# MATH 2

# UNIT 1

## Quadratics and Complex Numbers

# Georgia Performance Standards
# High School Mathematics
# Mathematics 2

| Georgia Performance Standards: Curriculum Map | | | | | |
|---|---|---|---|---|---|
| 1st Semester | | | 2nd Semester | | |
| | | | | | |
| Unit 1 | Unit 2 | Unit 3 | Unit 4 | Unit 5 | Unit 6 |
| Quadratics & Complex Numbers | Right Triangle Trigonometry | Circles & Spheres | Data Analysis & Probability | Step and Piecewise Functions | Linear and Quadratic Regression |
| 6 Weeks | 4 Weeks | 6 Weeks | 5 Weeks | 6 Weeks | 4 Weeks |
| MM2N1<br>MM2A3<br>MM2A4 | MM2G1<br>MM2G2 | MM2G3<br>MM2G4 | MM2D1 | MM2A1b<br>MM2A2<br>MM2A5 | MM2A1a, c<br>MM2D2 |
| | | | | | |

These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units. Standards listed are key standards for the units. All units will include the Process Standards.

# MATH 2 UNIT 1 CONTENT MAP
# QUADRATICS AND COMPLEX NUMBERS

**Unit 1 – Quadratics & Complex Numbers (6 Weeks)**

**Essential Questions: How do you analyze and graph quadratic functions in both standard and vertex forms? How do you solve quadratic equations and inequalities in one variable? How do you simplify and operate with complex numbers? How do you relate finite arithmetic series to quadratic functions?**

**Lesson 1 – Quadratic Functions (5 hours)**
**Essential Question:** How do you analyze and graph quadratic functions of the forms $f(x) = ax^2 + bx + c$ (standard form) and $f(x) = a(x – h)^2 + k$ (vertex form)?

**Lesson 2 – Complex Numbers (5 hours)**
**Essential Question:** How do you represent and operate using complex numbers?

**Lesson 3 – Quadratic Equations and Inequalities (9 hours)**
**Essential Question:** How do you solve quadratic equations and inequalities graphically and algebraically and describe their solutions?

**Lesson 4 – Finite Arithmetic Series and Relationship to Quadratic Functions (4 hours)**
**Essential Question:** What are finite arithmetic series, how do you calculate their sums, and how do they relate to quadratic functions?

**Summarization, Review, & Evaluation of Unit 1 (7 Hours)**

# Mathematics 2 – Unit 1: Quadratic Functions

**INTRODUCTION:**

Students are first introduced to quadratic functions in Mathematics 1, where they study characteristics of the basic function $f(x) = x^2$ and learn to solve simple quadratic equations that can be put in the form $x^2 + bx + c = 0$. Through exploration of many real world situations which are represented by quadratic functions, students extend their previous study to include in-depth analysis of general quadratic functions in both standard form, $f(x) = ax^2 + bx + c$, and vertex form, which is introduced in this unit. Students extend their knowledge of solving quadratic equations through factoring and learn the quadratic formula, which can be used to solve any quadratic equation. Study of the quadratic formula introduces the complex numbers so students learn about the arithmetic of complex numbers. Students make connections between algebraic results and characteristics of the graphs of quadratic function and apply this understanding in solving quadratic inequalities. In addition they explore sums of terms of arithmetic sequences as examples of quadratic functions. This work provides a foundation for modeling data with quadratic functions, a topic that will be explored later in Mathematics II.

**ENDURING UNDERSTANDINGS:**

- The graph of any quadratic function is a vertical and/or horizontal shift of a vertical stretch or shrink of the basic quadratic function $f(x) = x^2$.
- The vertex of a quadratic function provides the maximum or minimum output value of the function and the input at which it occurs.
- Every quadratic equation can be solved using the Quadratic Formula.
- The discriminant of a quadratic equation determines whether the equation has two real roots, one real root, or two complex conjugate roots.
- The complex numbers are an extension of the real number system and have many useful applications.

**KEY STANDARDS ADDRESSED:**

**MM2N1. Students will represent and operate with complex numbers.**

a. Write square roots of negative numbers in imaginary form.
b. Write complex numbers in the form $a + bi$.
c. Add, subtract, multiply, and divide complex numbers.
d. Simplify expressions involving complex numbers.

**MM2A3. Students will analyze quadratic functions in the forms $f(x) = ax^2 + bx + c$ and $f(x) = a(x - h)^2 + k$.**

a. Convert between standard and vertex form.
b. Graph quadratic functions as transformations of the function $f(x) = x^2$.
c. Investigate and explain characteristics of quadratic functions, including domain, range, vertex, axis of symmetry, zeros, intercepts, extrema, intervals of increase and decrease, and rates of change.
d. Explore arithmetic series and various ways of computing their sums.
e. Explore sequences of partial sums of arithmetic series as examples of quadratic functions.

**MM2A4. Students will solve quadratic equations and inequalities in one variable.**

a. Solve equations graphically using appropriate technology.
b. Find real and complex solutions of equations by factoring, taking square roots, and applying the quadratic formula.
c. Analyze the nature of roots using technology and using the discriminant.
d. Solve quadratic inequalities both graphically and algebraically, and describe the solutions using linear inequalities

**RELATED STANDARDS ADDRESSED:**

**MM2P1. Students will solve problems (using appropriate technology).**

a. Build new mathematical knowledge through problem solving.
b. Solve problems that arise in mathematics and in other contexts.
c. Apply and adapt a variety of appropriate strategies to solve problems.
d. Monitor and reflect on the process of mathematical problem solving.

**MM2P2. Students will reason and evaluate mathematical arguments.**

a. Recognize reasoning and proof as fundamental aspects of mathematics.
b. Make and investigate mathematical conjectures.
c. Develop and evaluate mathematical arguments and proofs.
d. Select and use various types of reasoning and methods of proof.

**MM2P3. Students will communicate mathematically.**

a. Organize and consolidate their mathematical thinking through communication.
b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
c. Analyze and evaluate the mathematical thinking and strategies of others.
d. Use the language of mathematics to express mathematical ideas precisely.

**MM2P4. Students will make connections among mathematical ideas and to other disciplines.**

a. Recognize and use connections among mathematical ideas.
b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
c. Recognize and apply mathematics in contexts outside of mathematics.

**MM2P5. Students will represent mathematics in multiple ways.**

a. Create and use representations to organize, record, and communicate mathematical ideas.
b. Select, apply, and translate among mathematical representations to solve problems.
c. Use representations to model and interpret physical, social, and mathematical phenomena.

**UNIT OVERVIEW:**

This unit focuses on quadratic functions, equations, and inequalities and requires understanding of many topics from Mathematics I. In completing the unit, students will use all of the skills and concepts acquired in achieving the algebra standards in Mathematics I and apply understanding of the coordinate plane and use of mathematical argument from the geometry standard. Tasks in the unit assume an understanding of the characteristics of functions studied in Mathematics I, especially of the basic function $f(x) = x^2$ and transformations involving vertical shifts, stretches, and shrinks, as well as reflections across the x- and y-axes. They build on the ability to solve simple non-linear equations especially simple rational equations, quadratic equations that can be solved by finding square roots, and quadratic equations of the form $x^2 + bx + c = 0$. Students also need experience expressing the solutions to linear inequalities in one variable and working with sequences, both in recursive and closed form.

The unit begins with an applied problem that explores a new type of function transformation, the horizontal shift, and combinations of this transformation with those studied in Mathematics I. Students work with quadratic functions that model the behavior of objects that are thrown in the air and allowed to fall subject to the force of gravity to learn to factor general quadratic expressions completely over the integers and to solve general quadratic equations by factoring. Students then continue their study of objects in free fall to learn to find the vertex of the graph of any polynomial function and to convert the formula for a quadratic function from standard to vertex form. Next students explore quadratic inequalities graphically, apply the vertex form of a quadratic function to find real solutions of quadratic equations that cannot be solved by factoring, and then use exact solutions of quadratic equations to give exact values for the endpoints of the intervals in the solutions of quadratic inequalities

After students have learned to find the real solutions of any quadratic equation, they develop the concept of the discriminant of a quadratic equation, learn the quadratic formula, and then explore the complex numbers as non-real solutions of quadratic equations. Basic arithmetic operations on complex numbers are explored so that students have the ability to verify complex solutions to quadratic equations and understand that they come in conjugate pairs. The unit ends with an exploration of arithmetic series, the development of formulas for calculating the sum of such series, and applications of the concepts to counting possible pairs from a set of objects, counting the number of diagonals of a polygon, and to understanding the definition of polygonal numbers.

Throughout the unit there is an emphasis on problem solving and mathematical reasoning. The applications in the unit include most of the standard types of application problems involving quadratic functions and equations. The order of development allows students to develop mathematical arguments for the theorems of the unit, especially to explain why the graph of every quadratic function is a translation of the graph of the basic function $f(x) = x^2$ and to justify the quadratic formula.

Throughout the unit there is a focus on integration of algebraic and graphical viewpoints and multiple representations of the same mathematical concepts. Throughout the unit it is important to:

- Promote student use of multiple representations of concepts and require students to explain how to translate information from one representation to another. Such activities especially include requiring students to explain how their equations represent the physical situation they are intended to model.
- Emphasize sketching quadratic graphs by hand to reveal important features and using understanding of characteristics of quadratic graphs to select appropriate viewing windows when using graphing technology.
- In the problem solving process, distinguish between solving the equation in the mathematical model and solving the problem.

As a final note, we observe that completing the square is a topic for Mathematics III and is not used in this unit.

**Formulas and Definitions:**

Horizontal shift: A rigid transformation of a graph in a horizontal direction, either left or right.

Complete factorization over the integers: Writing a polynomial as a product of polynomials so that none of the factors is the number 1, there is at most one factor of degree zero, each polynomial factor has degree less than or equal to the degree of the product polynomial, each polynomial factor has all integer coefficients, and none of the factor polynomial can written as such a product.

Vertex form of a quadratic function: A formula for a quadratic equation of the form $f(x) = a(x - h)^2 + k$, where *a* is a nonzero constant and the vertex of the graph is the point (*h*, *k*).

Discriminant of a quadratic equation: The discriminant of a quadratic equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, is the number $b^2 - 4ac$.

**Theorems:**

For $h = \frac{-b}{2a}$ and $k = f\left(\frac{-b}{2a}\right)$, $f(x) = a(x-h)^2 + k$ is the same function as $f(x) = ax^2 + bx + c$.

The graph of any quadratic function can be obtained from transformations of the graph of the basic function $f(x) = x^2$.

Quadratic formula: The solution(s) of the quadratic equation of the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers with $a \neq 0$, is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The discriminant of a quadratic equation is positive, zero, or negative if and only if the equation has two real solutions, one real solution, or two complex conjugate number solutions respectively.

**TASKS**:
The remaining content of this framework consists of student tasks or activities. The first is intended to launch the unit. Each activity is designed to allow students to build their own algebraic understanding through exploration. The last task is a culminating task, designed to assess student mastery of the unit. There is a student version, not included in the Student Version of the unit, as well as a Teacher Edition version that includes notes for teachers and solutions.

**RESOURCES NEEDED BY THE TEACHER FOR THE LESSONS IN THIS UNIT:**
Elmo or Overhead Projector for each teacher, Classroom set of Graphing Calculators, Classroom set of Algebra Tiles, Classroom set of small cubes such as Algeblocks, Classroom set of individual marker boards (blank on one side and a grid on the other) and markers for students to use, Coordinate Grids, Colored Pencils, Rulers, Markers, Roll of Graph Paper with Inch Squares, Pad of Quad Paper, Glue Sticks, Scissors, Post-it Notes, Construction Paper, Poster Board, Copies of all Handouts for Students, Copies of the Standards for Students, Large Copy of the Standards to Post on the Wall

**RESOURCES NEEDED BY THE STUDENTS FOR THE LESSONS IN THIS UNIT:**
Notebook with at least 10 dividers for the introduction, individual lessons, and culminating activities, pencils, notebook paper, graph paper

Note: A copy of the standards for this unit should be given to the students with discussion to be held throughout the unit concerning their meaning and relation to the learning tasks of the day. A large copy of the standards should be posted in the classroom and referred to regularly as the various standards are discussed. Vocabulary should be emphasized by using word walls and flash cards or vocabulary sheets in the students' notebooks. Students will need individual copies of all handouts in the lessons of the unit. Students should keep a math notebook with all materials in it.