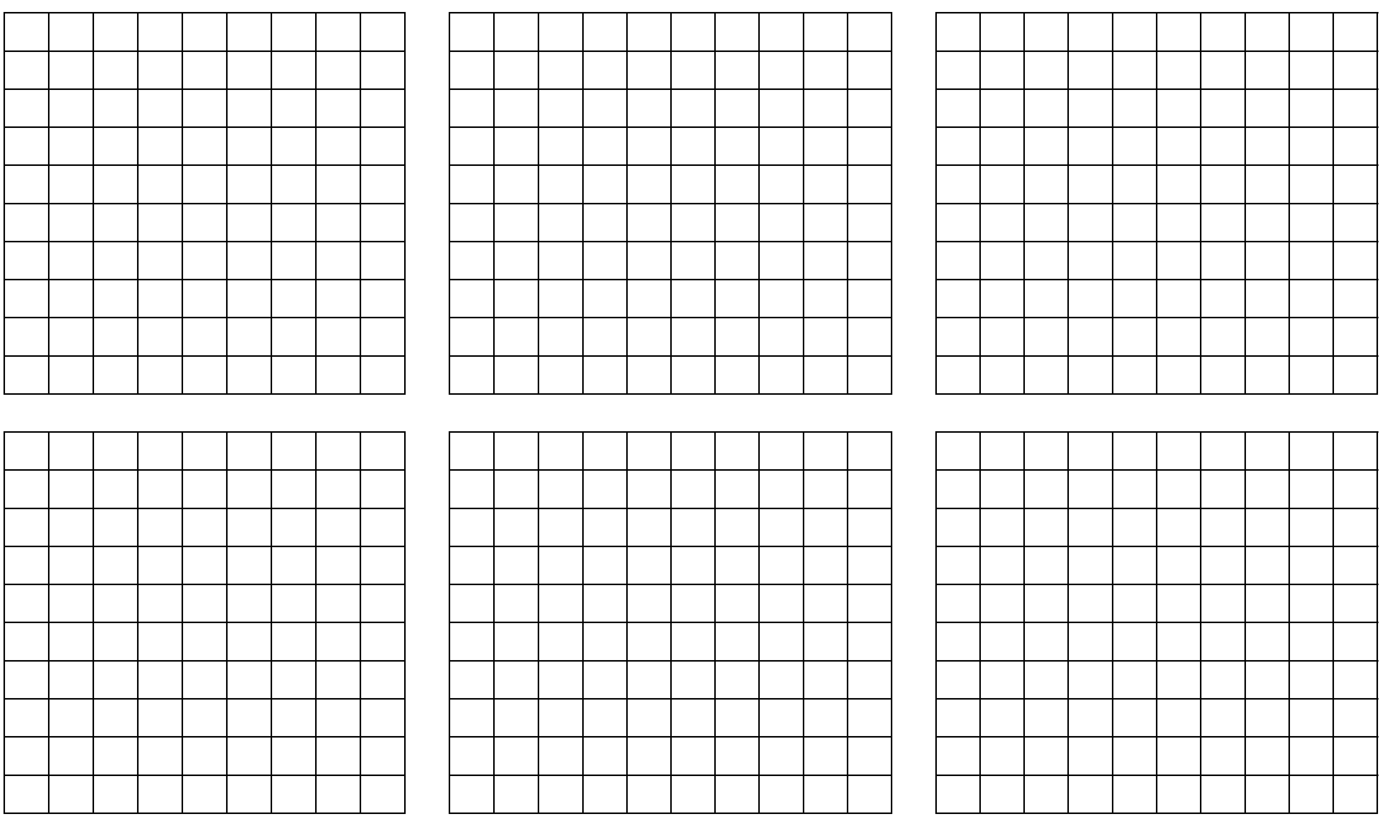


The graph of the function k
for the nonnegative part of the domain.

f. In Mathematics I, you have studied six basic functions:

$$f(x) = x,\ f(x) = x^2,\ f(x) = x^3,\ f(x) = \sqrt{x}\ ,\ f(x) = |x|, \text{ and } f(x) = \frac{1}{x}$$

Classify each of these basic functions as even, odd, or neither. Draw graphs and justify your answer.



Attachment #3c

g. For each basic function you classified as even, let *g* be the function obtained by shifting the graph down five units, and determine whether *g* is even, odd, or neither. Draw graphs.

Attachment #3c

h. For each basic function you classified as odd, let *h* be the function obtained by shifting the graph up three units, and determine whether *h* is even, odd, or neither. Draw graphs.

Attachment 3d

Sample Homework for Logo Symmetry Learning Task

Unit: 5

The activity you are about to do is called a card sort. You are given a set of cards that contain statements, equations, and graphs each of which is associated with a particular kind of symmetry.

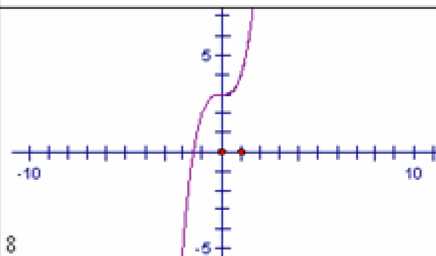
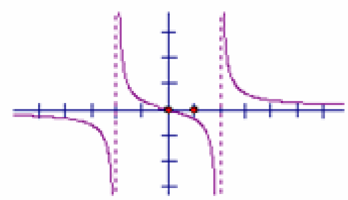
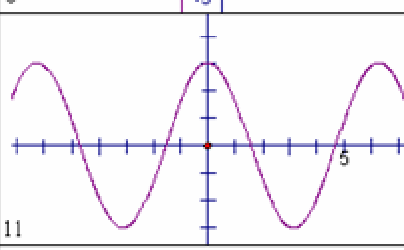
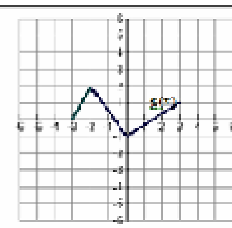
You are to place each numbered card in the appropriate column on the page labeled *Symmetry Card Sort Table*. (You might want to cut the cards apart.)

When you decide where to place a card, **justify your reasoning** using words, pictures, graphs, tables, or algebraic notation.

You have several blank cards. Use these cards to create a graph, equation, or statement to match **each** category.

Attachment 3d

Symmetry Cards

$f(x) = \frac{4}{x-2}$ 1	If the point (a,b) is on the graph of f(x), then the point (-a,b) is on the graph of f(x). 2	f(-x) = -f(x) 3
f(-x) = f(x) 4	$f(x) = \frac{x^2}{x^2-4}$ 5	If (a,b) is on the graph of f(x) then (b,a) is on the graph of f(x). 6
If the point (a,b) is on the graph of f(x), then the point (-a,-b) is on the graph of f(x). 7	 8	 9
$f(x) = 2x^5$ 10	 11	 12
13	14	15

Attachment 3d

Symmetry Card Sort Table		
Y-axis Symmetry (even functions)	Origin Symmetry (odd functions)	Other or No Symmetry (neither even nor odd)

Acquisition Lesson Planning Form
Math I, Unit 5, Concept 1, Session 2
Key Standards: MM1A1 h,i
Time Allotted: 100 minutes

Essential Question: Session 2 Functions

How can we determine where f(x) = g(x)?

Activating Strategies: (Learners Mentally Active)

Warm-Up: Show how to find the point of intersection for the following system of equations.

$$5x + 3y = 15$$
$$x + 4y = 3$$

Acceleration/Previewing: (Key Vocabulary)
OPENING

Key Vocabulary: Leading coefficient, Quadratic, Factors, Degree, Parabola
Use graphing organizer (Attachment 5)

Graph

Demonstrate the following problem using graphing calculator:

$f(x) = x^2 - 5$ This equation has a *degree* of 2, so it is called a *quadratic equation*.

$g(x) = 5x+9$ This equation has a *degree* of 1, so it is called a linear function.

Looking at the graph of these two equations, how would you determine the values of x that make f(x) = g(x)? The x coordinate(s) of the intersecting point(s).

Looking at the points of intersection, what would f(x) and g(x) represent? The y coordinate(s) of the intersecting point(s).

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)
WORK SESSION:

Attachment: What Happens At An Intersection? Task (attachment 4)

Collaborative pairs

- Interpret the solutions of the equation f(x) = g(x) as the x-values of the intersection point(s) of the graph y = f(x) and y = g(x). (See attachment #4 for examples and student problems)
  - Students will graph and determine the intersection(s) g(x) and f(x) using a graphing calculator
  -

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

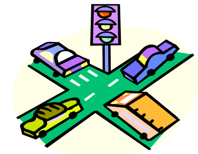
Attachment 4 contains a graphic organizer and student problems on solutions of the graphs of f(x) and g(x) by completing the problems graphically (on paper and graphing calculator)

STUDENTS SHARE OUT WORK DURING WORK SESSION OR CLOSING!

Summarizing Strategies: Learners Summarize & Answer Essential Question
CLOSING

Answer EQ: How can we determine where f(x) = g(x)?
Quiz

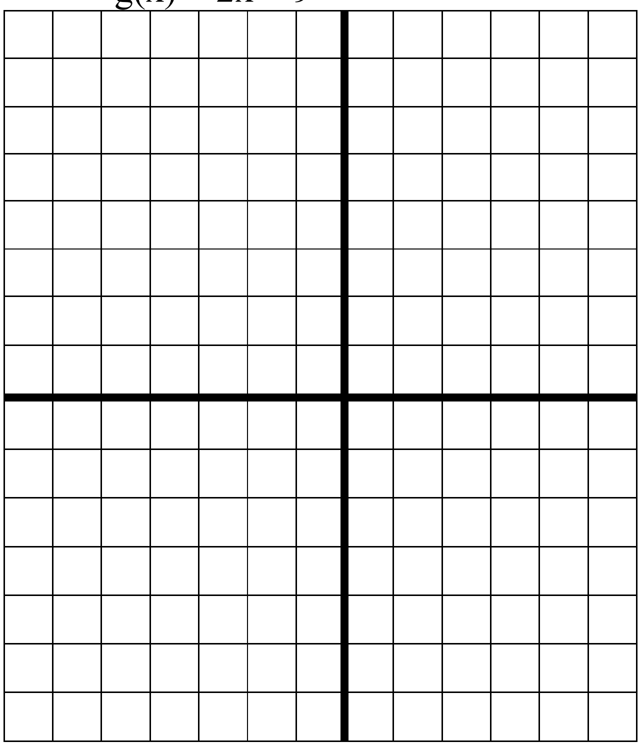
What Happens At An Intersection?



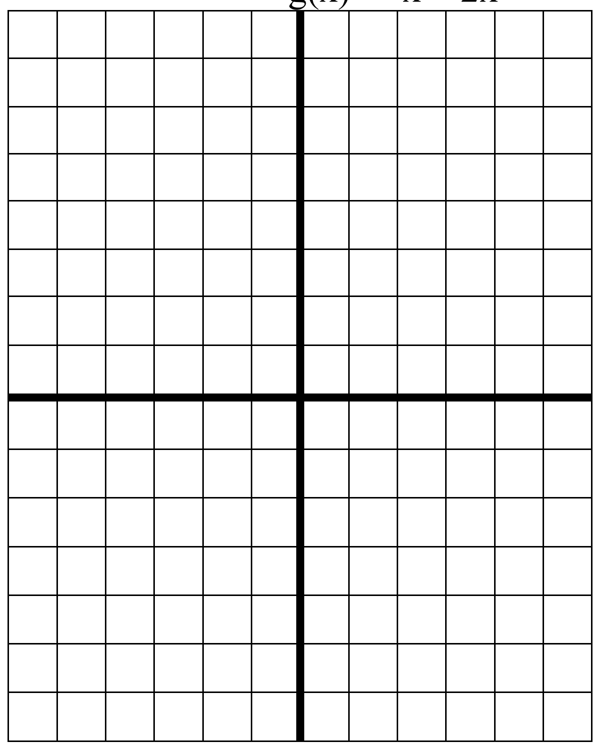
Attachment 4

Find where f(x) = g(x). Represent your answer(s) with graphs. Graph f(x) and g(x) in different colors.

1. $f(x) = x^2$
   $g(x) = 2x + 9$



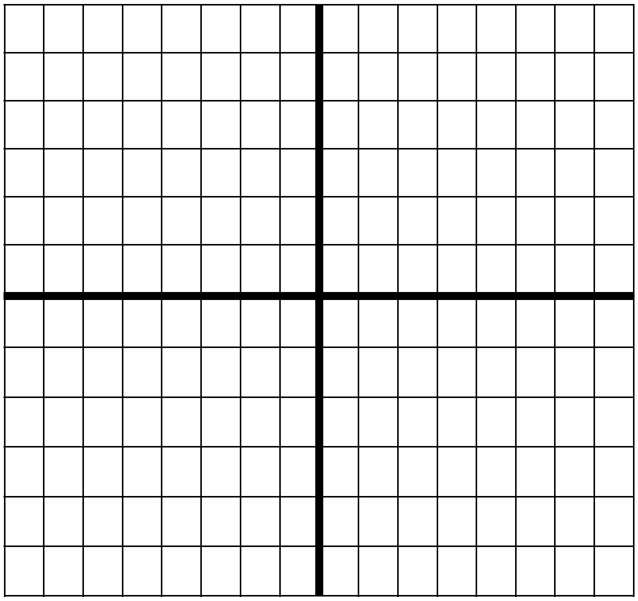
2. $f(x) = x^2 – x – 1$
   $g(x) = -x + 2x$



f(x) = g (x) where x = _________________
Why?

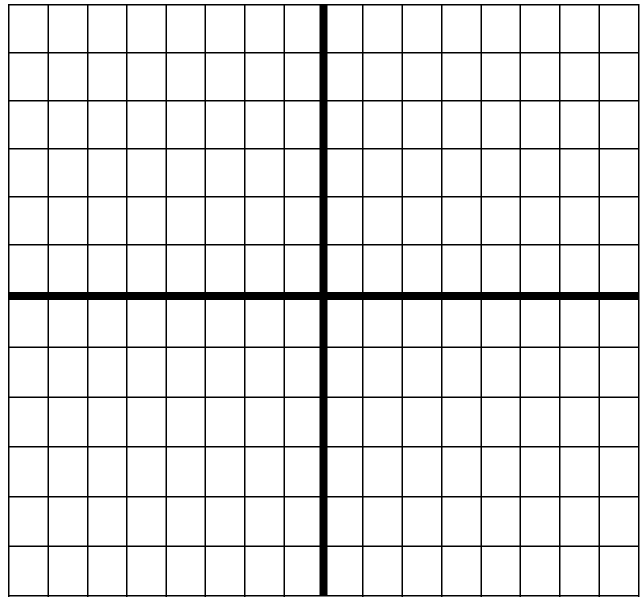
f(x) = g (x) where x = _____________
Why?

3. $f(x) = 2x^2 - x + 1$
   $g(x) = x^2 - x + 5$



f(x) = g (x) where x = __________________
Why?

4. $f(x) = x^2 - 2x - 3$
   $g(x) = x - 6$



f(x) = g (x) where x = _____________
Why?

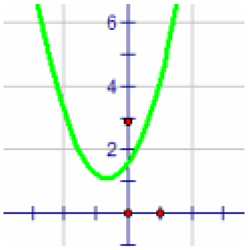
Attachment 3b

Function Families

Name_________________________________ Period_______ Date______________

Determine which graphs are Even, Odd, or Neither and justify your answers.

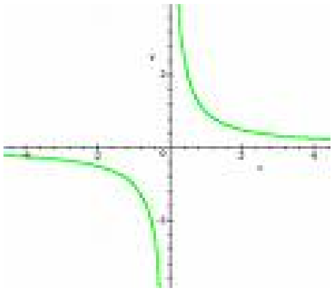
1. **Quadratic Function**



2. Square Root Function

3. Cubic Function

4. Reciprocal Function 1/x



5. If $f(x) = 5x - 6$ and $g(x) = x^2$, find **where** $\mathbf{f(x) = g(x)}$.

Acquisition Lesson Planning Form

Math I Unit 5 Algebra in Context

Unit 5 Concept 2 Quadratic Equations
Key Standards: MM1A3c
Sessions 3, 4, 5, 6
Time allotted: 4 hours

Essential Question: Session 3 Quadratic Equations

How do we model quadratic expressions and equations?
How do you determine if the function is linear?
How can you determine if a function is quadratic?
How do you determine an appropriate domain for a function?

Materials:

Activating Strategies: (Learners Mentally Active)

Warm-up

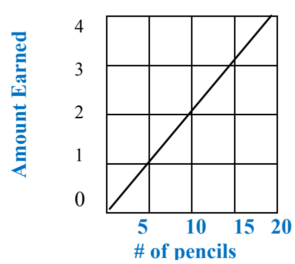
John sells 5 pencils for $1. Create a table showing the relationship between the number of pencils (*p*) sold and the amount earned (*A*). Show the relationship between the number of pencils (p) and the amount earned (A) using functional notation.

The average rate of change is constant, so the graph is linear. This rate of change is called the slope and is found by the slope formula $(y_1 - y_2) / (x_1 - x_2)$.

p	A
5	1
10	2
15	3
20	4

Slope = (2-1) / (10-5) = 1/5

The slope tells me that John earns $1 for every 5 pencils sold.



The linear equation showing the relationship between *p* and *A* is

A(p) = (1/5)(p)

Acceleration/Previewing: (Key Vocabulary)

Maximum value, **minimum value, domain, average rate of change, slope formula, linear equations, quadratic equations, restricted domain**

OPENING

Session 1:

Teacher models the following problem solving process using italicized vocabulary.

Amy has 16 popsicle sticks to build a picture frame. If she cannot break the popsicle sticks, what is the *maximum* area of her picture?

a) **How might we approach this problem? (*See if students can come up with a plan)***
b) **Draw a picture.**
c) **If the width is represented by *w,* what expression would represent the length?**
d) **If we let *w* be the independent value, what set would represent the *domain*?**
(Reinforce the use of set notation {w│0 < w < 16, w є integers} and talk about restricting the domain.)
e) **Write an expression to represent the length in terms of the width?**
f) **What equation would you use to find the area of the picture in terms of the width?**
g) **Is this a linear or quadratic equation? Justify your answer.**
h) **Make a table and graph showing the relationship between the width and area of the picture.**
i) **Is the graph *discrete* or *continuous*? Why?**

j) **The Area values represent the range. What set would represent the domain?**
(Reinforce the use of set notation $\{A \mid 0 < A \leq 4, A \in \text{integers}\}$)

k) **How would you solve for the average rate of change of Area with respect to the width when the width goes from 1 to 4?**
l) **What is the maximum and/or minimum area? How do you know?**
m) **What conjecture would you make about determining the maximum area of a rectangle? area value?**
n) **Over what interval does the graph increase? Decrease?**
(You may introduce interval notation using parentheses and brackets.)
Interval of increase is $(-\infty,4]$ and interval of decrease is $[4, \infty)$

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)
WORK SESSION:

Paula's Peaches Scenario 1: (attachment 5.2-1)

- **Have students read through Paula's Peaches.**
- **Students "think" individually and answer questions *a* through *e*.**
- **Students "pair" with their partner and "share" their answers. If one disagrees with the other, a discussion should take place to determine the correct answer.**
- **After each pair have compiled their answers, direct the groups to pair with another group "pairs squared" and share their answers.**
- **Each group should compare answers and choose the best answer and prepare to discuss with the class. One or more groups may be called upon to share with the class.**

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Extending/Refining strategies:
Have students explore the concept further by working 10, 11, 12 and 13 from the Unit 5 Framework version of Paula's peaches.

Summarizing Strategies: Learners Summarize & Answer Essential Question

Reflection questions for "ticket out the door" (5.2-5)

How do you determine if the function is linear?
How can you determine if a function is quadratic?
How do you determine an appropriate domain for a function?

Paula's Peaches Learning Task

Attachment 5.2 – 1

Paula is a peach grower in central Georgia and wants to expand her peach orchard. In her current orchard, there are 30 trees per acre and the average yield per tree is 600 peaches. Data from a local agricultural experiment station indicates that if Paula plants more than 30 trees per acre, once the trees are in production, the average yield of 600 peaches per tree will decrease by 12 peaches for each tree over 30. She needs to decide how many trees to plant in the new section of the orchard. Throughout this task assume that, for all peach growers in this area, the average yield is 600 peaches per tree when 30 trees per acre are planted. This yield will decrease by 12 peaches per tree for each additional tree per acre.

Scenario 1

Paula believes that algebra can help her determine the best plan for the new section of her orchard and begins by developing a mathematical model of the relationship between the number of trees per acre and the average yield in peaches per tree.

a. Use a mathematical model to determine if the relationship is linear or nonlinear. Explain your reasoning.

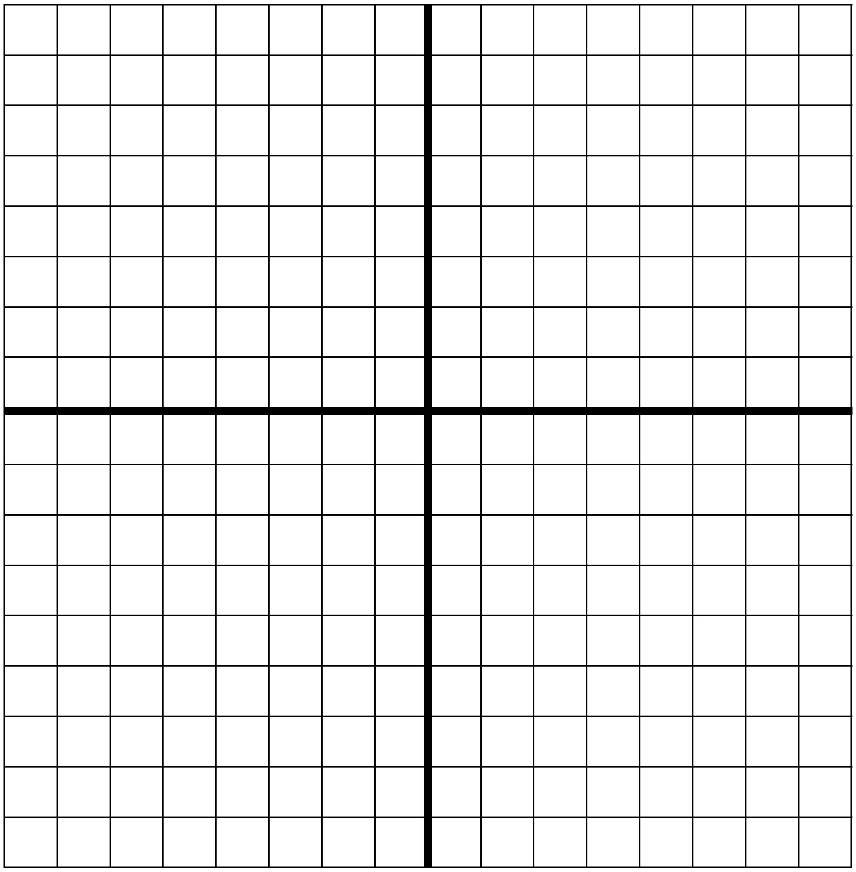
Trees per acre	peaches per tree
30	600
31	?
32	?
33	?
34	?

b. If Paula plants 6 more trees per acre, what will be the average yield in peaches per tree?

c. What is the yield in peaches per tree if she plants 42 trees per acre?

d. Let T be the function for which the input *x* is the number of trees planted on each acre and T(*x*) is the average yield in peaches per tree. Write a model (equation) for T(*x*) in terms of *x* and express it in simplest form. Explain how you can tell that your formula is correct.

e. Draw a graph of the function T values that you found in the previous table. Given that the information from the agricultural experiment station applies only to increasing the number of trees per acre, what is an appropriate domain for the function T?



Unit 5 Concept 2 Quadratic Equations
Key Standards: MM1A3c, MM1A1d
Sessions 3, 4, & 5
Time allotted: 3 hours

Essential Question: Session 4 Quadratic Equations

How do we model quadratic expressions and equations?
How do you determine if the function is linear?
How can you determine if a function is quadratic?
How do you determine an appropriate domain for a function?

Materials:
Scissors, tape, construction paper, rulers, chart paper, graphing calculators, materials 5.2-1

Activating Strategies: (Learners Mentally Active)

Daily warm-up: 5 minutes

Georgia, Massachusetts, Connecticut, New Hampshire, South Carolina, Virginia, New York, and Maryland all became states in the same year. They became the 4th through the 11th states of our United States. Solve this puzzle to learn the year.

- **My tens and units digits are the same number. Learn them by evaluating this expression:**
  $$7-4\div 12+8\div 6$$
- **My hundreds digit is equal to the value of *y* in this equation:** $5(y-2)=2y+11$
- **The sum of all of my digits is equal to** $\sqrt{625}-1$

What year am I?

__________	__________	__________	__________
Thousands	**Hundreds**	**Tens**	**Units**

\- -

Acceleration/Previewing: (Key Vocabulary)
Maximum value, minimum value, domain, average rate of change, slope formula, linear equations, quadratic equations, restricted domains

OPENING

Give each student and $8\frac{1}{2}"\times 11"$ **piece of colored paper. Have them make a tray by cutting congruent squares from each corner. Then fold the sides up and tape them. Have each student determine the volume of his/her tray. Put a chart on the board to keep track of the dimensions and volume for each tray.**

1. **Is there a maximum volume?**
2. **Is there a minimum volume?**
3. **Find the average rate of change of volume with respect to the height when the height increases from .5 to 1**
4. **Is the relationship between the volume and height linear? Explain your reasoning.**

5. **Find the interval of increase and explain how this relates to the problem.**
6. **Find the interval of decrease and explain how this relates to the problem.**
7. **Let V be the function that expresses the relationship between the volume and height where x represents the height of the tray and V(x) represents the volume of the tray. Write a formula for V(x) in terms of x and express your answer in expanded form.**
8. **Define the domain and range.**
9. **Graph this function and determine if the graph should be discrete or continuous. Explain how you know.**
10. **What points on the graph would represent the zeroes or x-intercepts. What do they represent in the problem?**

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)
WORK SESSION:

Paula's Peaches Scenario 2: (attachment 5.2-2)

- **Have students work in pairs to complete scenario 2. Then direct students to "pair-square" to check work, discuss answers, make revisions.**
- **Have chart paper/graph paper prepared so students can plot points if needed to decide if the relationship is linear or nonlinear.**
- **Allow 20 minutes for students to work together and complete *(a)* through *(i).* Select a couple of pairs to present their work to the class.**
- **Allow students to use graphing calculators to graph the data.**

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Extending/Refining strategies:
Have students explore the concept further by working problems 4 and 5 from the Unit 5 Frameworks Task "Paula's Peaches".

Summarizing Strategies: Learners Summarize & Answer Essential Question

Reflection questions for "ticket out the door" (Attachment 5.2-5)

Scenario 2 **Attachment 5.2-2**

Since her income from peaches depends on the total number of peaches she produces, Paula realized that she needed to take a next step and consider the total number of peaches that she can produce <u>per acre.</u>

a. With the current 30 trees per acre, what is the yield in total peaches per acre?

b. Complete the table and determine the yield if Paula plants 36 trees per acre.

Trees per acre	**Total peaches per acre**
30	
31	
32	
33	
34	
35	
36	

c. What is the yield (total peaches per acre) if she plants 42 trees?

d. Find the average rate of change of peaches per acre with respect to number of trees per acre when the number of trees per acre increases from 30 to 36. Write a sentence to explain what this number means.

e. Find the average rate of change of peaches per acre with respect to the number of trees per acre when the number of trees per acre increases from 36 to 42. Write a sentence to explain what this number means.

f. Is this relationship between number of trees per acre and yield in peaches per acre linear? Explain your reasoning.

g. Let Y be the function for which the input *x* is the number of trees planted on each acre and the output Y(*x*) is the total yield in peaches per acre. Write a formula for Y(*x*) in terms of *x*.

h. Calculate Y(30), Y(36), and Y(42). What is the meaning of these values? How are they related to your answers to part a through c?

i. What is the relationship between the domain for the function T and the domain for the function Y? Explain.

Ticket out the door

Attachment 5.2 – 5

1. How do you determine if the function is linear?

2. How can you determine if the function is quadratic?

3. How do you determine an appropriate domain for a function?

Acquisition Lesson Planning Form

Math 1Unit 5 Concept 2 Quadratic Equations

Key Standards addressed in this lesson: MM1A3c

Sessions 3, 4, 5, 6

Time allotted: 3 hours

Essential Question: Session 5

How do we factor quadratic expressions with leading coefficient of 1?
Materials 5.2-3 and 5.2-4

Activating Strategies: (Learners Mentally Active)
OPENING

Acceleration/Previewing: (Key Vocabulary)
Maximum value, domain, rate of change, slope formula, linear equations, quadratic equations

Review: Describe what set of numbers would represent the domain and would the graph be discrete or continuous.

1. **Number of People**
2. **Length of a room**
3. **Ounces of milk in a glass**
4. **Number of M&M's in a bowl**
5. **Miles on a car**

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)
WORK SESSION

Have students work in pairs on Practice Worksheet 5.2-3

Have students work individually problems assigned from class textbook as needed.

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Extending/Refining strategies:
Have students explore the concept further by working 10, 11, 12 and 13 from the framework version of Paula's peaches.

Summarizing Strategies:
CLOSING

Learners Answer Essential Question

Homework: Attachment 5.2-4 or Assessment for MM1A1d

Attachment 5.2-3

Unit 5 Name________________

Practice WS 1 **Concept 2**

Modeling Quadratic Equations

1. A tour bus carries tourists through the historic district of Savannah, Georgia. The charge is currently \$8.00 per person and the bus services 300 passengers per day. The company owner estimates that the company would lose 20 passengers a day for each \$1 increase in the fare.

a. If the tour bus charges \$8 per passenger, how many passengers will be serviced per day?

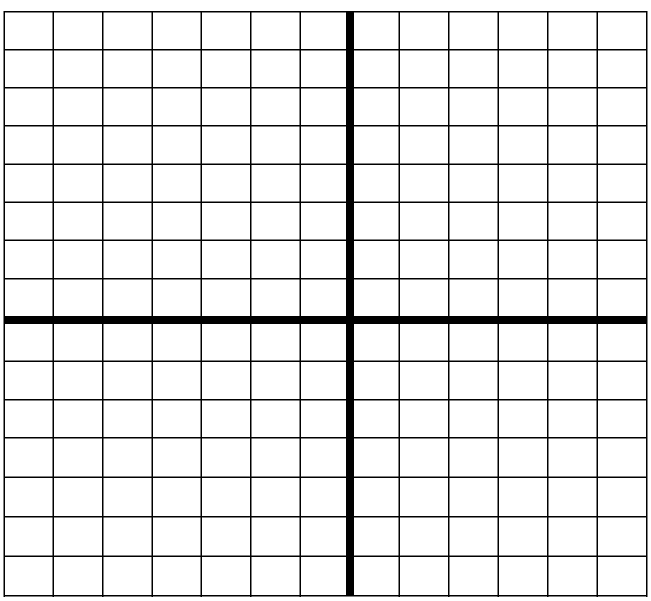
b. If the tour bus raises the fare \$1.00, how many passengers will the tour bus serve in a day?

c. How many passengers will be served daily if the fare increases by \$2.00?

d. Make a chart showing the charge and the number of passengers served daily up to a charge of \$15.

Fare	**Daily passengers**
8	300
9	
10	
11	
12	
13	
14	
15	

e. Is this relationship linear or nonlinear? Justify your answer graphically.



Note: A graphing calculator may be used to show the relationship. Use Stat plot, List L1 represents the fare, List L2 represents the passengers. Use L1 for X List and L2 for Y List ; use the window Xmin 5, Xmax 20, Xscl 1, Ymin 150, Ymax 400, Yscl 25, Res 1.

f. Given that the information from the tour bus company applies only to increasing the fare per passenger, what is an appropriate domain for the function?

g. Let T be the function for which x is the fare (charge) per passenger and $T(x)$ represents the decrease in passengers serviced per day after the price increase. Write a formula for $T(x)$ in terms of x and express it in simplest form. Explain how you know your formula is correct.

2. The income from the tour bus company depends on the total number of passengers. The owner decides to take a next step and find the fare that will maximize the income for the tour bus.

a. With the current charge, \$8 per passenger, what is the total income per day?

b. If the fare is increased to \$10 per passenger, what is the total income per day?

c. If the fare is increased to \$12 per passenger, what is the total income per day?

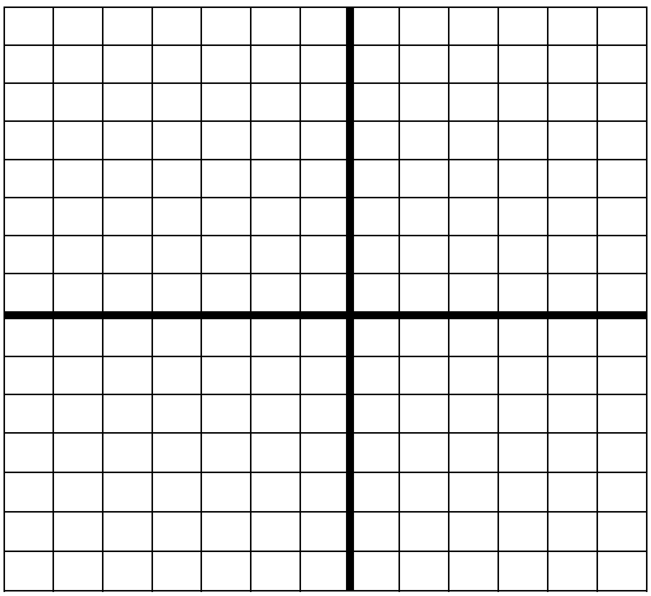
d. Make a chart showing the charge and the total income per day at that charge.

Fare	**Daily Income**
8	\$2400
9	
10	
11	
12	
13	
14	
15	

e. Find the average rate of change of total income with respect to fare charged per passenger when the fare per passenger increases from \$8 to \$10. Write a sentence to explain the meaning of this number.

f. Find the average rate of change of total income with respect to fare charged per passenger when the fare per passenger increases from \$10 to \$12. Write a sentence to explain the meaning of this number.

g. Is the relation between fare charged and total daily income a linear relationship? Justify your answer graphically.



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NOTE: A graphing calculator may be used to show the relationship. Use the Stat feature. L1 represents the fare, L2 represents the passengers, L3 represents the income. For the plot, leave XList as L1, change YList to L3. Use the window: Xmin 5, X max 20, Xscl 1, Ymin 2200, Ymax 2800, Yscl 200, Yres 1

h. Let B be the function that expresses this relationship, that is, the function for which the input x is the fare per passenger and the output B(x) is the total income. Write a formula for B(x) in terms of x.

i. Calculate B(8) , B(10), and B(12) using the formula found in (h).
What is the meaning of these values? How are these values related to your answers in (a) – (c)?

B(8) = __________ B(10)= __________ B(12) = __________

j. What is the relationship between the domain for the function T and the domain for the function B? Explain.

Attachment (5.2 -4)

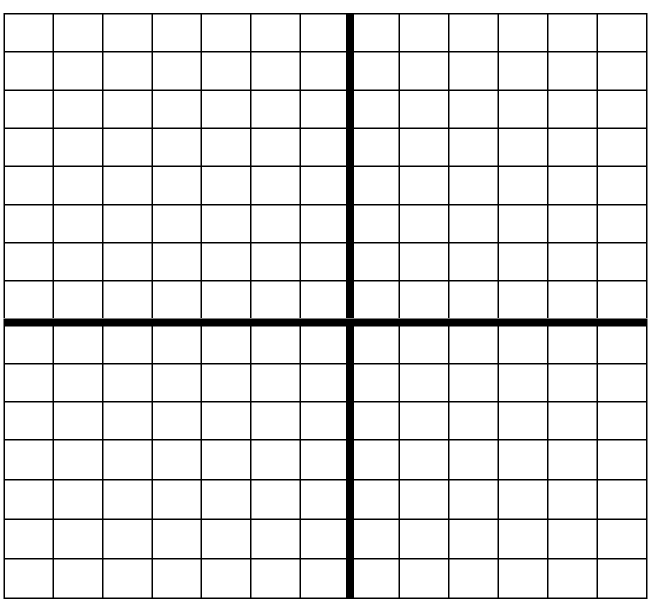
Homework — Unit 5 Concept 2 — Name__________________

A clothing store sells 40 pairs of jeans daily at $30 each. The owner figures that for each $3.00 increase in price, 2 fewer pairs will be sold each day. What should the price be changed to in order to maximize income? Complete exercises below to discover the answer.

a. Make a chart showing the current jean price and the number of jeans expected to sell at that price. Include jean prices from $30 to $60. Is this relationship linear or nonlinear? How can you tell?

Jean Price	Number sold
$30	
$33	
$36	
$39	
$42	
$45	
$48	
$51	
$54	
$57	
$60	

b. Graph the data from your chart. Does the graph justify your decision on linear or non- linear?

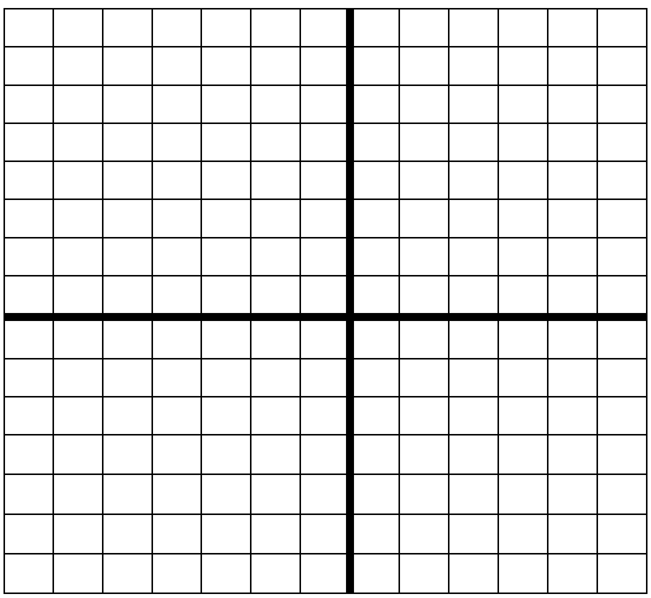


c. Let P be the function for which x is the price of a pair of jeans and $P(x)$ represents that decrease in jean sales after the price increases. Write a formula for $P(x)$ in terms of x and express it in simplest form. Explain how you know your formula is correct.

d. The income from the jean sales depends on the total number of jeans sold. The owner now needs to decide what price to charge to maximize income from jean sales. Complete a chart showing the income from jean sales when the price increases from \$30 to \$60

Jean Price	Income from jean sales
\$30	
\$33	
\$36	
\$39	
\$42	
\$45	
\$48	
\$51	
\$54	
\$57	
\$60	

e. Graph the information in the chart above. Is this relationship linear or nonlinear?



f. Let J be the function that expresses this relationship, that is, the function for which the input x is the price for a pair of jeans and the output $J(x)$ is the total income from the jean sales. Write a formula for $J(x)$ in terms of x.

g. Calculate J(30), J(39), J(45) and J(51). What is the meaning of these values? How are these values related to you answer in (d)?

h. What is the relationship between the domain for the function P and the domain for the function J? Explain.

Acquisition Lesson Planning Form
Unit 5 Concept 2 Quadratic Equations
Key Standards addressed in this lesson: MM1A3c
Sessions 3, 4, 5, 6
Time allotted: 3 hours

Essential Question: Session 6

How do we factor quadratic expressions with leading coefficient of 1?
Materials 5.2-6 thru 5.2-8, calculator, ruler

Activating Strategies: (Learners Mentally Active)
Form a rectangle using 1 x^2 -piece, 5 x-pieces, and 6 ones.

Acceleration/Previewing: (Key Vocabulary)
Maximum value, domain, rate of change, slope formula, linear equations, quadratic equation

OPENING

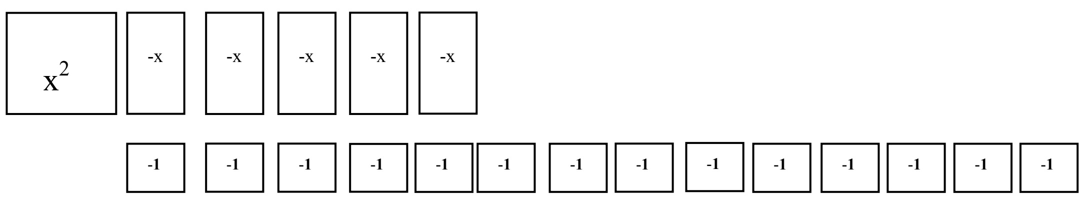
Explain what the algebra tiles represent. Show how the rectangle formed helps you find the factors of the expression $x^2 + 5x + 6$. The dimensions of the rectangle are (x + 3) and (x+ 2).

Model: f(x) = g(x)

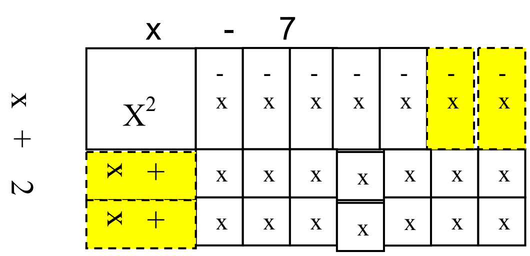
$x^2 - 5 = 5x+9$ Set = 0 and make the *leading coefficient* 1.

$x^2 - 5x - 14 = 0$ Use algebra tiles to model factoring and use the *Zero Product Property*

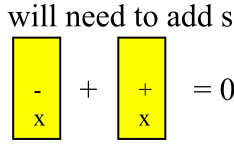
If a●b = 0, then either a=0 or b=0.



Use these tiles to form a rectangle. Remember that only sides of the same length can touch. If a piece has to touch a positive and a negative, then the piece must be negative because positive x negative = negative. This will help you determine the *factors* (side lengths).



In order to complete the rectangle, I will need to add some zeros.



Symbols

$x^2 - 5x - 14 = 0$
$(x-7)(x+2)=0$
$x-7 = 0$ or $x+2 = 0$
$x = 7$ or $x = -2$

The factors of $x^2 -5x -14$ are (x-7) and (x+2).

So the *zeros, intercepts*, or solutions of the equation $x^2 - 5x - 14 = 0$ are 7 and -2.
Look at the graph of this equation and tell why you think the solutions are also called *zeroes* or *intercepts*. What is the shape of this graph called? (*Parabola*)

Direct Instruction on factoring quadratics with leading coefficient of 1 using the graphic organizer on factoring (5.2 - 7)

From the framework…

"The steps in items 3 and 4 outline a method of solving equations of the form $x^2 + bx + c = 0$. These equations are called ***quadratic equations*** and an expression of the form $x^2 + bx + c$ is called a ***quadratic expression***. In general, quadratic expressions may have any nonzero coefficient on the x^2 term, but in Mathematics I we focus on quadratic expressions with coefficient 1 on the x^2 term. An important part of this method for solving quadratic equations is the process of rewriting an expression of the form $x^2 + bx + c$ in the form $(x + m)(x + n)$. The rewriting step is an application of Identity 1 from Unit 2. The identity tells us that the product of the numbers *m* and *n* must equal *c* and that the sum of *m* and *n* must equal *b*. In Mathematics I, we will apply Identity 1 in this way only when the values of *b*, *c*, *m*, and *n* are integers."

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

Work in collaborative pairs on Worksheet "Factor Practice". (Attachment 5.2 – 8)

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Have students work individually problems assigned from class textbook as needed.

Extending/Refining strategies:
Have students explore the concept further by working 10, 11, 12 and 13 from the framework version of Paula's peaches.

Summarizing Strategies: Learners Summarize & Answer Essential Question

Ticket out the door to summarize : 3-2-1 summary

Give 3 facts that you learned about quadratics today.

Find 2 factors of the expression $x^2 + 11x – 12$.

List 1 concept that you need to know more about.

Factoring Graphic Organizer Attachment 5.2-7

first last

$$x^2 \pm (m+n)x \pm mn$$

If the last sign is a **plus** then both factors will have the __________ signs…the first sign will tell you which one to choose.

x^2 ◇+◇ $(m+n)x$ (+) mn x^2 ⬠-⬠ $(m+n)x$ (+) mn

now re-write as binomial factors

(x ◇ m)(x ◇ n) (x ⬠ m) (x ⬠ n)

If the last sign is a **minus** then both factors will have _______________signs….For quadratic expressions and equations that have a leading coefficient of 1 for the x^2 term, the first sign will tell the sign of the factor with the larger value between m and n.

FOR THESE EXAMPLES ASSUME THAT m > n (What do you suppose is true if m = n?)

X^2 ◇+◇ $(m+n)x$ (-) mn x^2 ⬠-⬠ $(m+n)x$ (-) mn

now re-write as binomial factors

(x ◇ m)(x - n) (x ⬠ m)(x + n)

HINT: Use divisibility rules to determine possible factor combinations of the product mn.

Example: In the expression $x^2 + 19x + 60$ (m + n) = ______ mn = _______

Start by finding the possible factor pairs of mn ….fill in the missing factor

1 * ___ 2 * ___ 3 * ___ 4 * ___ 5 * ____ 6 * ___ 7 * ___ (what happened?)

10 * ___	12* ___	15* ____	20 * ___	30* ___	60* ____	Why is there a box around these?

Which of these factor pairs adds up to (m + n)? ________ so your expression factors as

(x + ____) (x + ____)! *HINT: Always remember to use mental multiplication to __________ your factoring!!!*

Attachment 5.2-8

Factor Practice   Name: ___________________

For the following expressions in the form $x^2 + bx + c$, rewrite the expression as a product of factors of the form $(x + m)(x + n)$. Verify each answer by drawing a rectangle with sides of length $(x + m)$ and $(x + n)$, respectively, and showing geometrically that the area of the rectangle is $x^2 + bx + c$. Use algebra tiles.

a. $x^2 + 3x + 2$

b. $x^2 + 6x + 5$

c. $x^2 + 5x + 6$

d. $x^2 + 7x + 12$

e. $x^2 + 8x + 12$

f. $x^2 + 13x + 36$

In the items above, the values of *b* and *c* were positive. Now factor each of the following quadratic expressions of the form $x^2 + bx + c$ where *c* is positive but *b* is negative. Verify each answer by multiplying the factored form to obtain the original expression.

a. $x^2 - 8x + 7$ Check
m●n = ____
m + n = _____
The factors are
(   )(   )

b. $x^2 - 9x + 18$ Check
m●n = _____
m+n = ______
The factors are
(   )(   )

c. $x^2 - 4x + 4$ Check
m●n = ____
m + n = _____
The factors are
(   )(   )

d. $x^2 - 8x + 15$ Check
m●n = _____
m+n = ______
The factors are
(   )(   )

e. $x^2 - 11x + 24$

m●n = ____
m + n = _____
The factors are
( )( )

Check

f. $x^2 - 11x + 18$

m●n = _____
m+n = ______
The factors are
( )( )

Check

g. $x^2 - 12x + 27$

m●n = ____
m + n = _____
The factors are
( )( )

Check

Factor each of the following quadratic expressions of the form $x^2 + bx + c$ where c is negative. Verify each answer by multiplying the factored form to obtain the original expression.

a. $x^2 + 6x - 7$

m●n = ____
m + n = _____
The factors are
( )( )

Check

b. $x^2 - 6x - 7$

m●n = _____
m+n = ______
The factors are
( )( )

Check

c. $x^2 + x - 42$

m●n = ____
m + n = _____
The factors are
( )( )

d. $x^2 - x - 42$

m●n = _____
m+n = ______
The factors are
( )( )

e. $x^2 + 10x - 24$

m●n = ____
m + n = _____
The factors are
( )( )

f. $x^2 - 10x - 24$

m●n = _____
m+n = ______
The factors are
( )( )

Acquisition Lesson Planning Form

Math 1 Unit 5 Concept 2 Quadratic Equations
Key Standards addressed in this lesson: MM1A3c,d
Sessions 7 & 8
Time allotted: 2 hours

Essential Question Session 7

How do we solve quadratic equations with leading coefficient of 1 using Zero-Factor Property?

Materials: 5.2 – 9

Activating Strategies: (Learners Mentally Active)

Acceleration/Previewing: (Key Vocabulary)
Zero Factor Property, Multiplicative Property of Equality

OPENING

Warm-up
"If two numbers have a product equal to zero, what must be true about the numbers?" Give number examples to support your argument.

Guide students through Paula's Preserved Peaches (Attachment5.2-6)
and Paula's Peaches Scenario 3: (attachment 5.2-6)

- **The teacher will emphasize the Zero Factor Property.**
- **The teacher will emphasize the Multiplicative Property of Equality**

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)
WORK SESSION

- **Students will work on Skill Worksheet "Solving Quadratic Equations" in pairs. (Attachment 5.2-9)**
- **After each pair have compiled their answers, direct the groups to pair with another group "pairs squared" and share their answers.**
- **Each group should compare answers and choose the best answer and prepare to discuss with the class. One or more groups may be called upon to share with the class.**

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Extending/Refining strategies:
Have students explore the concept further by working 10, 11, 12 and 13 from the framework version of Paula's peaches.

Summarizing Strategies:
Learners Summarize & Answer Essential Question

Attachment 5.2-6

Paula's Peaches Preserved….. Writing Quadratic Models …recap of Paula's Peaches Question 1 and 2 and introduction into writing quadratic models. (5.2 -5)

review

Paula is a peach grower in central Georgia and wants to expand her peach orchard. In her current orchard, there are 30 trees per acre and the average yield per tree is 600 peaches. Data from the local agricultural experiment station indicates that if Paula plants more than 30 trees per acre, once the trees are in production, the average yield of 600 peaches per tree will decrease by 12 peaches for each tree over 30. She needs to decide how many trees to plant in the new section of the orchard. Throughout this task assume that, for all peach growers in this area, the average yield is 600 peaches per tree when 30 trees per acre are planted and that this yield will decrease by 12 peaches per tree for each additional tree per acre.

In scenario 1d) we determined T to be the function for which the input x is the number of trees planted on each acre and T(x) is the average yield in peaches per tree. Write this formula for T(x) in terms of x and express it in simplest form. ____________________

In scenario 2 g) we determined Y to be the function for which the input x is the number of trees planted on each acre and the output Y(x) is the total yield in peaches per acre. Write a formula for Y(x) in terms of x and express your answer in expanded form.

____

Paula's Peaches Scenario 3

Paula is curious to know whether there is another number of trees per acre that will give the same yield per acre as when she plants 30 trees per acre.

a. Write an equation that expresses the requirement that *x* trees per acre yields the same total number of peaches per acre as planting 30 trees per acre.

____

b. Use the algebraic rules for creating equivalent equations to obtain an equivalent equation with an expression in *x* on one side of the equation and 0 on the other.

____

c. Multiply this equation by an appropriate rational number so that the new equation is of the form $x^2 + bx + c = 0$. Explain why this new equation has the same solution set as the equations from parts a and b.

____

…***To be continued***…We will find out the answer to Paula's curiosity in a later lesson….

TEACHER'S PAGE ***Paula's Peaches Preserved*****…..** Writing Quadratic Models …recap of Paula's Peaches Question 1 and 2 and introduction into writing quadratic models (5.2 -5T)

review

Paula is a peach grower in central Georgia and wants to expand her peach orchard. In her current orchard, there are 30 trees per acre and the average yield per tree is 600 peaches. Data from the local agricultural experiment station indicates that if Paula plants more than 30 trees per acre, once the trees are in production, the average yield of 600 peaches per tree will decrease by 12 peaches for each tree over 30. She needs to decide how many trees to plant in the new section of the orchard. Throughout this task assume that, for all peach growers in this area, the average yield is 600 peaches per tree when 30 trees per acre are planted and that this yield will decrease by 12 peaches per tree for each additional tree per acre.

In question 1d) we determined T to be the function for which the input x is the number of trees planted on each acre and T(x) is the average yield in peaches per tree. Write this formula for T(x) in terms of x and express it in simplest form. ______________________

T(x) = 600 – 12(x – 30) = 600 – 12x + 360 = 960 – 12x

In question 2 g) we determined Y to be the function for which the input x is the number of trees planted on each acre and the output Y(x) is the total yield in peaches per acre. Write a formula for Y(x) in terms of x and express your answer in expanded form.

____

Y(x) = x(960 – 12x) or Y(x) = 960x – 12x²

3. Paula wants to know whether there is a different number of trees per acre that will give the same yield per acre as the yield when she plants 30 trees per acre.
   d. Write an equation that expresses the requirement that *x* trees per acre yields the same total number of peaches per acre as planting 30 trees per acre.

   ____

   Equation: $960x - 12x^2 = 18000$ or $x(960 - 12x) = 18000$

   e. Use the algebraic rules for creating equivalent equations to obtain an equivalent equation with an expression in *x* on one side of the equation and 0 on the other.

   ____

$$x(960 - 12x) = 18000$$
$$960x - 12x^2 = 18000$$
$$-12x^2 + 960x - 18000 = 0$$

   f. Multiply this equation by an appropriate rational number so that the new equation is of the form $x^2 + bx + c = 0$. Explain why this new equation has the same solution set as the equations from parts a and b.

   ____

$$-\frac{1}{12}(-12x^2 + 960x - 18000) = -\frac{1}{12}(0)$$
$$x^2 - 80x + 1500 = 0$$

Solving Quadratic Equations (5.2 -9)

Unit 5 Concept 2 Name ______________________________

Solve the following equations using the Zero Factor Property. Verify your work by checking each solution in the original equation.

1. $x^2 - 6x + 8 = 0$

2. $x^2 - 15x + 36 = 0$

3. $x^2 + 28x + 27 = 0$

4. $x^2 - 3x - 10 = 0$

5. $x^2 + 2x - 15 = 0$

6. $x^2 - 4x - 21 = 0$

7. $x^2 - 7x = 0$

8. $x^2 + 13x = 0$

For each equation below, find an equivalent equation in the form $x^2 + bx + c = 0$. Then solve.

9. $6x^2 + 12x - 48 = 0$

10. $x^2 - 8x = 9$

11. $\mathrm{p}^2 - 3(\mathrm{p} + 2) = 4$

12. $3x^2 = 21x - 30$

13. $4x^2 + 24 = 20x$

14. $x(x - 11) + 30 = 0$

15. $\frac{1}{2}x(x + 8) = 10$

16. $(x + 1)(x + 5) + 3 = 0$

17. $(x + 5)^2 = 49$

18. $(2x + 3)(x + 4) = x + 24$

Session 8

Unit 5 Concept 2 Assessment form A (5.2 -10A)

Name ______________________
Date_____________

I Writing quadratic models

A community center is building a patio along two adjacent sides of its flower bed. The flower bed is rectangular with a width of 50 feet and a length of 100 feet. The patio will extend the same width on each side of the two sides of the flower bed.

A. Draw a sketch indicating the dimensions of the flower bed and patio area.

B. Express each side as a polynomial expression. Use **x** to represent the width of the patio.

C. Write a polynomial expression which represents the combined area of the patio and flower bed.

D. The combined area of the patio and flower bed area should be 8400 square feet. Write an equation to represent this area.

E. Re-write this equation into the $x^2 + bx + c = 0$ form. ______________________

F. Solve the equation to determine the width of the patio.

II. Solve quadratics of the form $x^2 + bx + c = 0$.

Solve.

1. $x^2 + 8x + 7 = 0$	2. $x^2 - 5x - 36 = 0$	3. $-6m + m^2 + 5 = 0$
3. $w^2 - 13w = -42$	5. $x(x + 17) = -60$	7. $w(w+8) + 15 = 0$
7. $p^2 - 3(p + 2) = 4$	8. $2n^2 - 10n + 8 = 0$	9. $8x^2 + 48 = 40x$

Acquisition Lesson Plan
Math 1 Unit 5 Concept 3 Radical Equations
Key Standards MM1A3b
Time Allotted: 2hours

Session 9

Standards(s): MM1A3b: Students will solve simple equations: Solve equations involving radicals such as $\sqrt{x}+b=c$ using algebraic techniques

Essential Question:
What restrictions must be considered when solving equations with radicals under the set of real numbers?

Activating Strategies: (Learners Mentally Active) –
OPENING

Warm- up

1. $\sqrt{28}$ 2. $\sqrt{2}*\sqrt{18}$ 3. $\dfrac{\sqrt{17}}{\sqrt{25}}$

Activating Strategy

Puzzle Pairs – (see attachment 5.3.1)
Give groups 10-20 minutes to complete activity. Then, have an anchor chart at the front of the room for each group to present one problem and explain to the class.

Acceleration/Previewing: (Key Vocabulary)
Domain
Range
Restricted Domain
Restricted Range
Extraneous solutions

Review key vocabulary from Math 1 Unit 2 Concept 4:
Radical expression
Radical
Index (pre-view cub roots/fourth roots, etc only)
Rationalizing denominators

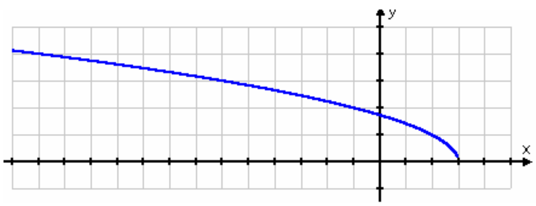
Previewing Vocabulary – use graphic organizer (attachment 5.3.2) day before class for students who need acceleration (those who may be in a math support class)

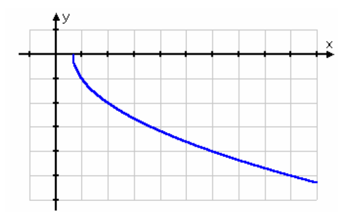
Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)
WORK SESSION

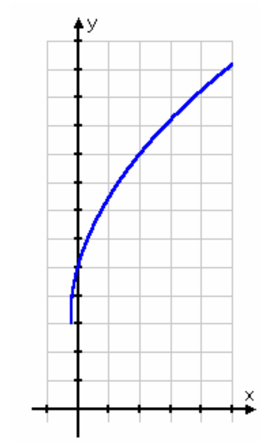
1. Real Number chart (umbrella) – (attachment 5.3.3)
2. Complete the graphic organizer (attachment 5.3.4)

A. Assign Real Numbers Project as homework (attachment 5.3.5)
B. Give approximately a week for students to complete on own – see attached

3. Remind students how to find domain and range given a graph

1. 

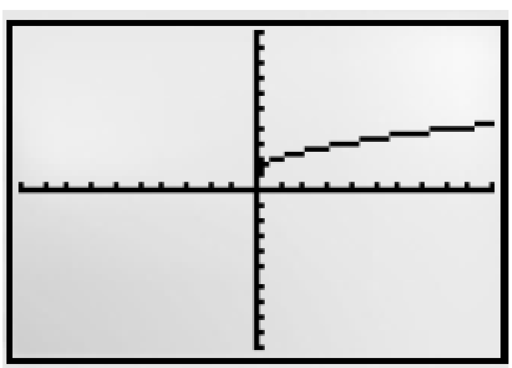
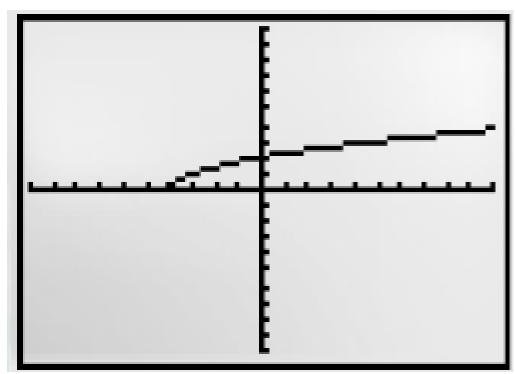
2. 

3. 

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Complete problems on domain and range

Determine the domain and range of the following:

1. 
2. 

3. $h(x) = \sqrt{x+3}$ 4. $g(x) = \sqrt{2x}$

Summarizing Strategies: Learners Summarize & Answer Essential Question

Draw the graph of f(x) = $^{1}/_{x}$ using a table. Find the domain and range of the relation. Is this graph a function?

Attachment 5.3.1

Puzzle Pairs

Directions: In groups of 4, arrange the pieces so that the problems match their answers. On a sheet of paper, draw 3 columns. In column 1 and 2 arrange the pieces so that the problems match their answers. In column 3, work out the problem to demonstrate your understanding.

Example:

$\sqrt{45x^3}$	$3x\sqrt{5x}$	$\sqrt{45x^3}$ $\sqrt{9*5*x*x*x}$ $3x\sqrt{5x}$

$\sqrt{72}$	$\sqrt{81c^2d^4}$	$9cd^2$	$\frac{8-2\sqrt{5}}{11}$
$\sqrt{2}\cdot\sqrt{10}$	$\sqrt{\frac{12}{b^2}}$	$\frac{2\sqrt{3}}{b}$	$2\sqrt{5}$
$2\sqrt{3}\cdot 3\sqrt{15}$	$\frac{2}{4+\sqrt{5}}$	$6\sqrt{2}$	$18\sqrt{5}$

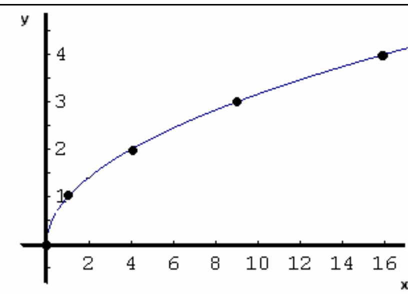
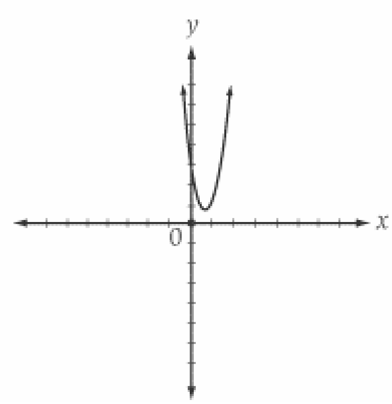
Previewing Vocabulary

Attachment 5.3.2

Concept: _______________

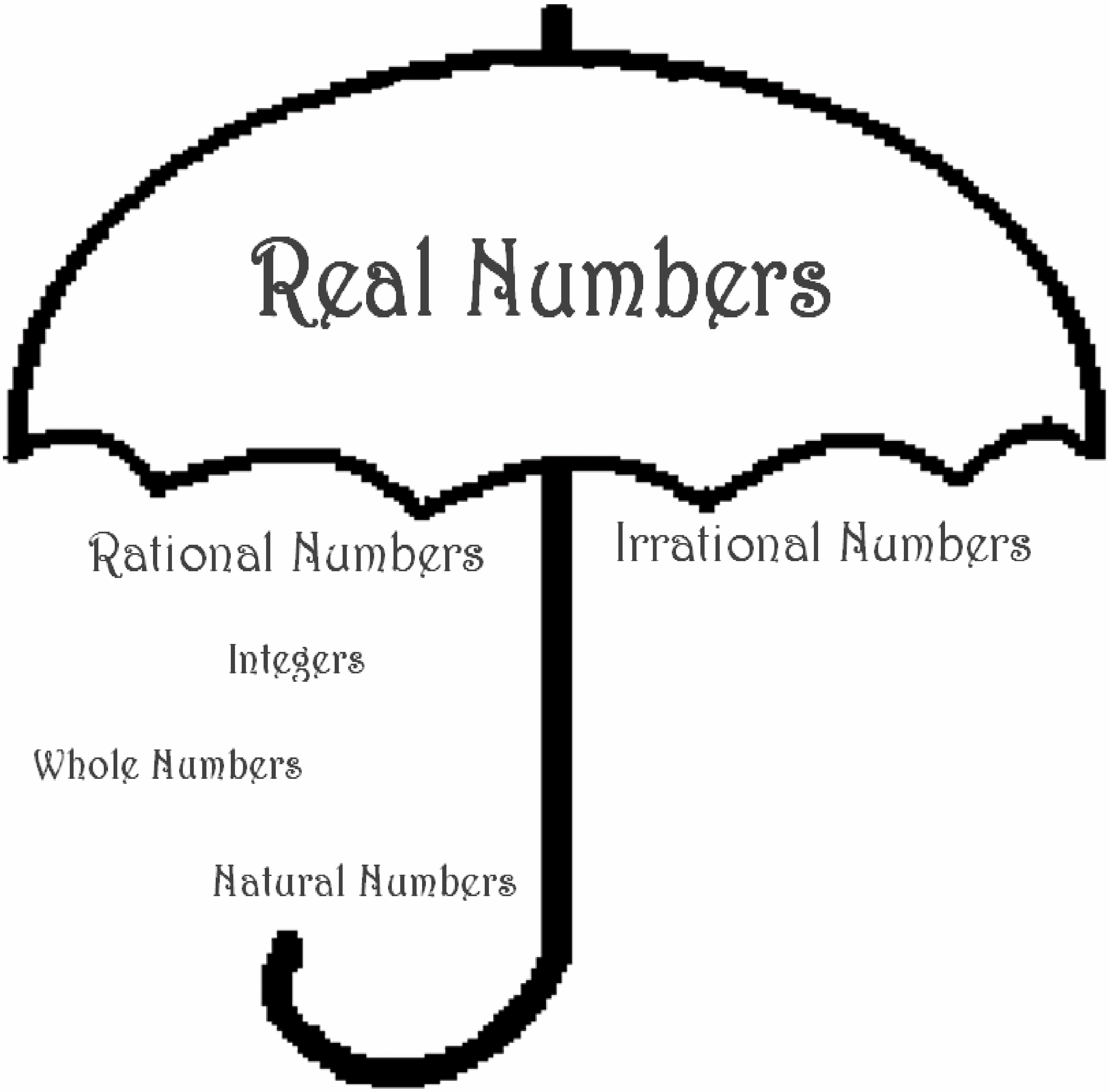
Word	Symbol/Example	Meaning	'What it is like'

Previewing Vocabulary Attachment 5.3.2Answer Key
Concept: Equations with Radicals

Word	Symbol/Example	Meaning	'What it is like'
Domain	D	The values of x	
Range	R	The values of y	
Restricted Domain			 Notice that the smallest value of x is 0 and the largest value is infinity (since the graph goes on forever) Therefore, the restricted domain is $0 \leq x \leq \infty$
Restricted Range			 Notice that the smallest value of y is 1 and the largest value is infinity (since the graph extended forever) Therefore, the restricted

			range is $1 \leq x \leq \infty$
Radical Expression	Y = √(x) + 1	An expression that contains one or more terms involving a radical	A number sentence that has √ in it.
Index	$\sqrt[n]{x}$		$\sqrt[2]{x}$, $\sqrt[3]{x}$, $\sqrt[4]{x}$
Radical Sign	Y = √x	The sign √	√
Rationalizing Denominators		process by which a fraction containing radicals in the denominator is rewritten to have only rational numbers in the denominator.	$\frac{7}{\sqrt{13}} = \frac{7 \times \sqrt{13}}{\sqrt{13} \times \sqrt{13}} = \frac{7\sqrt{13}}{13}$
Extraneous Solutions		A solution of the simplified form of an equation that does not satisfy the original equation.	Consider the equation $y - 5 = 4\sqrt{y}$. Square both sides and solve the equation for y, we get, y = 1 and 25. Check that only y = 25 satisfies the original equation, not y = 1. So, y = 1 is an extraneous solution.
Radical	Y = √x (highlight or color radical)	An expression that contains a radical sign	Y = √49x

Number System Umbrella



The Number System

Type	Symbol	Definition	Example	Real World Example
Reals	R	all the numbers that you use in everday life	…, -½, -0.25, 0, 3, 234,….	Any number in the world
Rational	Q	any number that can be expressed as a ratio m/n, where m and n aren't integers and n is not zero	…-3, -2/9, 0, 4, 15…	anywhere fractions can be found - measuring cup or bank account
Integers	Z	negative and positive whole numbers	…, -2, -1, 0, 1, 2….	Coordinate plane money
Whole	W	natural numbers including 0	0, 1, 2, 3, 4….	look at phone keypad, calculator, stop watch etc.
Natural	N	counting numbers	1, 2, 3, 4, 5…	Natural life: a baby can't be zero, a clock (ie: 1 minute, 15 minutes, etc)
Irrational	I	a number that cannot be written as a fraction; a non-terminiating or non-repeating number	2.12346651358105141..	π

The Number System

Type	Symbol	Definition	Example	Real World Example
Reals	R			
Rational	Q			
Integers	Z			
Whole	W			
Natural	N			
Irrational	I			

Real Number Systems Project

- This is an individual project.
- It must be on a poster or PowerPoint.
- You must name the *system*; give the *definition*, *characteristics*, *symbol*, and a *number* representation (*example*) of the system.
- Each system must have *2 real world pictures* to represent where we find these numbers.
- The project must be *creative* and *well organized*.
- You must include the rubric with your project.

Real Number Systems Project Rubric

Task	Total Points	Points Earned
Rubric Attached	**6 points**	
Real Numbers Name, Definition, Characteristics, Symbol, & Number Examples	**10 points**	
Rational Numbers Name, Definition, Characteristics, Symbol, & Number Examples	**10 points**	
Irrational Numbers Name, Definition, Characteristics, Symbol, & Number Examples	**10 points**	
Integers Name, Definition, Characteristics, Symbol, & Number Examples	**10 points**	
Whole Numbers Name, Definition, Characteristics, Symbol, & Number Examples	**10 points**	
Natural Numbers Name, Definition, Characteristics, Symbol, & Number Examples	**10 points**	
Real World Pictures	**2pts each = 24 total**	
Creativity & Organization	**10 points**	
Totals	**100 points**	

Comments:

Acquisition Lesson Plan
Math 1 Unit 5 Concept 3 Radicals
Key Standards MM1A3b
Time Allotted: 2hours

Session 10

Standards(s): MM1A3b: Students will solve simple equations: Solve equations involving radicals suc as $\sqrt{x}+b=c$ using algebraic techniques

Essential Question

What restrictions must be considered when solving equations with radicals under the set of real numbers?

Activating Strategies: (Learners Mentally Active) –
OPENING

Warm-up

1. $\frac{3}{\sqrt{6x}}$ 2. $\frac{1}{3+\sqrt{2}}$ 3. $3\sqrt{3}+6\sqrt{27}$

Acceleration/Previewing: (Key Vocabulary)
Domain
Range
Restricted Domain
Restricted Range
Extraneous solutions

Review key vocabulary from Math 1 Unit 2 Concept 4:
Radical expression
Radical
Index (pre-view cubic roots/fourth roots, etc only)
Rationalizing denominators

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)
WORK SESSION

Demonstrate how to find domain and range of a radical equation

1. $h(x)=\sqrt{x-1}$ 2. $g(x)=\sqrt{x+6}$ 3. $f(x)=\sqrt{x}$ 4. $g(x)=\sqrt{x+2}$ 5. $h(x)=\sqrt{4-x}$

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Determine the domain and range of the following:

1. $f(x)=\sqrt{3x+2}$ 2. $g(x)=\sqrt{x-2}$ 3. $h(x)=\sqrt{5-x}$

Summarizing Strategies: Learners Summarize & Answer Essential Question When you graph a function by making a table and graphing ordered pairs, it is not always possible to connect the points you have graphed. Give an example of a function for which this is the case. (HINT: Consider a function for which there is an x-value that is not in the domain)

Acquisition Lesson Plan
Math 1 Unit 5 Concept 3 Radical Equations
Key Standards MM1A3b
Time Allotted: 2 hours

Session 11

Standards(s): MM1A3b: Students will solve simple equations: Solve equations involving radicals suc as $\sqrt{x}+b=c$ using algebraic techniques

Essential Question:
How would you solve an equation with radicals algebraically?

Activating Strategies: (Learners Mentally Active) – OPENING

Warm- up
1. 3x – 4 = 9x + 5
2. 2(x – 4) = 3x + 6
3. 16 – 2c = 4c – 2

Activating Strategy
Tell whether the given value is a solution of the equation
1. $4\sqrt{2x-3}=12$ x = 2
2. $\sqrt{4x+8}=\sqrt{6+2x}$ x = -1

Acceleration/Previewing: (Key Vocabulary)
Domain
Range
Restricted Domain
Restricted Range
Extraneous solutions

Review key vocabulary from Math 1 Unit 2 Concept 4:
Radical expression
Radical
Index (pre-view cubic roots/fourth roots, etc only)
Rationalizing denominators

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

I DO, YOU DO – Teacher will demonstrate and describe the steps to solve the odd equations and the students will complete the even follow up problem on their own. Be sure to check for extraneous solutions.

1. $\sqrt{x} = 7$
2. $\sqrt{r - 4} = 9$
3. $\sqrt{2t + 3} = 0$
4. $\sqrt{w} - 4 = 7$
5. $\sqrt{6t + 4} = -3$
6. $\sqrt{2x + 2} = \sqrt{3x - 5}$
7. $\sqrt{3m + 3} = \sqrt{5m - 1}$
8. $\sqrt{x + 7} = 5$
9. $\sqrt{9 - s} = 5$
10. $\sqrt{p} = -8$

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing

Summarizing Strategies: Learners Summarize & Answer Essential Question

Part 1: Explain how you would solve $\sqrt{2x-3} - \sqrt{x+3} = 0$

Answer: Add $\sqrt{x+3}$ *to each side of the equation. Square each side of the equation then solve for x*

Part 2: Explain why you need to isolate the nth root before raising it to the nth power to remove it. Consider the problem.

Answer: When the radical is isolated squaring will remove it. If it is not isolated then squaring it will simply produce another radical term in the equation.

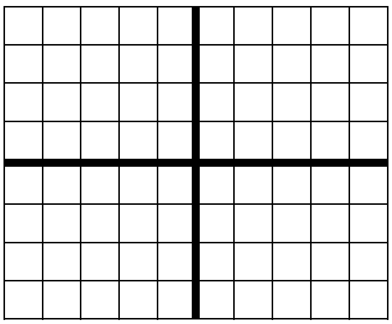
Homework: Solving Functions (attachment 5.3.6)

Attachment 5.3.6

Solving Functions

$F(x)= \frac{1}{2} x^3$
$G(x)= \frac{1}{2} x$

1. Graph each equation using your graphing calculator. Draw the graph on your paper. Where do these lines intersect?



2. Explain what these intersections mean in terms of g(x) and f(x)?

Solve Algebraically

$f(x)= \frac{1}{2} x^3$ When will $f(x) = g(x)$
$g(x)= \frac{1}{2} x$

Student Practice: Find where f(x) = g(x)

1. $f(x)=\sqrt{x-1}$ $g(x)=x-3$

2. $f(x)=x^3+2x^2$ $g(x)= -2x$

3. $f(x)= 3x^2-2$ $g(x)= 2x+3$

4. $f(x)= x^3-7$ $g(x)= \sqrt{x-1}$

5. $f(x)= x^2-3x$ $g(x)= x+2$

6. $f(x)= -\sqrt{x+10}$ $g(x)= x-7$

Solving Functions

(Answer Key)

$F(x)= \frac{1}{2} x^3$
$G(x)= \frac{1}{2} x$

1. Graph each equation using your graphing calculator. Where do these lines intersect?
These lines intersect at (-1, -1/2) (0,0) (1, ½)
2. Explain what these intersections mean in terms of g(x) and f(x)?
These ordered pairs satisfy both equations.

Solve Algebraically

$\frac{1}{2} x^3= \frac{1}{2} x$
$\frac{1}{2} x^3-\frac{1}{2} x=0$
$\frac{1}{2} x(x^2-1)=0$
$\frac{1}{2} x(x+1)(x-1)=0$
$\frac{1}{2} x=0$ or $x+1=0$ or $x-1=0$
$x= 0$ $x= -1$ $x= 1$

Student Practice: Find where f(x) = g(x)

1. $f(x)=\sqrt{x-1}$ $g(x)= x-3$
$\sqrt{x-1} = x - 3$
$(\sqrt{x-1})^2 = (x-3)^2$
$x - 1 = x^2 - 6x + 9$
$0= x^2 - 7x + 10$
$0 = (x - 5)(x - 2)$
$x = 5$ or $x = 2$
*Answer $x = 5$

2. $f(x)= 2x^3+4x^2$ $g(x)= -2x$
$x^3+2x^2 = -2x$
$2x^3+4x^2 + 2x = 0$
$2x(x^2+2x + 1) = 0$
$x (x +1)(x+1)= 0$
$x = 0$ or $x = -1$

3. $f(x)=3x^2-2$ $g(x)=2x^2+7$
$3x^2 - 2 = 2x^2 + 7$
$x^2 - 9 = 0$
$(x+3)(x - 3) = 0$
$x = -3$ or $x = 3$

4. $F(x)= x-7$ $g(x)=\sqrt{x-1}$
$x-7 = \sqrt{x-1}$
$(x-7)^2 = (\sqrt{x-1})^2$
$x^2 - 14x + 49 = x - 1$
$x^2 - 15x + 50 = 0$
$(x - 5)(x - 10) = 0$
$x = 5$ or $x = 10$
*Answer is $x = 10$

5. $f(x)= x^2-4x$ $g(x)= -(x+2)$
$x^2 - 4x = -(x + 2)$
$x^2 - 3x + 2 = 0$
$(x - 2)(x - 1) = 0$
$x = 2$ or $x = 1$

6. $F(x)= -\sqrt{2x + 10}$ $g(x)= x - 7$
$-\sqrt{2x + 10} = x - 7$
$(-\sqrt{2x + 10})^2 = (x - 7)^2$
$2x + 10 = x^2 - 14x + 49$
$0 = x^2 - 16x + 39$
$0 = (x - 3)(x - 13)$
$x = 3$ or $x = 13$
*Answer $x = 3$

*When you raise both sides of an equation to an even power, the resulting equation may have a solution that is not a solution of the original equation. This is called an extraneous solution of the equation.

Acquisition Lesson Plan
Math 1 Unit 5 Concept 3 Radical Equations
Key Standards MM1A3b
Time Allotted: 2 hours

Session 12

Standards(s): MM1A3b: Students will solve simple equations: Solve equations involving radicals suc as $\sqrt{x} + b = c$ using algebraic techniques

Essential Question:

How would you solve an equation with radicals algebraically?

Activating Strategies: (Learners Mentally Active) –
OPENING

Warm -up

1. 2/3x + 3y = 10y – 1
2. ½ (4x + 2) < 1/3(3x + 9)
3. 1.7m + 16.8 ≥ 25.8 – 0.55m

Acceleration/Previewing: (Key Vocabulary)
Domain
Range
Restricted Domain
Restricted Range
Extraneous solutions

Review key vocabulary from Math 1 Unit 2 Concept 4:
Radical expression
Radical
Index (pre-view cub roots/fourth roots, etc only)
Rationalizing denominators

Previewing Vocabulary – use graphic organizer (see attached) day before class for students who need acceleration (those who may be in a math support class)

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)
WORK SESSION

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Solve the equations. Check for extraneous solutions.

$\sqrt{f} = 7$ $\sqrt{-x} = 5$ $\sqrt{g} - 6 = 3$ $\sqrt{5a} + 2 = 0$ $\frac{\sqrt{d}}{3} = 4$

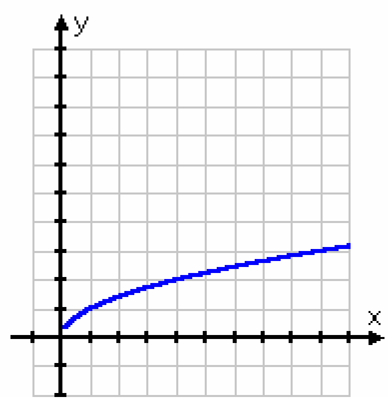
$\sqrt{5p} = 10$ $\sqrt{4y} = 6$ $\sqrt{2c - 1} = 5$ $\sqrt{3k - 2} = 4$ $x = \sqrt{x + 2}$

Summarizing Strategies: Learners Summarize & Answer Essential Question

Quiz:

1. Find the domain and range of the following radical equation. Put your answer in set notation.
   $\sqrt{2x - 5} = -7$

2. State the domain and range of the following graph:



Solve each equation and check your solution:

3. $\sqrt{x} + 5 = 8$
4. $\sqrt{m + 3} + 2 = 6$
5. $\sqrt{n + 2} = n - 4$

Session 13

Unit 5 Concept 3 Assessment Form NAME ________________

Math 1 DATE ________ Period ___

Section 1 – Skills

Factor the following completely.

1. $x^2 - 5x + 6$
2. $x^2 + 8x + 12$
3. $x^2 + 3x - 18$

Solve the following equations

4. $x^2 + 7x - 30 = 0$
5. $x^2 + 12x + 20 = 0$
6. $x^2 - 11x - 12 = 0$

Solve the following equations.

7. $\sqrt{x} - 3 = 2$
8. $\sqrt{-x} + 6 = 13$
9. $\frac{3}{\sqrt{2}} + \sqrt{x} = \frac{5\sqrt{2}}{2}$
10. $16 + \sqrt{x} = 20$

Section 2 – Application

11. Suppose that you had a room where the width of the room was 5 units more than the length. Write the length and width of the rooms as two algebraic expressions and find the product of them to have an expression representing the area of the room.

12. If the length of the room was 16 ft, what would the area of the room be?

13. Suppose that the width of the room was increased by an additional 4 units. Write an expression representing the new area (product of the room).

14. If the length of the room was 12 ft, what would the area of the room be?

15. Suppose that the area of a room is given in the expression $x^2 - 4x$ where 'x' represents the length of the room. Find the algebraic expression for the width of the room.

16. Suppose that you are shipping vases in a square shaped box that contains 'x' rows and columns of vases in each box. If you increased the length of the box to allow 3 more vases per row and increased the width of the box to add 1 extra column per box, write an algebraic expression representing the amount of vases in each box.

17. Suppose that a room has an area with the expression $x^2 + 10x + 16$. Use factorization to find expressions representing the length and width of the room.

Acquisition Lesson Plan
Math 1 Unit 5 Concept 4 Rational Equations
Key Standards MM1A3d
Time Allotted: 2 hours

Essential Question: Session 14
How do variables in the denominator impact solutions when solving rational equations?
Activating Strategies: (Learners Mentally Active) **K-W-L (WHAT DO YOU KNOW-WHAT DO YOU WANT TO KNOW-WHAT DID YOU LEARN)** **Work only the K and W (Attachment 1)** **About rationals, quadratics, linear equations. (students should have prior knowledge on quadratics; rational numbers-adding, subtracting, multiplying, and dividing rational expressions; and linear equations)** **Acceleration/Previewing: (Key Vocabulary)** **Excluded Values**
Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers) **Direct Instruction on excluded values (what makes the denominator equal to zero) with guided practice.** **Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)** **Independent Practice on excluded values in the denominator of an expression & simple equations.**
Summarizing Strategies: Learners Summarize & Answer Essential Question **Finish KWL (L section)**

Rational Equations

What We Know	What We Want to Find Out	What We Learned	How Can We Learn More

Acquisition Lesson Plan
Math 1 Unit 5 Concept 4 Rational Equations
Key Standards MM1A3a,d
Time Allotted: 6 hours

Essential Question: Session 15
How would you solve a rational equation that can be simplified into a linear or quadratic equation?
Activating Strategies: (Learners Mentally Active) **OPENING**
Acceleration/Previewing: (Key Vocabulary) **Rational Equation, restricted domain** **Direct Instruction & Guided Practice on rational equations with <u>numerical denominators.</u>** **Review Work Problems using attachments 5.4.1, 5.4.2, 5.4.3, 5.4.4, 5.4.5** **OR** **Review Unit 5 Framework Problem 1 from Shadows and Shapes Task**
Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers) **WORK SESSION**
Work in pairs. **Reference the Flow Chart (attachment 5.4.7)** **Using "Work Done Problems – Set 2" (attachment 5.4.5)** **Choose 5 of the 14 problems to work.** **OR** **Work Unit 5 Framework "Shadows and Shapes Task" Problems 2 & 4** **<u>STUDENTS WILL SHARE OUT THEIR ANSWERS WITH WHOLE CLASS.</u>** **Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)** **What has to be true before you can add or subtract rational expressions with denominators?** **How can you simplify a rational equation by removing denominators?** **How do variables in denominators affect domains**
Summarizing Strategies: **CLOSING**
Learners Summarize & Answer Essential Question **Use Flow chart to summarize these sessions. (attachment 5.4.7) Have students use "paired-heads" to fill in flow chart or create flow chart. See following pages for flow chart example.**
HOMEWORK
Practice Finding LCD (attachment 5.4.8) **Or "Shadows and Shapes Task" from Frameworks Unit 5 problems 6&7**

Work Done Problems

This involves finding the rate or the time required for doing certain things when two or more individuals or objects work together. The time required to complete a job together is always less than the time required for the fastest object to complete the job alone.

The direction below can be used for solving problems involving rate of doing some job.

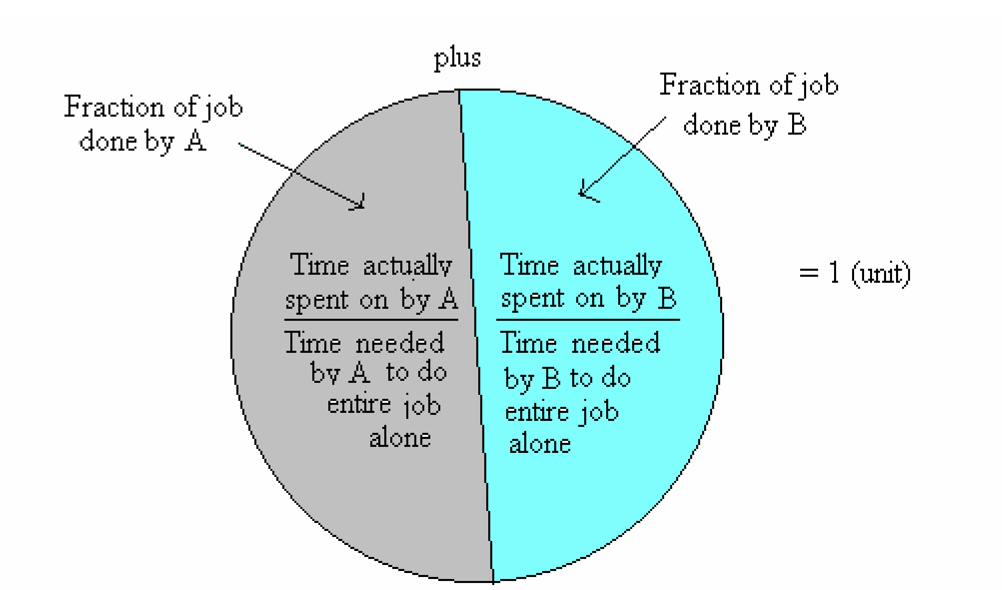
First, find the fractional part of the job done by each individual, or each object. The fractional part of the job done by each individual is equal to the amount of time the individual actually worked or spent on the job *divided by* the amount of time required to do the entire job alone.

Fractional part of job done by each individual = $\dfrac{\text{Time actually spent on doing the job}}{\text{Time needed to do entire job alone}}$

Time actually spent on doing the job = time spent with other plus time spent alone.

1. Find the relationship between the fractional parts. Usually, the sum of each fractional part over all the individuals equals one.

2. The unit "one" represents the entire job. Set up an equation and solve for the unknowns.

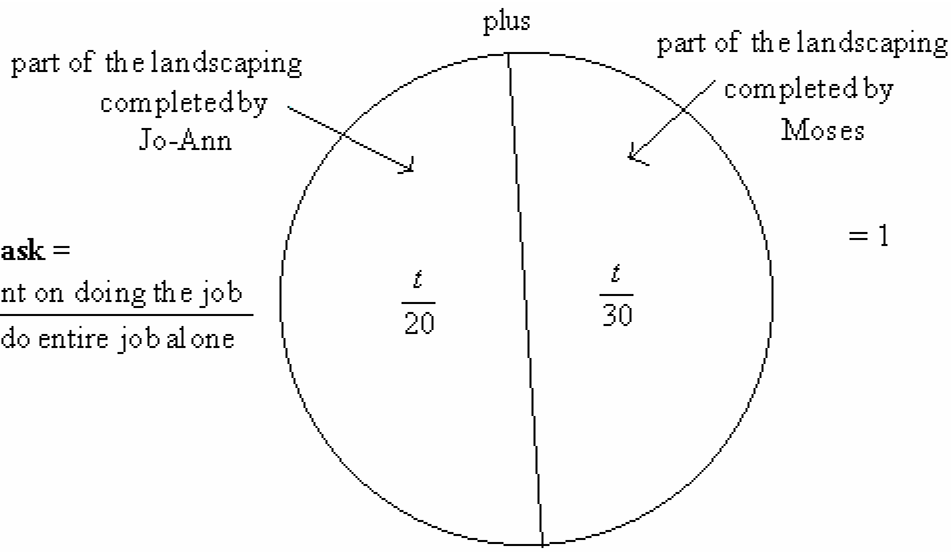


Attachment 5.4.2

EXAMPLE 1

Jo-Ann Newgen can landscape Ami's yard by himself in 20 hours. Moses Cray can landscape the same yard by himself in 30 hours. How long will it take them to landscape the yard if they work together?

Solution:
Let *t* = the time, in hours, for both Jo-Ann and Moses to landscape the yard together. We will construct a circle to help us in finding the part of the task completed by Jo-Ann and Moses in *t* hours.



Part of Task = $\dfrac{\text{Time actually spent on doing the job}}{\text{Time needed to do entire job alone}}$

$$\frac{t}{20} \quad + \quad \frac{t}{30} \quad = \quad 1$$

Now multiply each term of the equation by LCD (20, 30) = 60

$$\mathbf{60}\left(\frac{t}{20}+\frac{t}{30}\right) = \mathbf{60} \cdot 1$$

$$3t + 2t = 60$$

$$5t = 60$$
$$t = 12$$

Therefore, it will take 12 hour to complete the entire job if they work together.

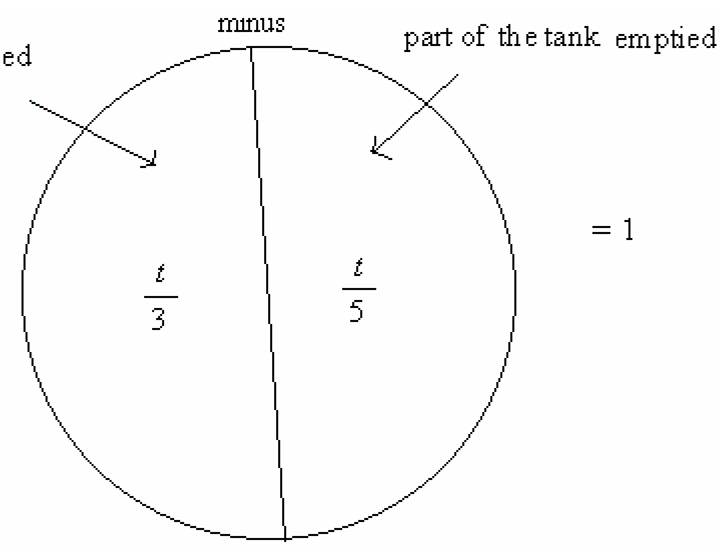
EXAMPLE 2 **Attachment 5.4.3**

At the Spring Hill Water Treatment Plant, one pipe can fill water tank in 3 hours and another pipe can empty it in 5 hours. If the valves to both pipes are open, how long will it take to fill the tank?

Solution:

Let t = amount of time to fill the tank with both valves open.

Part of Task = $\dfrac{\text{Time actually spent on doing the job}}{\text{Time needed to do entire job alone}}$



As one pipe is filling, the other is emptying the tank. Thus, the pipes are working against each other. Therefore instead of adding the parts of the task, as was done in the previous example where the people worked together, we will subtract the parts of the task.

$$\frac{t}{3} - \frac{t}{5} = 1$$

Now multiply each term of the equation by LCD (3, 5) = 15

$$\mathbf{15}\left(\frac{t}{3} - \frac{t}{5}\right) = 15 \bullet 1$$

$$5t - 3t = 15$$

$$2t = 15 \quad \text{so that,} \quad t = 7\frac{1}{2}$$

Therefore, it will take $7\frac{1}{2}$ hours to fill the tank tire job if the valves to both pipes are open.

Alternative method: *Attachment 5.4.4*

Pipe	**Rate of Work**	**Time Actually Worked** (hours)	**Part of Task =** $\frac{\text{Time actually spent on doing the job}}{\text{Time needed to do entire job alone}}$
Pipe filling tank	$\frac{1}{3}$	t	$\frac{t}{3}$
Pipe emptying tank	$\frac{1}{5}$	t	$\frac{t}{5}$

As one pipe is filling, the other is emptying the tank. Thus, the pipes are working against each other. Therefore instead of adding the parts of the task, as was done in the previous example where the people worked together, we will subtract the parts of the task.

$\begin{pmatrix}\text{part of tank} \\ \text{filled in t hours}\end{pmatrix}$ - $\begin{pmatrix}\text{part of tank} \\ \text{emptied in t hours}\end{pmatrix}$ = 1 (total tank filled)

$$\frac{t}{3} - \frac{t}{5} = 1$$

$$\mathbf{15}\left(\frac{t}{3} - \frac{t}{5}\right) = 15 \bullet 1$$

$5t - 3t = 15$

and $2t = 15$

so that, $t = 7\frac{1}{2}$

The tank will be filled in $7\frac{1}{2}$ hours.

Attachment 5.4.5

Work Done Problems – Set 2

a) Tell what variables and expressions represent.

b) Write equation for solving and show work.

c) Write a sentence explaining the answer.

1. Winnie can do a job in 3 hours, and Rita needs 5 hours. How long will it take if they work together?

2. An inlet pipe can fill an empty tank in 6 hours. An outlet pipe can empty the tank in 8 hours. Through an error both pipes are left on. How long will it take for the tank to fill?

3. Jennifer and Mike want to paint Jennifer's studio apartment. Jennifer can do it in 5 hours, and Mike can do it in 6 hours. How long will it take them working together?

4. Nicola can paint a room in 8 hours when working alone. If Gail helps her, the total job takes 6 hours. How long can it take Gail if she worked alone?

5. Jeannette and Ivan are refinishing an antique chest. Working alone, Jeannette can do the job in 12 hours. When Ivan helps, the total job takes only 8 hours. How long will it take Ivan if he worked alone?

6. A bathtub can be filled with water in 20 minutes. The drain can empty it in 30 minutes. How long will it take to fill the bathtub if both the water is on and the drain is open?

7. A cold-water tap can fill a sink in 8 minutes, and a hot-water tap can fill it in 12 minutes. How long would it take to fill the sink with both taps open?

8. It will take Lull 6 days to clear her lot. Her son, Nike, can do the same job in 5 days. How long will it take them to clear the lot if they work together?

9. It takes a small pipe three times longer to fill a gasoline tank than a larger pipe and together they can fill it in 6 hours, how long would it take the larger pipe to fill the tank?

10. David can process an order twice as fast as Landa, and working together they can do the work in 3 hours. How long would it take each working together?

11. A man and his son worked for 3 hours painting his garage. Then the boy quit and the man finishes the job in 45 minutes. If the man can paint it alone in 6 hours, how long will it take the boy to paint it working alone?

12. A steam shovel capable of making a certain excavation in four days broke down after 1.5 days on the job. How long will it take a smaller machine to finish the job if it can make the whole excavation in 6 days?

13. A tank can be filled by one pipe in 4 hours and emptied by another in 5 hours. The water is allowed to run for 2 hours with both pipes open and then the outlet is closed. How long will it take the tank to fill?

14. Mike and Jennie drove to their grandmother's place the past Thanksgiving holiday. If Mike drove the entire trip, the trip would take 10 hours. Jennie is a faster driver. If Jennie drove the entire trip, the trip would take 8 hours. After Mike had been driving for 4 hours, Jennie took over the driving. How long will it take Jennie to reach their final destination?

Application problem for monomial denominators: Attachment 5.4.6

1. An airplane flies 480 miles against the wind to another city and then returns with the wind. The speed of the wind is a constant 15 miles per hour.
   a. Write an equation that gives the total flying time t (in hours) as a function of the airplane's average speed r (in miles per hour) in still air.
   b. Find the total flying time if the airplane's average speed in still air is 300 miles per hour

Application problem for binomial denominators:

2. You were always told that if you start counting when you see a bolt of lightning and stop counting when you hear the thunder, you could figure out how far the storm is from you. Each second represents a mile's distance between you and the storm.

In science class, however, you learned that the time between seeing the lightning and hearing thunder was not only a function of distance, but also of temperature.

The time (t) between seeing the lightning and hearing the thunder = $\dfrac{d}{1.09T + 1050}$.

In this equation, d is the distance you are from the lightening in feet and T is the temperature in Fahrenheit.

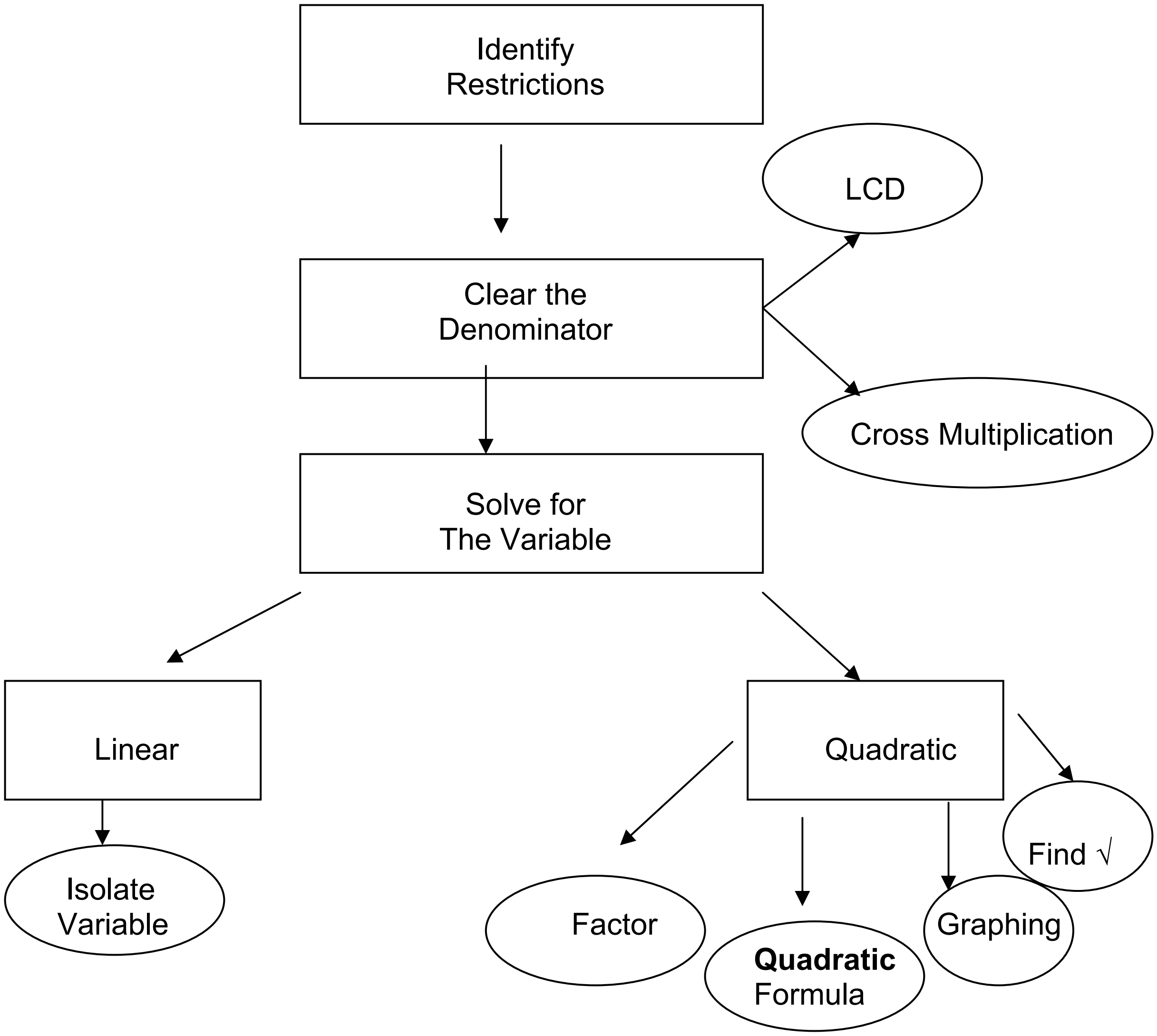
Suppose you know t = 10 seconds and d = 2 miles. What is the temperature in Fahrenheit?

(Solution is 5.5 degrees)

3. 8/(x-5) = 3/x

4. (-4)/(a-4) = 3 - a/(a-4)

5. Solving Rational Equations Flow Chart Attachment 5.4.7



Attachment 5.4.8

Practice Finding the LCD of rational expressions.

Expression	LCD	Solution
$\frac{x}{5} + \frac{x}{5}$		5
$\frac{2x}{3} + \frac{3}{5}$		15
$\frac{2x}{4} + \frac{x}{3}$		12
$\frac{3x}{8} + \frac{5x}{4}$		8
$\frac{3}{x} + \frac{4}{x}$		X
$\frac{3}{x} + \frac{4}{5}$		5x
$\frac{3}{x} + \frac{2}{7x}$		7x
$\frac{3}{x+1} + \frac{4}{5}$		5(x +1) or 5x + 5
$\frac{3}{X-2} + \frac{2}{x}$		X(x – 2) or x^2 – 2x
$\frac{3}{x+1} + \frac{3}{x-2}$		(x +1)(x – 2) or x^2 – x – 2

Acquisition Lesson Plan
Math 1 Unit 5 Concept 4 Rational Equations
Key Standards MM1A3d
Time Allotted: 6 hours

Essential Question: Session 16
How would you solve a rational equation that can be simplified into a linear or quadratic equation?
Activating Strategies: (Learners Mentally Active) **OPENING**
Acceleration/Previewing: (Key Vocabulary) **Rational Equation, restricted domain** **Direct Instruction & Guided Practice on rational equations with monomial denominators. Discuss restrictions on domain.** Direct Instruction on ohms: Unit 5 Frameworks Resistance Learning Task pg.56-57 Guide students through problems 1,2,3 of Resistance Learning Task
Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers) **WORK SESSION** **Students work in pairs on Resistance Learning Task Problems 4 & 5**
Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)
Summarizing Strategies: Learners Summarize & Answer Essential Question **CLOSING**
How do variables in the denominator affect the solution of a problem?

Acquisition Lesson Plan
Math 1 Unit 5 Concept 4 Rational Equations
Key Standards MM1A3d
Time Allotted: 6 hours

Essential Question: Session 17
How would you solve a rational equation that can be simplified into a linear or quadratic equation?
Activating Strategies: (Learners Mentally Active) **OPENING**
Acceleration/Previewing: (Key Vocabulary) **Rational Equation, restricted domains** **Have students graph (x^2 – 1)/(x + 1)** **Sketch the graph.** **What does the graph look like?** **What does y equal when x = -1? Why do you think this is?** **Direct Instruction & Guided Practice on solving rational equations that have binomial factors in the denominator.** **Ex) 4/(a – 2) = 4** **Guide students through problem 6 of Resistance Learning Task**
Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers) **WORK SESSION**
Students work in pairs on Resistance Learning Task problem 7 **STUDENTS SHARE THEIR WORK WITH THE CLASS.** **Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)**
Summarizing Strategies: **Quiz** **Attachment 5.4.11**

QUIZ 5.4.11

Name_____________________________________ Period _______ Date ____________________

A marketing executive traveled 810 miles on a corporate jet in the same amount of time that it took to travel an additional 162 miles by helicopter. The rate of the jet was 360 mph greater than the rate of the helicopter. Find the rate of the jet.

	Distance	**Rate**	**Time**
Jet			
Helicopter			

Acquisition Lesson Plan
Math 1 Unit 5 Concept 4 Rational Equations
Key Standards MM1A3d
Time Allotted:
6 hours

Essential Question: Session 18
How would you solve a rational equation that can be simplified into a linear or quadratic equation?
Activating Strategies: (Learners Mentally Active) **OPENING**
Acceleration/Previewing: (Key Vocabulary) **Rational Equation, restricted domain** **Direct Instruction & Guided Practice on solving rational equations that factor and cancel out during the process of solving the radical equation.** **Ex) $(x^2 + 2x + 2)/(x - 1) = (2x + 3)/(x - 1)$** **Ex) $x/(x+2) = 6/(x+5)$**
Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers) **WORK SESSION**
Students work individually on related type problems from textbook. **They share their work with a partner and make needed revisions.** **Students share their work with the class.** **Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)**
Summarizing Strategies: Learners Summarize & Answer Essential Question **Ticket out the Door** **Solve and show your work. $x/(x-2) = 3/(x-4)$**

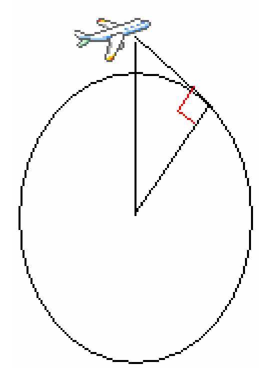
Acquisition Lesson Plan
Math 1 Unit 5 Concept 4 Rational Equations
Key Standards MM1A3d
Time Allotted: 6 hour

Essential Question: Session 19 - 21
Activating Strategies: (Learners Mentally Active) **OPENING**
Today you will begin your Culminating task for Unit 5 – Fairfield Aviation **Remember to: a) Define all variables and expressions** **b) Justify all work (Zero Property, Multiplicative prop. Of Equality)** **c) Write solutions using complete sentences and using the language of the standards.**
Acceleration/Previewing: (Key Vocabulary)
Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers) **WORK SESSION**
“Fairfield Aviation Culminating Task” **Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)**
Summarizing Strategies: Learners Summarize & Answer Essential Question **CLOSING**

Culminating Task: Fairfield Aviation

Name __________________________________Period_______Date______________

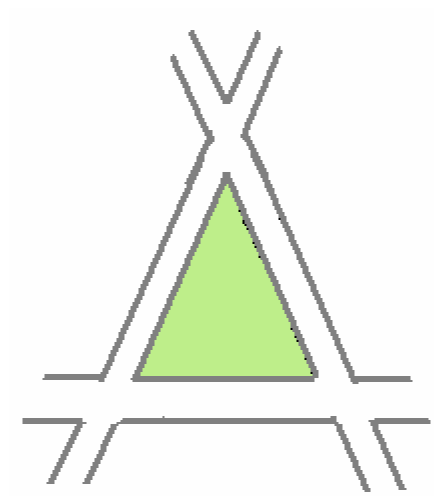
1. Fairfield Aviation specializes in providing sightseeing flights, charter flights, and flight instruction. Their sightseeing flights always have a concern of visibility since customers will demand refunds if they do not have ample scenic views. When the weather is clear, the line of sight from the plane to the furthest visible point on the ground makes a right angle with a radius of the Earth drawn to that point, as shown in the figure at the right, where the height of the plane relative to the radius of the Earth is greatly exaggerated to keep the drawing small and still represent the relationships. For the items below, where applicable, assume that the radius of the Earth is 3963 miles.



   a. At the altitude of 1.52 miles above the Earth, how far to the nearest mile can passengers see to the horizon?

   b. When the plane is on its return to the airport and has an altitude of 4000 feet, by how many miles is the sight distance reduced from the sight distance at 8000 feet (1.52 miles)?

2. There is a place at the airport at which Fairfield Aviation offers flight instruction where the area bordered by three taxiways has the shape of an equilateral triangle, as shown in the figure at the right. Student pilots sometimes forget about this area, and there have been a few times when students almost collided with another plane. The airport has asked Fairfield Aviation to find a solution. Airport documents indicate that the area enclosed by the taxiways is 27063 square feet. Fairfield Management has decided that marking the area with a short reflective fence is the best solution. They want to fence an equilateral triangle whose sides are, to the nearest foot, 10 feet shorter than the sides of the actual equilateral triangular area. Using shorter sides will allow the fence to be erected inside the triangular area with the fence near but not touching the edge of each taxiway.



   a. Let x equal to a side of the equilateral triangle. Express the height (altitude) in terms x.

   b. Solve for x using the area formula. Use the expression from part a for h.

   c. How much fence should they buy to enclose the desired area?

3. One of the popular charter flight options is a vacation package offered by Fairfield Aviation in conjunction with a major resort. Fairfield requires a minimum of 20 people to schedule the trip, and the resort limits the number of participants to a maximum of 50. If there are 20 people, the charge is $1500 per person, but there is a price reduction of $25 for **each additional person.** If the charge for a particular group booking this option was $37,500, how many people were in the group? Justify your answer algebraically and graphically. (Hint: After obtaining your quadratic equation, divide both sides of your equation by your leading coefficient.)

4. One afternoon two student pilots who are taking lessons from Fairfield Aviation instructors left the airport at the same time flying in opposite directions. Each flew at an airspeed of 175 miles per hour. One flew in the same direction in which the wind was blowing; the other was flying directly into the wind.
   a. Express the time it took the student pilot flying with the wind to fly 80 miles as a function of the wind speed.

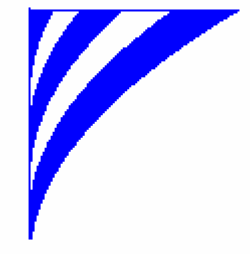
   b. Express the time it took the student pilot flying against the wind to fly 60 miles as a function of wind speed.

   c. What is a reasonable domain for the functions in parts a and b?

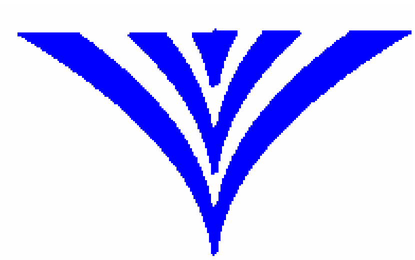
   d. The instructors had their students check in with the tower at the same time. At that time, the student flying with the wind had flown 80 miles while the student flying against the wind had flown 60 miles. What is the speed of the wind? Justify your answer algebraically and graphically. Explain how the graphs of the functions in parts a and b are related to the question of wind speed.

5. One day a Fairfield Aviation pilot flew a charter to deliver an organ transplant from Atlanta to Jackson, MS. For the 350 mile flight from Atlanta to Jackson and for the return flight, the pilot flew at an average air speed of 200 miles per hour. The return flight with the wind took only 70% of the time for the flight from Atlanta to Jackson into the wind, and the wind speed was consistent over the time period for both flights. To the nearest mile per hour, what was the speed of the wind?

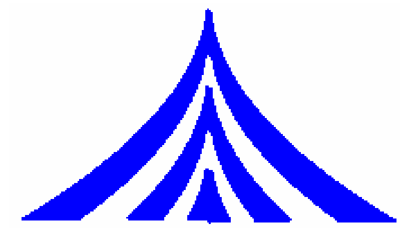
6. Fairfield Aviation has contracted with a graphics design firm to create a new company logo. The graphic designer, Petra, has proposed three possible designs for the logo, each based on one basic shape. This shape is shown below as Petra has proposed that it appear in the logo and as she created it in the coordinate system using translations of the graph of the function $y = \sqrt{x}$.



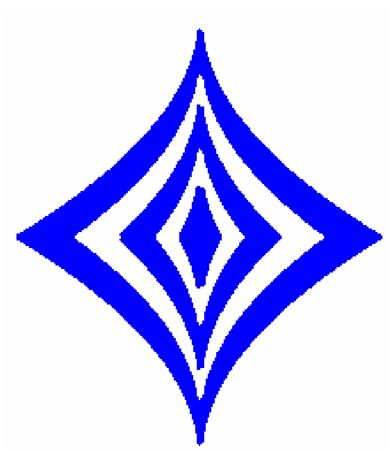
a. The three proposed logos are shown below. Discuss the symmetry of each shape.



Shape A



Shape B



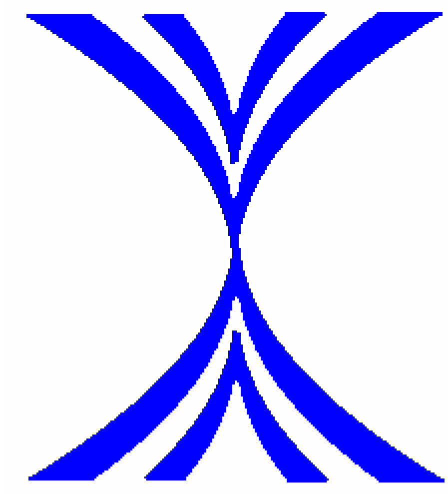
Shape C

b. The mathematical formula for the basic shape uses the portion of the graph of the equation $y = \sqrt{x}$, and of some vertical shifts of this graph, bounded on the left by the *y*-axis and above by a horizontal line, as shown in Petra's graph above. Write the mathematical definition that creates the sections of the basic shape and indicate how the domains of the square root function and its vertical shifts should be restricted so that the graph does not extend above the bounding horizontal line.

c. What transformation of the basic shape is required to create the other half of Shape A? Write a mathematical definition for Shape A.

d. What transformation of Shape A is required to create Shape B? Write a mathematical definition for shape B.

e. The CEO of Fairfield Aviation did not want any of the proposed shapes for the logo but saw how to modify Shapes A and B and combine them to make the logo design shown at the right. Create a mathematical definition for this logo.

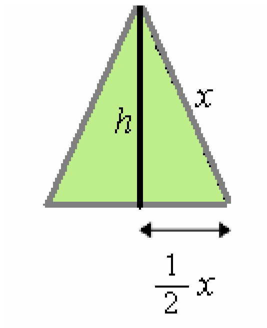


Math I Unit 5 Algebra in Context
Teacher Notes:

1. The students should be allowed to use graphing calculators for this test.

2. Eliminate part b of number 1 for math support students or if time is a factor.

3. For problem 2, you may want to provide the math support students with the following diagram.



4. For problem 3, provide the following statement to the math support students.
   Let x = the number of people beyond 20.

5. For problem 4, provide the Beaufort Wind Scale information to all students, so that they can apply the domain to the real world problem. May wish to provide additional help to math support students or give partial credit for domain of less than 175.
6. For problem 4, provide the correct window settings for the graphing calculator to the math support class. Remind the other students to consider the algebraic solution when setting the window dimensions.
7. Problem 5 can be omitted if necessary for time constraints. If not omitted, then you may wish to explain the meaning of the word charter to the math support class.
8. For problem 7b, provide a chart to fill in answers for math support class.

Mathematical Definition of Basic Shape			
Square Roots of Graphs (Equations)	Restriction of Domain	Boundary Lines	Restrictions on the domain and on the range
		x =	$\le$ x $\le$
		y =	$\le$ y $\le$

9. Omit 7e for all students except accelerated students.

Rubric for Fairfield Aviation Culminating Task

Task Value	Expectations	Point Earned
1. a. 4 points b. 4 points (optional)	a. * Find the correct number of miles to horizon. b. * Find the number of miles sight distance is given new altitude.	
2. a. 4 points b. 4 points c. 4 points	a. * Express height in terms of x. b. * Solve for x (side length of equilateral triangle). c. * Determine amount of fence needed.	
3. 12 points	a. * Find how many people in the group. * Justify answer algebraically and graphically.	
4. a. 4 points b. 4 points c. 4 points d. 6 points	a. *Express time as a function of wind speed for 80 miles. b. *Express time as a function of wind speed for 60 miles. c. *Provide reasonable domain for a and b. d. *Find speed of wind. *Justify algebraically and graphically. *Explain how the graphs of part a and b related to the question of wind speed.	
5. 12 points	a. *Find the speed of the wind.	
6. a. 2 points b. 10 points c. 6 points d. 6 points e. 12 points (optional)	a. * Describe the symmetry of each shape. b. * Write mathematical functions that create the basic shape. * Indicate how the domain and range of each function should be restricted. c. * Describe the transformation of the basic shape to create the other half of shape A. * Write mathematical functions that create shape A. d. *Describe the transformation of the shape A to create the other half of shape B. *Write mathematical functions that create shape B. e. *Write mathematical function that creates the given logo.	