Acquisition Lesson Planning Form
Key Standards addressed in this Lesson: MM2A4b & MM2A4c
Time allotted for this Lesson: 9 hours

Essential Question: LESSON 3 – Solving Quadratic Equations and Inequalities
How do you solve quadratic equations and inequalities both graphically and algebraically?
Activating Strategies: (Learners Mentally Active)
Use collaborative pairs to review factoring trinomials of the form $ax^2 + bx + c = 0$ with $a \neq 1$ and the GCF $\neq 1$.
Acceleration/Previewing: (Key Vocabulary)
Factor by grouping, Zero Factor Property, quadratic function, binomial, quadratic equation, quadratic formula, standard form of quadratic equation, discriminant, radical expression, quadratic inequality
Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)
• Have a place in your room for the word wall. As you reach each term, have it written on a card and let a student place it on the wall. This works well with KWL chart. The first couple of minutes of each class could be used to let the students review the words on the wall. • Mini Lesson on using the graphing calculator to solve equations and inequalities. (Pages 3 & 4) Be sure to input as two separate equations such as $y = 19000$ and $y = 960x - 12x^2$ to solve $19000 = 960x - 12x^2$. Also discuss with students how you know if a quadratic opens up or down, and how that determines if there is a minimum or maximum. Differentiate between what the max/min value is, and where it occurs. • Small groups using guided distributed practice: Do Paula's Peaches Task #1 - #7 (Page 11). Students share results with the class. • Just the Right Border Learning Task. Teacher-led discussion of the problem and part #1. • Mini Lesson on the Quadratic Formula (Page 5) before part #2. Practice problems on page 6 are available for distributed guided practice. Mini Lesson on the Discriminant (Page 7) should be done before part #3. Students work in small groups on the remainder of the Just the Right Border Learning Task #2 and #3 (Page 15). • Students complete parts #5 – 7 from Just the Right Border Learning Task in groups and share with the class.
Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)
• What are some special products and their factoring patterns?

- How can you solve a quadratic equation which will not factor?
- How does the discriminant affect the solutions of a quadratic equation?
- How does the value of a affect the direction of opening of the curve?
- How are “at least” and “at most” represented in the inequality sign?
- How do the zeros of the function determine the intervals of solution?

Extending/Refining Strategies:

Task: Just the Right Border part 4 and alternate part 4 can be assigned homework or guided practice

Task: Just the Right Border parts # 8 - 9 can be completed as time permits

Compare and contrast the solution of a quadratic equation to a quadratic inequality?

Complete applications of quadratic inequalities worksheet

Summarizing Strategies: Learners Summarize & Answer Essential Question

Quick Write using terms discussed from Word Wall.

Ticket Out the Door: students solve a quadratic inequality both graphically and algebraically

TOD Page 8

TOD You Decide Model (Pages 9 & 10)

The Graphing Calculator, Equations, and Inequalities

Part 1: Solving Equations with the Graphing Calculator:

Problem 1

$4x^2 - 6x - 18 = 0$

Procedure

Problem 2

$8 - 4(x - 3)^2 = 12 - 4x$

Part 2: Solving Inequalities with the Graphing Calculator:

Problem 1

$4x^2 - 6x < 18$

$9 - 3(x - 5) \geq 6 - x$

Procedure

Problem 2

$4x^2 > 25$

$-2x^2 - 5x \leq 7$

How do you solve quadratic inequalities?

Algebraic Method	Graphing Method

Solve each of the following algebraically. Check your solution graphically.

1. $x^2 - 5x + 6 \geq 0$
2. $x^2 - 11x + 5 \leq 0$
3. $-x^2 - 9x + 36 > 0$
4. $-3x^2 + x + 7 < 0$

Quadratic Formula

If $ax^2 + bx + c = 0$,
then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Procedure:

$6x^2 + 4x - 2 = 0$	$3x^2 = 6x - 2$	$16 = 3x - 4x^2$

Practice Problems for the Quadratic Formula

1. $6x^2 + x = 15$

2. $2x^2 = 12x - 18x$

3. $x^2 - 6x + 10 = 0$

4. $2x^2 + 8x = 5$

5. $-2x^2 - 5x + 6 = 0$

6. $8x^2 + 6x = 9$

7. $3x^2 = 4x – 5$

8. $6 – 5x = -2x^2$

What does the discriminant of a quadratic equation tell us about the solutions?

D (discriminant) = $b^2 - 4ac$

$4x^2 - 2x + 5 = 0$	If D ___ 0, then the equation has _________________________ roots.
$3x^2 - 6x + 3 = 0$	If D ___ 0, then the equation has _________________________ roots.
$5x^2 + 7x + 2 = 0$	If D is a ___________________ and D ___ 0, then the equation has _________________________ roots.
$3x^2 - 4x - 5 = 0$	If D is a ___________________ and D ___ 0, then the equation has _________________________ roots.

TOD Name: ____________ Date: ______

Solve by factoring:

$4x^2 - 1 = 0$

$2x^3 - 3x^2 - 20x = 0$

Solve by extracting roots:

$6x^2 - 48 = 0$

$-2(x - 3)^2 = -48$

TOD Name: ____________ Date: ______

Given the discriminant, determine the number and type of roots for the quadratic equation:

Discriminant	Number of Solutions	Type of Solutions Use all the terms below which apply: Rational, Irrational, Complex, Conjugates
19		
-42		
0		
25		

Ticket Out the Door   Name: ________________

YOU DECIDE!

Choose one of the following problem sets to work. Show all steps you take. The point value is above each problem set.

Level 3 (30 points)

Solve by graphing	Solve algebraically
$3x^2 + 8x + 9 < 0$	$-4(3x - 1)^2 + 9 > 0$

Level 2 (20 Points)

Solve by graphing	Solve algebraically
$6x^2 + 7x \geq 20$	$x^2 + 10x + 25 < 81$

Level 1 (10 Points)

Solve by graphing	Solve algebraically
$4x^2 - 64 > 0$	$2x^2 - 18 \leq 0$

Ticket Out the Door Name: ________________

YOU DECIDE!

Choose one of the following problem sets to work. Show all steps you take. The point value is above each problem set.

Level 3 (30 points)

Solve by Factoring	Solve by Extracting Roots
$3x^2 + 8x + 9 = 0$	$-4(3x - 1)^2 + 9 = 0$

Level 2 (20 Points)

Solve by Factoring	Solve by Extracting Roots
$6x^2 + 7x = 20$	$(x + 5)^2 = 81$

Level 1 (10 Points)

Solve by Factoring	Solve by Extracting Roots
$4x^2 - 64 = 0$	$18 - 2x^2 = 0$

Paula's Peaches Learning Task: Part 2

In this task, we revisit Paula, the peach grower who wanted to expand her peach orchard last year. In the established part of her orchard, there are 30 trees per acre with an average yield of 600 peaches per tree. Data from the local agricultural experiment station indicated that if Paula chose to plant more than 30 trees per acre in the expanded section of orchard, when the trees reach full production several years from now, the average yield of 600 peaches per tree would decrease by 12 peaches per tree for each tree over 30 per acre. In answering the questions below, remember that the data is expressed in averages and does mean that each tree produces the average number of peaches.

1. Let x be the number of trees Paula might plant per acre in her new section of orchard and let $Y(x)$ represent the predicted average yield in peaches per acre. Write the formula for the function Y. Explain your reasoning, and sketch a graph of the function on an appropriate domain.

2. Paula wanted to average at least as many peaches per acre in the new section of orchard as in the established part.

   a. Write an inequality to express the requirement that, for the new section, the average yield of peaches per acre should be at least as many peaches as in the established section.

   b. Solve the inequality graphically. Your solution should be an inequality for the number of trees planted per acre.

   c. Change your inequality to an equation, and solve the equation algebraically. How are the solutions to the equation related to the solution of your inequality?

3. Suppose that Paula wanted to a yield of at least 18,900 peaches.

   a. Write an inequality to express the requirement of an average yield of 18,900 peaches per acre.

b. Solve the equation $x^2 - 80x + 1575 = 0$ by factoring.

c. Solve the inequality from part a. Explain how and why the solutions from part b are related to the solution of the inequality?

Solving $x^2 - 80x + 1575 = 0$ by factoring required you to find a pair of factors of 1575 with a particular sum. With a number as large as 1575, this may have taken you several minutes. Next we explore an alternative method of solving quadratic equations that applies the vertex form of quadratic functions. The advantages of this method are that it can save time over solving equations by factoring when the right factors are hard to find and that it works with equations involving quadratic polynomials that cannot be factored over the integers.

4. Consider the quadratic function $f(x) = x^2 - 80x + 1575$.

a. What are the *x*-intercepts of the graph? Explain how you know.

b. Rewrite the formula for the function so that the *x*-intercepts are obvious from the formula.

c. There is a third way to express the formula for the function, the vertex form. Rewrite the formula for the function in vertex form.

d. Use the vertex form and take a square root to solve for the *x*-intercepts of the graph. Explain why you should get the same answers as part a.

e. Explain the relationship between the vertex and the *x*-intercepts.

5. Consider the quadratic equation $x^2 + 4x - 3 = 0$.

a. Show that the quadratic polynomial $x^2 + 4x - 3$ cannot be factored over the integers.

b. Solve the equation by using the vertex form of the related quadratic function and taking a square root.

c. Approximate the solutions to four decimal places and check them in the original equation.

d. How are the *x*-intercepts and axis of symmetry related? Be specific.

6. Suppose that Paula wanted to grow at least 19000 peaches.

a. Write an inequality for this level of peach production.

b. Solve the inequality graphically.

c. Solve the corresponding equation algebraically. Approximate any non-integer solutions to four decimal places. Explain how to use the solutions to the equation to solve the inequality.

7. Suppose that Paula wanted to grow at least 20000 peaches.

a. Write an inequality for this level of peach production.

b. What happens when you solve the corresponding equation algebraically?

c. Solve the inequality graphically.

d. Explain the connection between parts b and c.

The method you used to solve the equations in items 5 and 6 above can be applied to solve: $ax^2+bx+c=0$ for any choice of real number coefficients for *a, b,* and *c* as long as $a \neq 0$. When this is done in general, we find that the solutions have the form $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$. And $x = \frac{-b-\sqrt{b^2-4ac}}{2a}$. This result is called the quadratic formula and usually stated in summary form as follows.

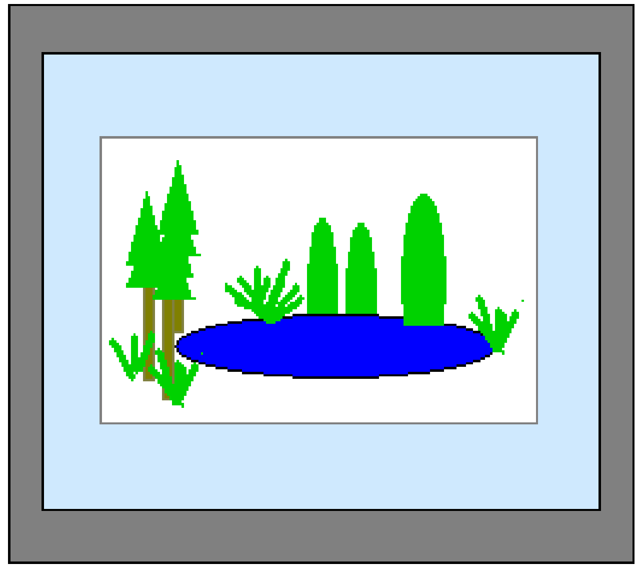
The Quadratic Formula: If $a\,x^2+bx+c=0$ with $a \neq 0$, then

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

Using the Quadratic Formula is more straightforward and, hence, more efficient than the method of putting the quadratic expression in vertex form and solving by taking square roots. An upcoming task, *Just the Right Border Learning Task,* will explore using the Quadratic Formula to solve quadratic equations.

Just the Right Border Learning Task

1. Hannah is an aspiring artist who enjoys taking nature photographs with her digital camera. Her mother, Cheryl, frequently attends estate sales in search of unique decorative items. Last month Cheryl purchased an antique picture frame that she thinks would be perfect for framing one of Hannah's recent photographs. The frame is rather large, so the photo needs to be enlarged. Cheryl wants to mat the picture. One of Hannah's art books suggest that mats should be designed so that the picture takes up 50% of the area inside the frame and the mat covers the other 50%. The inside of the picture frame is 20 inches by 32 inches. Cheryl wants Hannah to enlarge, and crop if necessary, her photograph so that it can be matted with a mat of uniform width and, as recommended, take up 50% of the area inside the mat. See the image at the right.



   a. Let x denote the width of the mat for the picture. Write an equation in x that models this situation.

   b. Put the equation from part a in the standard form $ax^2 + bx + c = 0$. Can this equation be solved by factoring over the integers?

   c. The quadratic formula can be used to solve quadratic equations that cannot be solved by factoring over the integers. Using the simplest equivalent equation in standard form, identify a, b, and c from the equation in part b and find $b^2 – 4ac$ then substitute these values in the quadratic formula to find the solutions for x. Give exact answers for x and approximate the solutions to two decimal places.

   d. To the nearest tenth of an inch, what should be the width of the mat and the dimensions for the photo enlargement?

2. The quadratic formula can be very useful in solving problems. Thus, it should be practiced enough to develop accuracy in using it and to allow you to commit the formula to memory. Use the quadratic formula to solve each of the following quadratic equations, even if you could solve the equation by other means. Begin by identifying *a*, *b,* and *c* and finding $b^2 - 4ac$; then substitute these values into the formula.

a. $4z^2 + z - 6 = 0$

e. $7x^2 = 10x$

b. $t^2 + 2t + 8 = 0$

f. $\frac{t}{2} + \frac{7}{t} = 2$

c. $3x^2 + 15x = 12$

g. $3(2p^2 + 5) = 23p$

d. $25w^2 + 9 = 30w$

h. $12z^2 = 90$

3. The expression $b^2 - 4ac$ in the quadratic formula is called the **discriminant** of the quadratic equation in standard form. All of the equations in item 2 had values of *a*, *b*, and *c* that are rational numbers. Answer the following questions for quadratic equations in standard form when ***a*, *b*, and *c* are rational numbers**. Make sure that your answers are consistent with the solutions from item 2.

a. What is true of the discriminant when there are two real number solutions to a quadratic equation?

b. What is true of the discriminant when the two real number solutions to a quadratic equation are rational numbers?

c. What is true of the discriminant when the two real number solutions to a quadratic equation are irrational numbers?

d. Could a quadratic equation with rational coefficients have one rational solution and one irrational solution? Explain your reasoning.

e. What is true of the discriminant when there is only one real number solution? What kind of number do you get for the solution?

f. What is true of the discriminant when there is no real number solution to the equation?

4. There are many ways to prove the quadratic formula. One that relates to the ideas you have studied so far in this unit comes from considering a general quadratic function of the form $f(x) = ax^2 + bx + c$ putting the formula for the function in vertex form, and then using the vertex form to find the roots of the function. Such a proof does not require that *a*, *b*, and *c* be restricted to rational numbers; *a*, *b*, and *c* can be any real numbers with $a \neq 0$. Why is the restriction $a \neq 0$ needed?

Alternate item 4 There are many ways to show why the quadratic formula always gives the solution(s) to any quadratic equation with real number coefficients. You can work through one of these by responding to the parts below . Start by assuming that each of *a*, *b*, and *c* is a real number, that $a \neq 0$, and then consider the quadratic equation $ax^2 + bx + c = 0$..

a. Why do we assume that $a \neq 0$?

b. Form the corresponding quadratic function, $f(x) = ax^2 + bx + c$ and put the formula for $f(x)$ in vertex form, expressing k in the vertex form as a single rational expression.

c. Use the vertex form to solve for *x*-intercepts of the graph and simplify the solution. Hint: Consider two cases, $a > 0$ and $a < 0$, in simplifying $\sqrt{a^2}$.

5. Use the quadratic formula to solve the following equations with real number coefficients. Approximate each real, but irrational, solution correct to hundredths.

a. $x^2 + \sqrt{5}\,x + 1 = 0$

b. $3t^2 + 11 = 2\sqrt{33}\,t$

c. $3q^2 - 5q + 2\pi = 0$

d. $9w^2 = \sqrt{13}\,w$

6. Verify each answer for item 5 by using a graphing utility to find the *x*-intercept(s) of an appropriate quadratic function.

   a. Put the function for item 5, part c, in vertex form. Use the vertex form to find the *t*-intercept.

   b. Solve the equation from item 5, part d, by factoring.

7. Answer the following questions for quadratic equations in standard from where *a*, *b*, and *c* are real numbers.

   a. What is true of the discriminant when there are two real number solutions to a quadratic equation?

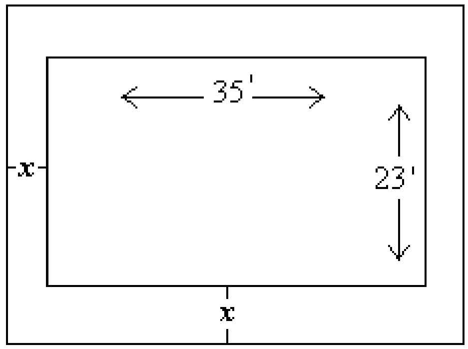
   b. Could a quadratic equation with real coefficients have one rational solution and one irrational solution? Explain your reasoning.

   c. What is true of the discriminant when there is only one real number solution?

   d. What is true of the discriminant when there is no real number solution to the equation?

   e. Summarize what you know about the relationship between the determinant and the solutions of a quadratic of the form $ax^2 + bx + c = 0$ where *a*, *b*, and *c* are real numbers with $a \neq 0$ into a formal statement using bi-conditionals.

8. A landscape designer included a cloister (a rectangular garden surrounded by a covered walkway on all four sides) in his plans for a new public park. The garden is to be 35 feet by 23 feet and the total area enclosed by the garden and walkway together is 1200 square feet. To the nearest inch, how wide does the walkway need to be?



9. In another area of the park, there will be an open rectangular area of grass surrounded by a flagstone path. If a person cuts across the grass to get from the southeast corner of the area to the northwest corner, the person will walk a distance of 15 yards. If the length of the area is 5 yards more than the width, to the nearest foot, how much distance is saved by cutting across rather than walking along the flagstone path?