Acquisition Lesson Planning Form
Key Standards addressed in this Lesson: MM2A3a, MM2A3b, MM2A3c
Time allotted for this Lesson: 5 hours

Essential Question: LESSON 1 – QUADRATIC FUNCTIONS
How do you analyze and graph quadratic functions of the forms $f(x) = ax^2 + bx + c$ (standard form) and $f(x) = a(x - h)^2 + k$ (vertex form)?
Activating Strategies: (Learners Mentally Active)
Use GO #1 Families of Quadratic Functions graphic organizer as a review of quadratic concepts from Math 1. Students work in collaborative pairs to complete the organizer, The teacher might need to discuss the y-intercept if the students were not taught this in Math 1. They should do the problems by hand, and check with a graphing calculator. Discuss with the entire group Use #2 – #5 of the Protein Bar Learning Task, Part 1 as activating strategies/review problems on a subsequent day.
Acceleration/Previewing: (Key Vocabulary)
Quadratic Function, standard form, vertex form, horizontal shift, vertical shift, reflection, vertical stretch, vertical shrink, vertex, axis of symmetry, domain, range, zeros, intercepts, extrema, intervals of increasing and decreasing, and rates of change (slope). Have the students make foldables with these words. Students can brainstorm all the things they remember from last year about these words.
Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)
Note: Much of this is a review from Math 1. Review characteristics of the quadratic function, factoring methods for quadratics, simplifying square roots, and solving quadratics by factoring and by extracting roots. Use the graphic organizers #2 - #4 as needed for your class. • Pairs complete parts 1- 6 of "Henley's Chocolates Learning Task." (Page 20) Choose pairs to explain each part. Ones share with twos the patterns noticed in "Henley's Chocolates Learning Task" #5 a-f. • Small groups of 3-4 work on #6 and #7 of "The Protein Bar Toss Learning Task, Part 1". (Page 23) • A mini lesson on factoring trinomials with coefficients of x^2 other than 1 and factoring four terms by grouping is needed before the students reach #8 in the task. The Matching Factors Activity beginning on page 10 can be used to reinforce the concepts of factoring. • Groups complete "The Protein Bar Toss, Part 1". Then share parts with the entire class. • Pairs complete "The Protein Bar Toss, Part 2, Learning Task," #1 - #8. (Page 29) Pairs should explain each part.

- Mini lesson, if needed, using Graphic Organizer #2: Properties of a Function, to illustrate the characteristics of a function, particularly stressing intervals of increase and decrease.
- Pairs complete #9 from the Protein Bar Toss, Part 2, Learning Task. Discuss with the entire group.
- Mini lesson using Graphic Organizers #5 and #6 on converting between forms before completing part #10 from the Protein Bar Toss, Part 2, Learning Task.
- Pairs complete the remainder of The Protein Bar Learning task. Think-Pair-Share. Share answers and answer questions.

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

What are the differences between the transformations of f(x) when graphing
a. f(-x) and –f(x);
b. f(x + a) and f(x) + a

How does the height a ball rises when thrown into the air correspond to the equation of the function representing this motion?

If it takes a ball t seconds to rise to its peak when tossed into the air, how long does it take before it hits the ground?

If the point (-3, 5) is on the graph of a quadratic function with axis of symmetry x = 1, what is another point on the quadratic function?

Extending/Refining Strategies:

Complete parts 7-10 of "Henley's Chocolates Learning Task."

Task: Protein Bar Toss (Question #13) Using what students have learned, students work in pairs to complete #13-15 to extend knowledge on solving quadratic equations algebraically in real-life situations.

Summarizing Strategies: Learners Summarize & Answer Essential Question

Ticket out the door. Teacher will put 2 quadratic equations such as $y = x^2$ and $y = (x + 3)^2$ on the board, and have students explain in writing the transformation that took place.

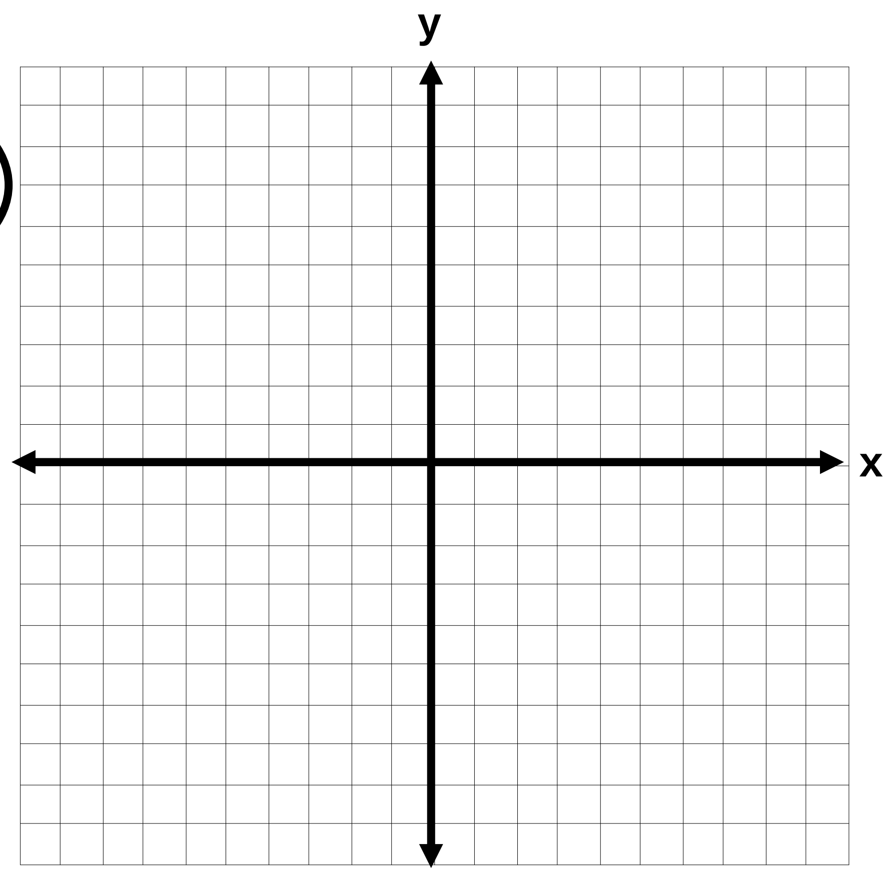
Other TODs as needed on pages 20 and 21.

GO #1: Do You Remember?

Families of Quadratic Functions

Complete the table below for each of the indicated functions and draw each in a different color on the graph to the right. As you graph each function, discuss the following questions with your partner:

1. How are the lines alike?
2. How are they different?
3. What transformation is occurring?



$f(x) = x^2$	
x	f(x)
-3	
-2	
-1	
0	
1	
2	
3	
y-int =	

$f(x) = x^2 - 2$	
x	f(x)
-3	
-2	
-1	
0	
1	
2	
3	
y-int =	

$f(x) = (x - 2)^2$	
x	f(x)
-3	
-2	
-1	
0	
1	
2	
3	
y-int =	

$f(x) = \frac{1}{2}x^2$	
x	f(x)
-3	
-2	
-1	
0	
1	
2	
3	
y-int =	

$f(x) = -2x^2$	
x	f(x)
-3	
-2	
-1	
0	
1	
2	
3	
y-int =	

$f(x) = (-x + 2)^2$	
x	f(x)
-3	
-2	
-1	
0	
1	
2	
3	
y-int =	

.

GO #2: Properties of a Function:

Domain:

f(x) = ___________

Range:

Maximum:

Reflection:

Minimum:

Increasing:

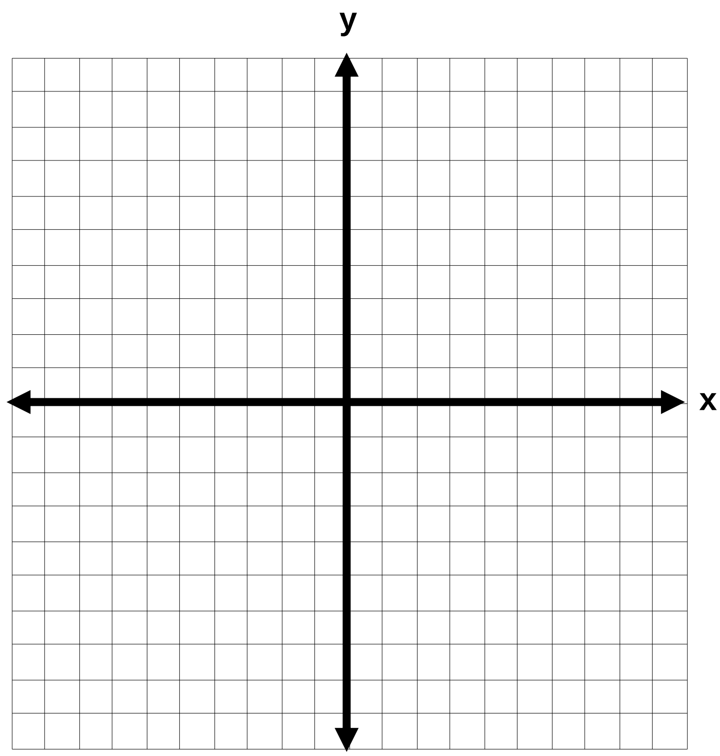
Decreasing:

What is the x-intercept?

What is the y-intercept?

GO #3: Parent Function: f(x) = _________

f(x) = _____	
x	f(x)



Describe:

VOCABULARY

x-intercepts (zeros)

Domain	Range

y-intercept

End Behavior

Intervals of Increase/Decrease

Max or Min

GO #4: Exploring Quadratic Functions -- Transformations

How do the transformations relate to the parent graph?

	$y = x^2 + 2$	$y = x^2 - 2$	$y = 2x^2$	$y = \frac{1}{2}x^2$	$y = -x^2$
How does the graph change?					
What is the range?					
What is the domain?					
What is the end behavior?					
What is the maximum / minimum?					
Identify the intervals for which the function is increasing / decreasing.					
What are the intercepts ?					

GO # 5: Quadratic Functions:

Vertex Form: $f(x) = a(x - h)^2 + k$ Standard Form: $f(x) = ax^2 + bx + c$

Converting Quadratic Equations from standard form into vertex form:

Standard Form: Factor the coefficient of x^2 from the first two terms.

$f(x) = -2x^2 + 16x - 27$

Take half the coefficient of x and square it. Add it inside the parentheses.

$= -2 (x^2 + 8x) - 27$

Add the opposite of that **value** to the constant.

$= -2(x^2 + 8x + 16) - 27 + 32$

Factor the trinomial, write it as a quantity squared, combine terms and you have Standard Form

$= =2(x + 4)^2 + 5$

GO #6: Quadratic Functions

Vertex Form: $f(x) = a(x - h)^2 + k$ Standard Form: $f(x) = ax^2 + bx + c$

Converting Quadratic Equations from vertex form into standard form:

Vertex Form: Square the binomial.

$f(x) = -2(x - 4)^2 + 5$

Distribute the coefficient of the trinomial..

$= -2(x^2 - 8x + 16) + 5$

Combine like terms.

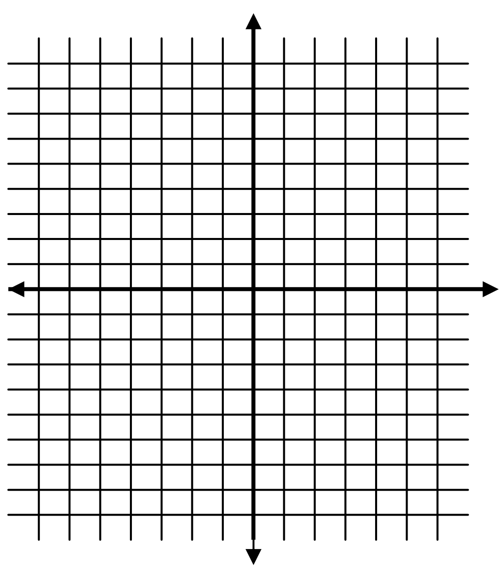
$= -2x^2 + 16x - 32 + 5$

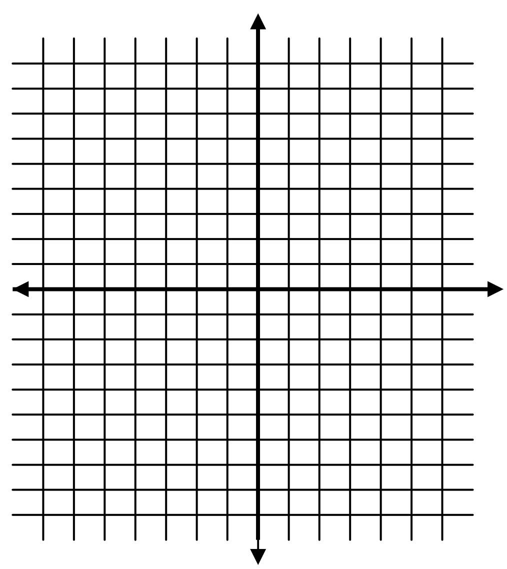
$= -2x^2 + 16x - 27$

Standard Form

GO #7: How do you graph quadratic functions?

	Vertex Form $f(x) = a(x-h)^2 + k$	Standard Form $f(x) = ax^2 + bx + c$
	$f(x) = 2(x + 5)^2 - 4$	$f(x) = -\frac{1}{2}x^2 - 2x + 1$
1. Find the vertex 2. Find and sketch the axis of symmetry. 3. Find two points on one side of the axis of symmetry. 4. Use symmetry to find two points on the opposite side of the axis of symmetry. 5. Connect with a smooth curve.		





Matching Factors Activity

Match the factors with the special product. Copy the pages onto cardstock, laminate them and then cut out the cards before use. You could use different colored cards for different sets so they can be kept separate easily if you wish. You might want to put the factors on one color and the products on a second color. The two sets of cards can be matched to practice multiplying polynomials and/or factoring quadratic expressions. This can be done by small groups of students, collaborative pairs, or by individual students working alone.

Polynomial Set 1

4x + 6	**3x – 9**	**6x – 8**
2x – 16	**8x + 24**	**3x – 36**
6x – 40	**4x – 44**	**12x + 80**
4x – 8	**9x – 9**	**3x – 12**
8x – 42	**12x – 60**	**9x - 12**
9x – 6	**16x – 4**	**12x – 8**

Factors Set 1

2(2x + 3)	**3(x – 3)**	**2(3x – 4)**
2(x – 8)	**8(x + 3)**	**3(x – 12)**
2(3x – 20)	**4(x – 11)**	**4(3x + 20)**
4(x – 2)	**9(x – 1)**	**3(x – 12)**
2(4x – 21)	**12(x – 5)**	**3(3x – 4)**
3(3x – 6)	**4(4x – 1)**	**4(3x – 2)**

Polynomial Set 2

$4x^2 – 1$	$4x^2 – 9$	$4x^2 – 25$
$4x^2 – 49$	$4x^2 – 81$	$9x^2 - 25$
$9x^2 – 1$	$9x^2 – 4$	$9x^2 – 16$
$x^2 – 10x + 25$	$4x^2 + 4x + 1$	$4x^2 – 4x + 1$
$9x^2 + 6x + 1$	$9x^2 – 6x + 1$	$4x^2 + 12x + 9$
$4x^2 – 12x + 9$	$9x^2+30x+25$	$16x^2–24x+9$

Factors Set 2

(2x+1)(2x–1)	**(2x+3)(2x-3)**	**(2x+5)(2x-5)**
(2x+7)(2x–7)	**(2x+9)(2x–9)**	**(3x+5)(3x-5)**
(3x+1)(3x-1)	**(3x+2)(3x-2)**	**(3x+4)(3x-4)**
$(x - 5)^2$	$(2x + 1)^2$	$(2x - 1)^2$
$(3x + 1)^2$	$(3x - 1)^2$	$(2x + 3)^2$
$(2x - 3)^2$	$(3x + 5)^2$	$(4x + 3)^2$

$9x^2$-30x+25	$16x^2$+8x+1	$16x^2$-8x+1
$16x^2$-24x+9	$16x^2$+40x+25	$16x^2$-40x+25
$16x^2$ – 1	$16x^2$ – 9	$16x^2$ – 25
$25x^2$ – 1	$25x^2$ – 4	$25x^2$ - 9
$25x^2$ – 16	$36x^2$ – 1	$36x^2$ – 25
$64x^2$ – 1	$64x^2$ – 9	$64x^2$ - 25

Factors Set 3

$(3x – 5)^2$	$(4x + 1)^2$	$(4x - 1)^2$
$(4x – 3)^2$	$(4x + 5)^2$	$(4x – 5)^2$
$(4x+1)(4x-1)$	$(4x+3)(4x-3)$	$(4x+5)(4x-5)$
$(5x+1)(5x-1)$	$(5x+2)(5x-2)$	$(5x+3)(5x-3)$
$(5x–4)(5x+4)$	$(6x-1)(6x+1)$	$(6x-5)(6x+5)$
$(8x-1)(8x+1)$	$(8x+3)(8x-3)$	$(8x-5)(8x+5)$

Polynomials Set 4

$12x^2 + 35x + 18$	$15x^2 + 26x - 24$	$15x^2 – 23x + 4$
$3x^2 + 14x – 24$	$5x^2 – 7x - 12$	$2x^2 – 19x + 24$
$10x^2 – 13x + 4$	$15x^2 – 2x - 8$	$2x^2 – 5x - 18$
$15x^2 – 34x - 16$	$3x^2 – x - 24$	$21x^2 – 22x - 24$
$9x^2 + 18x + 8$	$18x^2 + 27x + 4$	$15x^2 + 2x - 24$
$10x^2 + 13x - 30$	$21x^2 + 10x - 16$	$15x^2 – 19x - 10$

Factors Set 4

(3x – 4)(x + 6)	**(5x –12)(x + 1)**	**(x – 8)(2x – 3)**
(5x – 4)(2x – 1)	**(5x – 4)(3x + 2)**	**(2x – 9)(x + 2)**
(3x – 8)(5x + 2)	**(3x + 8)(x – 3)**	**(7x –12)(3x + 2)**
(3x + 4)(3x + 2)	**(3x + 4)(6x + 1)**	**(3x + 4)(5x – 6)**
(2x + 5)(5x – 6)	**(7x + 8)(3x – 2)**	**(5x + 2)(3x – 5)**
(4x + 9)(3x + 2)	**(5x +12)(3x - 2)**	**(3x – 4)(5x – 1)**

Ticket Out the Door #1

Write the special products for each of the following:

$(3x - 4)(3x + 4) =$ ________________________

$(2x - 3)^2 =$ ____________________________

$(3x - 7)^2 =$ ____________________________

$4(x - 5)(2x + 7) =$ ________________________

$(2x - 3)(3x - 8) =$ ________________________

Ticket Out the Door #2

Write the factors for each of the following:

$9x^2 - 25 =$ ________________________

$25x^2 + 10x + 1 =$ ____________________________

$3x^2 - 18x - 81 =$ ____________________________

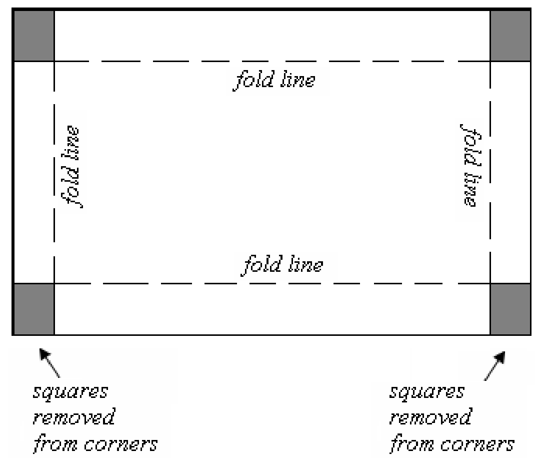
$3x^2 - 5x + 2 =$ ______________________

$15x^2 - 34x + 16 =$ ____________________

Henley's Chocolates Learning Task

Henley Chocolates is famous for its mini chocolate truffles, which are packaged in foil covered boxes. The base of each box is created by cutting squares that are 4 centimeters on an edge from each corner of a rectangular piece of cardboard and folding the cardboard edges up to create a rectangular prism 4 centimeters deep. A matching lid is constructed in a similar manner, but, for this task, we focus on the base, which is illustrated in the diagrams below.

Sketch 1

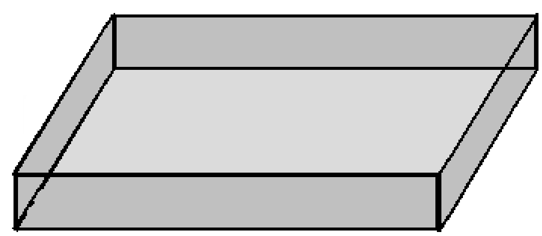


For the base of the truffle box, paper tape is used to join the cut edges at each corner. Then the inside and outside of the truffle box base are covered in foil.

Henley Chocolates sells to a variety retailers and creates specific box sizes in response to requests from particular clients. However, Henley Chocolates requires that their truffle boxes always be **4 cm deep** and that, in order to preserve the distinctive shape associated with Henley Chocolates, the bottom of each truffle box be a rectangle that is **two and one-half times as long as it is wide.**

1. Henley Chocolates restricts box sizes to those Which will hold plastic trays for a whole number of mini truffles. A box needs to be at least 2 centimeters wide to hold one row of mini truffles. Let L denote the length of a piece of cardboard from which a truffle box is made. What value of L corresponds to a finished box base for which the bottom is a rectangle that is 2 centimeters wide?
(Hint: Label dimensions of sketch 1.)

Sketch 2



2. Henley Chocolates has a maximum size box of mini truffles that it will produce for retail sale. For this box, the bottom of the truffle box base is a rectangle that is **50 centimeters long**. What are the dimensions of the piece of cardboard from which this size truffle box base is made?

3. a. Write an expression for the length and width of the rectangular base of the truffle box in sketch 2.

   b. Let *A* denote the area, in square centimeters, of the rectangular bottom of a truffle box base. Write a formula for *A* in terms of the length *L*, in centimeters, of the piece of cardboard from which the truffle box base is constructed.

4. The engineers responsible for box design studied the function *A* on the domain of all real number values of *L* in the interval from the minimum value of *L* **found in item 1** to the maximum value of *L* **found in item 2**. State this interval of *L* values as studied by the engineers at Henley Chocolates.

The next few items depart from Henley Chocolates to explore graph transformations that will give us insight about the function *A* for the area of the bottom of a mini truffle box. We will return to the function *A* in item 9.

5. **Use technology to graph** each of the following functions on the same axes with the graph of the parent function *f* defined by $y = x^2$ (use a standard viewing window). Use a new set of axes for each function listed below, but repeat the graph of *f* each time. For each function listed, describe a rigid transformation of the graph of *f* that results in the graph of the given function. Make a conjecture about the graph of $y = (x - h)^2$, where *h* is any real number.

   a. $y = (x - 3)^2$

   b. $y = (x - 6)^2$

   c. $y = (x - 8)^2$

   d. $y = (x + 2)^2$

   e. $y = (x + 5)^2$

   f. $y = (x + 9)^2$

6. For each pair of functions below, predict how you think the graphs will be related and then **use technology to graph** the two functions on the same axes and check your prediction.

   a. $y = x^2$ and $y = 3x^2$

   b. $y = 3x^2$ and $y = 3(x - 4)^2$

   c. $y = \frac{1}{2}x^2$ and $y = -\frac{1}{2}x^2 + 5$

   d. $y = -0.75x^2$ and $y = 0.75(x + 6)^2$

   e. $y = 2x^2$ and $y = -2(x - 5)^2 + 7$

Now we return to the function studied by the engineers at Henley Chocolates.

7. Using the results from 3c. , A = __________. For graphing purposes, let L=x. Now A = __________. Describe the transformations of $y = x^2$ that will produce the graph of A. **Use technology to graph** $y=x^2$ and your equation for A on the same set of axes.

8. Remember that you found the domain of the function in item 4, adjust your window to fit this domain. What is the range of the function *A*?

9. The engineers at Henley Chocolates responsible for box design have decided on two new box sizes that they will introduce for the next winter holiday season.

   a. The area of the bottom of the larger of the new boxes will be 640 square centimeters. Use the function A to write and solve an equation to find the length L of the cardboard needed to make this new box.

   b. The area of the bottom of the smaller of the new boxes will be 40 square centimeters. Use the function A to write and solve an equation to find the length L of the cardboard needed to make this new box.

10. How many mini-truffles do you think the engineers plan to put in the

    a. large box?

    b. small box?

The Protein Bar Toss Learning Task: Part 1

Blake and Zoe were hiking in a wilderness area. They came up to a scenic view at the edge of a cliff. As they stood enjoying the view, Zoe asked Blake if he still had some protein bars left, and, if so, could she have one. Blake said, “Here’s one; catch!” As he said this, he pulled a protein bar out of his backpack and threw it up to toss it to Zoe. But the bar slipped out of his hand sooner than he intended, and the bar went straight up in the air with his arm out over the edge of the cliff. The protein bar left Blake’s hand moving straight up at a speed of 24 feet per second. If we let t represent the number of seconds since the protein bar left the Blake’s hand and let $h(t)$ denote the height of the bar, in feet above the ground at the base of the cliff, then, assuming that we can ignore the air resistance, we have the following formula expressing $h(t)$ as a function of t.

$$h(t) = -16t^2 + 24t + 160$$

In this formula, the coefficient on the t^2-term is due to the effect of gravity and the coefficient on the t-term is due to the initial speed of the protein bar caused by Blake’s throw. In this task, you will explore, among many things, the source of the constant term.

1. In part 5 of *Exploring Functions with Fiona* , from Math 1 Unit 1, you considered a formula for the distance fallen by an object dropped from a high place. List some ways in which this situation with Blake and the protein bar differs from the situation previously studied.

2. **Use technology to graph the equation** $y = -16t^2 + 24t + 160$. Find a viewing window that includes the part of this graph that corresponds to the situation with Blake and his toss of the protein bar. What viewing window did you select?

3. What was the height of the protein bar, measured from the ground at the base of the cliff, at the instant that it left Blake’s hand? What special point on the graph is associated with this information?

4. If Blake wants to reach out and catch the protein bar on its way down, how many seconds does he have to process what happened and position himself to catch it? Justify your answer graphically and algebraically.

5. If Blake does not catch the falling protein bar, how long it take for the protein bar to hit the ground below the cliff? Justify your answer graphically. Then write a quadratic equation that you would need to solve to justify the answer algebraically.

The equation from item 5 can be solved by factoring, but it requires factoring a quadratic polynomial where the coefficient of the x^2-term is not 1. Our next goal is to learn about factoring this type of polynomial. We start by examining products that lead to such quadratic polynomials.

6. For each of the following, perform the indicated multiplication and use a rectangular model to show a geometric interpretation of the product as area for positive values of *x*.

   a. $(2x + 3)(3x + 4)$

   b. $(x + 2)(4x + 11)$

   c. $(2x + 1)(5x + 4)$

7. For each of the following, perform the indicated multiplication.

   a. $(2x – 3)(9x + 2)$

   b. $(3x – 1)(x – 4)$

   c. $(4x – 7)(2x + 9)$

The method for factoring general quadratic polynomial of the form $ax^2 + bx + c$, with *a, b,* and *c* all non-zero integers, is similar to the method learned in Mathematics I for factoring quadratics of this form but with the value of *a* restricted to *a* = 1. The next item guides you through an example of this method.

8. Factor the quadratic polynomial $6x^2 + 7x – 20$ using the following steps.
   a. Think of the polynomial as fitting the form $ax^2 + bx + c$.

   What is *a?* ____ What is *c*? ____ What is the product *ac*? ____

b. List all possible pairs of integers such that their product is equal to the number *ac*. It may be helpful to organize your list in a table. Make sure that your integers are chosen so that their product has the same sign, positive or negative, as the number *ac* from above, and make sure that you list all of the possibilities.

c. What is *b* in the quadratic polynomial given? _____ Add the integers from each pair listed in part b. Which pair adds to the value of *b* from your quadratic polynomial? We'll refer to the integers from this pair as *m* and *n*.

d. Rewrite the polynomial replacing *bx* with *mx+nx*. [*Note either m or n could be negative; the expression indicates to add the terms mx and nx including the correct sign.*]

e. Factor the polynomial from part d by grouping.

f. Check your answer by performing the indicated multiplication in your factored polynomial. Did you get the original polynomial back?

9. Use the method outlined in the steps of item 8 to factor each of the following quadratic polynomials. Is it necessary to always list all of the integer pairs whose product is *ac*?

a. $2x^2 + 3x – 54$

b. $4w^2 -11w + 6$

c. $3t^2 -13t – 10$

d. $8x^2 + 5x – 3$

e. $18z^2 +17z + 4$

f. $6p^2 – 49p + 8$

10. If you are reading this, then you should have factored all of the quadratic polynomials listed in item 9 in the form (*Ax+B)(Cx+D)*, where the *A, B, C,* and *D* are all integers.

a. Compare your answers with other students, or other groups of students. Did everyone in the class write their answers in the same way? Explain how answers can look different but be equivalent.

b. Factor $24q^2 – 4q – 8$ completely.

c. Show that $24q^2 – 4q – 8$ can be factored in the form (*Ax+B)(Cx+D)* where the *A, B, C,* and *D* are all integers, using three different pairs of factors.

d. How should answers to quadratic factoring questions be expressed so that everyone who works the problem correctly lists the same factors, just maybe not in the same order?

11. If a quadratic polynomial can be factored in the form (*Ax+B)(Cx+D)*, where the *A, B, C,* and *D* are all integers, the method you have been using will lead to the answer, specifically called the correct ***factorization***. As you continue your study of mathematics, you will learn ways to factor quadratic polynomials using numbers other than integers. For right now, however, we are interested in factors that use integer coefficients. Show that each of the quadratic polynomials below cannot be factored in the form (*Ax+B)(Cx+D)*, where the *A, B, C,* and *D* are all integers.

a. $4z^2 + z - 6$

b. $t^2 + 2t + 8$

c. $3x^2 + 15x - 12$

12. Now we return to our goal of solving the equation from item 5. Recall that you solved quadratic equations of the form $ax^2 + bx + c = 9$, with $a = 1$, in Mathematics I. The method required factoring the quadratic polynomial and using the Zero Factor Property. The same method still applies when $a \neq 1$, its just that the factoring is more involved, as we have seen above. Use your factorizations from items 9 and 10 as you solve the quadratic equations below.

a. $2x^2 + 3x – 54 = 0$

b. $4w^2 + 6 = 11w$

c. $3t^2 – 13t = 10$

d. $2x (4x + 3) = 3 + x$

e. $18z^2 + 21z = 4z – 1$

f. $8 – 13p = 6p(6 – p)$

g. $24q2 = 4q + 8$

13. Solve the quadratic equation from item 5. Explain how the solution gives an algebraic justification for your answer to the question.

14. Suppose the cliff had been 56 feet higher. Answer the following questions for this higher cliff.

a. What was the height of the protein bar, measured from the ground at the base of the cliff, at the instant that it left Blake's hand? What special point on the graph is associated with this information?

b. What is the formula for the height function in this situation?

c. If Blake wants to reach out and catch the protein bar on its way down, how many seconds does he have to process what has happened and position himself to catch it? Justify your answer algebraically.

d. If Blake does not catch the falling protein bar, how long it take for the protein bar to hit the ground below the cliff? Justify your answer algebraically.

15. Suppose the cliff had been 88 feet lower. Answer the following questions for this lower cliff.

a. What was the height of the protein bar, measured from the ground at the base of the cliff, at the instant that it left Blake's hand? What special point on the graph is associated with this information?

b. What is the formula for the height function in this situation?

c. If Blake wants to reach out and catch the protein bar on its way down, how many seconds does he have to process what has happened and position himself to catch it? Justify your answer algebraically.

d**.** If Blake does not catch the falling protein bar, how long it take for the protein bar to hit the ground below the cliff? Justify your answer algebraically.

The Protein Bar Toss, Part 2, Learning Task

In the first part of the learning task about Blake attempting to toss a protein bar to Zoe, you found how long it took for the bar to go up and come back down to the starting height. However, there is a question we did not consider: How high above its starting point did the protein bar go before it started falling back down? We're going to explore that question now.

1. So far in Mathematics I and II, you have examined the graphs of many different quadratic functions. Consider the functions you graphed in the Henley Chocolates task and in the first part of the Protein Bar Toss. Each of these functions has a formula that is, or can be put in, the form $y = ax^2 + bx + c$ with $a \neq 0$. When we consider such formulas with domain all real numbers, there are some similarities in the shapes of the graphs. The shape of each graph is called a *parabola*. List at least three characteristics common to the parabolas seen in these graphs.

2. The question of how high the protein bar goes before it starts to come back down is related to a special point on the graph of the function. This point is called the *vertex* of the parabola. What is special about this point?

3. In the first part of the protein bar task you considered three different functions, each one corresponding to a different cliff height. Let's rename the first of these functions as h_1, so that

$$h_1(t) = -16t^2 + 24t + 160..$$

   a. Let $h_2(t)$ denote the height of the protein bar if it is thrown from a cliff that is 56 feet higher. Write the formula for the function h_2.

   b. Let $h_3(t)$ denote the height of the protein bar if it is thrown from a cliff that is 88 feet lower. Write the formula for the function h_3.

   c. Use **technology** to graph all three functions, h_1, h_2, and h_3, on the same axes.

   d. Estimate the coordinates of the vertex for each graph.

   e. What number do the coordinates have in common? What is the meaning of this number in relation to the toss of the protein bar?

   f. The other coordinate is different for each vertex. Explain the meaning of this number for each of the vertices.

4. Consider the formulas for h_1, h_2, and h_3.

   a. How are the formulas different?

   b. Based on your answer to part a, how are the three graphs related? Do you see this relationship in your graphs of the three functions on the same axes? If not, restrict the domain in the viewing window so that you see the part of each graph you see corresponds to the same set of *t*-values.

5. In the introduction above we asked the question: How high above its starting point did the protein bar go before it started falling back down?

   a. Estimate the answer to the question for the original situation represented by the function h_1.

   b. Based on the relationship of the graphs of h_2 and h_3 to h_1, answer the question for the functions h_2 and h_3.

Estimating the vertex from the graph gives us an approximate answer to our original question, but an algebraic method for finding the vertex would give us an exact answer. The answers to the questions in item 5 suggest a way to use our understanding of the graph of a quadratic function to develop an algebraic method for finding the vertex. We'll pursue this path next.

6. For each of the quadratic functions below, find the *y*-intercept of the graph. Then find all the points with this value for the *y*-coordinate.

   a. $f(x) = x^2 – 4x + 9$

   b. $f(x) = 4x^2 + 8x – 5$

   c. $f(x) = -x^2 – 6x + 7$

   d. $f(x) = ax^2 + bx + c;\ a \neq 0$

7. One of the characteristics of a parabola graph is that the graph has a line of symmetry.

a. For each of the parabolas considered in item 6, use what you know about the graphs of quadratic functions in general with the specific information you have about these particular functions to find an equation for the line of symmetry.

b. The line of symmetry for a parabola is called the *axis of symmetry*. Explain the relationship between the axis of symmetry and the vertex of a parabola. Then, find the *x*-coordinate of the vertex for each quadratic function listed in item 6.

c. Find the *y*-coordinate of the vertex for the quadratic functions in item 6, parts a, b, and c, and then state the vertex as a point.

d. Describe a method for finding the vertex of the graph of any quadratic function given in the form $f(x) = ax^2 + bx + c$; $a \neq 0$..

8. Return to height functions h_1, h_2, and h_3.

a. Use the method you described in item 7, part d, to find the exact coordinates of the vertex of each graph.

b. Find the exact answer to the question: How high above its starting point did the protein bar go before it started falling back down?

9. Each part below gives a list of functions. Describe the geometric transformation of the graph of first function that results in the graph of the second, and then describe the transformation of the graph of the second that gives the graph of the third, and, where applicable, describe the transformation of the graph of the third that yields the graph of the last function in the list. For the last function in the list, expand its formula to the form $\mathbf{f(x) = ax^2 + bx + c}$ and compare to the function in the corresponding part of item 6 with special attention to the vertex of each.

a. $f(x) = x^2$; $f(x) = x^2 + 5$ $f(x) = 2x^2 + 5$

b. $f(x) = x^2$; $f(x) = 4x^2$ $f(x) = 4x^2 – 9$ $f(x) = 4(x + 1)^2 - 9$

c. $f(x) = x^2$; $f(x) = -x^2$ $f(x) = -x^2 + 16$ $f(x) = -(x + 3)^2 + 16$

10. For any quadratic function of the form **$f(x) = ax^2 + bx + c$**:

a. Explain how to get a formula for the same function in the form $f(x) = a(x – h)^2 + k$.

b. What do the *h* and *k* in the formula of part a represent relative to the function?

11. Give the **vertex form** of the equations for the functions h_1, h_2, and h_3 and verify algebraically the equivalence with the original formulas for the functions. Remember that you found the vertex for each function in item 8, part a.

12. For the functions given below, put the formula in the vertex form $y = a(x – h)^2 + k$, and give the equation of the axis of symmetry, and describe how to transform the graph of $y = x^2$ to create the graph of the given function.

a. $f(x) = 3x^2 + 12x + 13$

b. $f(x) = x^2 – 7x + 10$

c. $f(x) = -2x^2 + 12x - 24$

13. Make a dashed line for the axis of symmetry, and plot the vertex, *y*-intercept and the point symmetric with the *y*-intercept.

14. Which of the graphs that you drew in item 14 had *x*-intercepts?

a. Find the *x*-intercepts that do exist by solving an appropriate equation and then add the corresponding points to your sketch(es) from item 14.

b. Explain geometrically why some of the graphs have *x*-intercepts and some do not.

c. Explain how to use the vertex form of a quadratic function to decide whether the graph of the function will or will not have *x*-intercepts. Explain your reasoning.