Acquisition Lesson Planning Form
Plan for the Concept, Topic, or Skill – Characteristics of Conic Sections
Key Standards addressed in this Lesson: MM3G2a,b,c
Time allotted for this Lesson:

| |
|---|
| **Standard: MM3G2a,b,c:** Convert equations of conics by completing the square. Graph conic sections, identifying fundamental characteristics. Write equations of conic sections given appropriate information. |
| **Essential Questions**: How do I identify the characteristics of parabolas, ellipses, and hyperbolas graphically and algebraically centered on the origin? How do I identify the characteristics of parabolas, ellipses, and hyperbolas graphically and algebraically not centered on the origin? How are discriminants used to classify conic sections? |
| **Activating Strategies**:<br><br>Getting to Know Conic sections Learning Task: |
| **Acceleration/Previewing**: (Key Vocabulary)<br><br>Circle, parabola, ellipse, hyperbola, radius, center, major axis, minor axis, asymptote, vertices, foci, axis of symmetry, directrix, length of latus rectum |
| **Teaching Strategies**:<br>Several graphic organizers included for ellipses, hyperbolas, and graphing the conic sections<br>Guided notes for ellipses and hyperbolas worksheet |
| **Task**:<br>Parabolas Learning Task<br>Is it really an Ellipse? Learning Task<br>Hyperbolas Learning Task |
| **Distributed Guided Practice**: Formula Sheet for conics included. |
| **Extending/Refining Strategies**:<br>Let's Go Fishing Culminating Task |
| **Summarizing Strategies**:<br>For identifying the conics: Name that Conic Game |

# ELLIPSES

## Completing the square to find the standard form:

1. Write your x terms together, your y terms together and take your constant to the other side of the equal sign.
2. Take out any factors so that the coefficient of your $x^2$ and $y^2$ is 1.
3. Complete the square two times, once with x term and once with y term.
4. Add what you added on the left side to the right side also. Be sure to multiply first if you had to take out any common factors.
5. Write your trinomials as binomials and add up your constants on the right side of the equal sign.
6. Divide both sides by the constant on the right so that your equation equals 1.
7. Foci: $a^2 - b^2 = c^2$ (big – small denominator)

## Example:

$4x^2 + y^2 - 48x - 4y + 48 = 0$

## Writing the equation of an ellipse:

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ The major axis has the larger denominator.

1. We need to find our center (h, k).
2. We have to have our $a^2$, so we need to find a, the radius of our major axis. (The total length of our major axis = 2a.)
3. We also need our $b^2$, so we need to find b, the radius of our minor axis. (The total length of our minor axis = 2b.)
4. To write our equation, we must decide if the ellipse is vertical or horizontal, so sketch your graph to determine where $a^2$ and $b^2$ go in your equation.

## Example:

Equation

Vertices at (-2, 2) and (4, 2), co-vertices at (1,1) and (1, 3)

Midpoint: __________ Center:____________

Distance from (4,2) and center a =

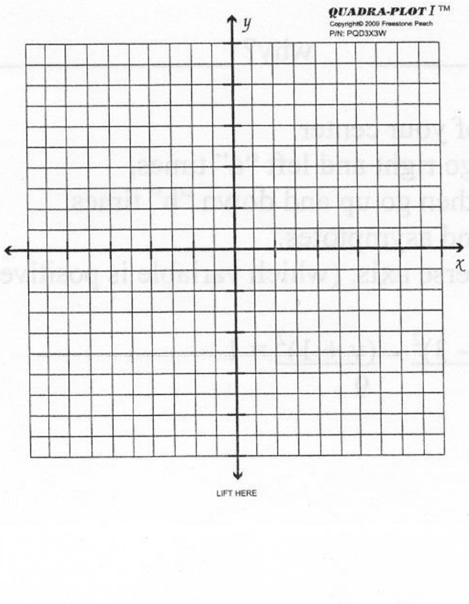
Distance from (3,1) and center b =

## Graphing:

1. Graph the point of your center.
2. From that point, go right and left "a" times, the go up and down "b" times.

## Example:

Graph $\frac{(x-4)^2}{4} + \frac{(y-4)^2}{9} = 1$



# HYPERBOLA

## Completing the square to find the standard form:

1. Write your x terms together, your y terms together and take your constant to the other side of the equal sign.
2. Take out any factors so that the coefficient of your $x^2$ and $y^2$ is 1.
3. Complete the square two times, once with x term and once with y term.
4. Add what you added on the left side to the right side also. Be sure to multiply first if you had to take out any common factors. (Watch for signs – negative in front means ______________
5. Write your trinomials as binomials and add up your constants on the right side of the equal sign.
6. Divide both sides by the constant on the right so that your equation equals 1.
7. Foci: $a^2$ $b^2 = c^2$

Example: $-25x^2 + 16y^2 - 150x - 96y - 481 = 0$

$16(y^2 - 6y + \_\_\_) - \_\_\_(x^2 + \_\_x + \_\_\_) = 481 + \_\_\_ + \_\_\_\_$

## Writing the equation of a hyperbola:

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ or $\frac{(\_\_ - \_\_)^2}{b^2} - \frac{(\_\_ - \_\_)^2}{a^2} = 1$

1. We need to find our center (h, k).
2. We have to have our $a^2$, so we need to find a, the radius of our t_________ axis.
3. We also need our $b^2$, so we need to find b.
4. To write our equation, we must decide if the hyperbola is vertical or horizontal, so sketch your graph to determine where $a^2$ and $b^2$ go in your equation.

## Example:

Vertices at (-3, 11) and (-3, -1), foci at (-3,12) and (-3, -2)

Equation

Midpoint:_________ Center_________

Distance from (-3,11) to center____________represents a, b, or c

Distance from (-3,12) to center ___________ represents a, b, or c

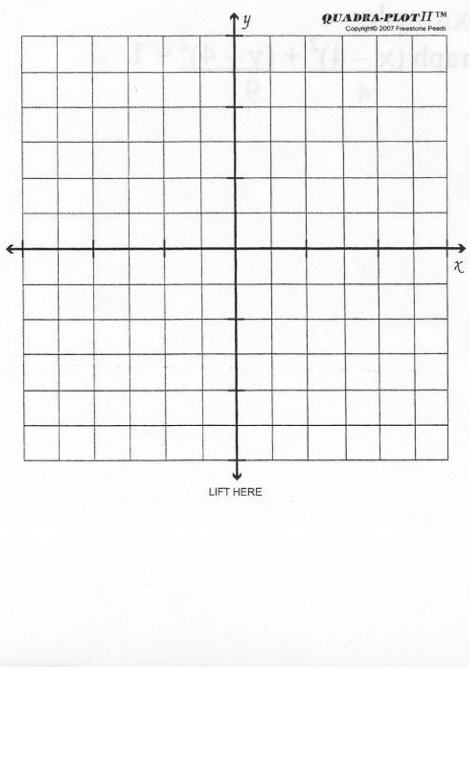
Find remaining variable $a^2 + b^2 = c^2$

Transverse axis? _______ why? ______________

## Graphing:

1. Graph the point of your center.
2. From that point, go right and left "a" times, then go up and down "b" times.
3. Draw rectangle and asymptotes.
4. Determine transverse axis. (which variable is positive)

Example: Graph $\frac{(x-3)^2}{4} - \frac{(y+1)^2}{9} = 1$



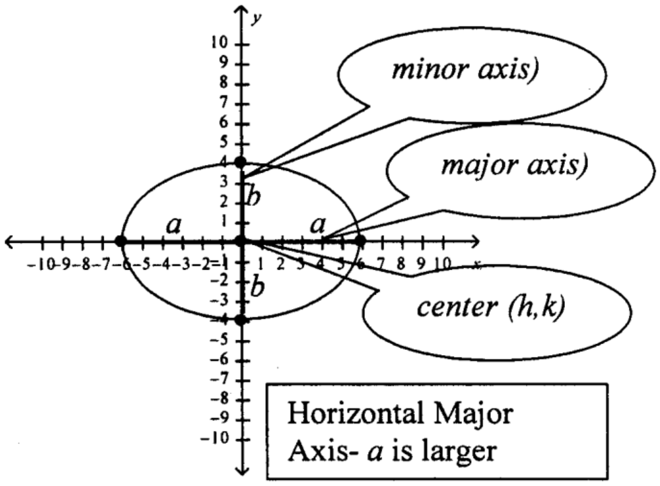
# 7-4 ELLIPSES

*NOTES WORKSHEET*

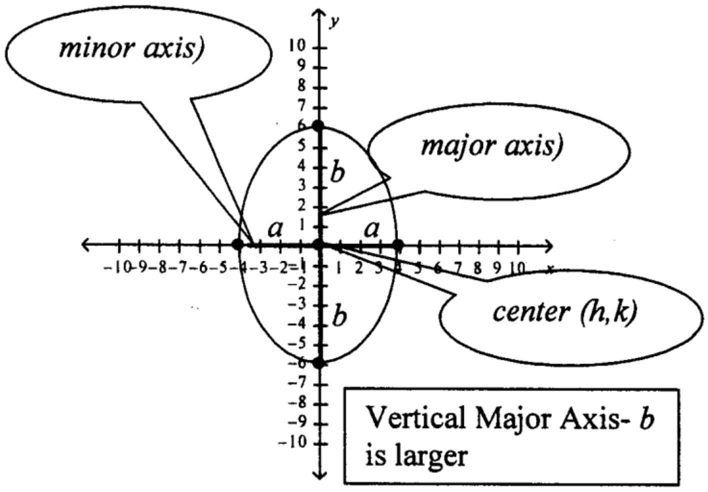
**NAME:** ______________________________________ **DATE:**_______________ **PERIOD:**___

**I. Graphing an ellipse on the graphing calculator.**

**PARTS OF AN ELLIPSE WITH HORIZONTAL MAJOR AXIS**




**PARTS OF AN ELLIPSE WITH VERTICAL MAJOR AXIS**




**ELLIPSE EQUATION:** $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

**Horizontal major axis**

**Length of major axis: 2a**

**Length of minor axis: 2b**

**Foci:** $c^2 = a^2 - b^2$ **(h ± c , k)**

**Verticies:** **(h ± a, k)**

**Co-vertices:** **(h, k ± b)**

**Vertical major axis**

**Length of major axis: 2b**

**Length of minor axis: 2a**

**Foci:** $c^2 = b^2 - a^2$ **(h, k ± c)**

**Verticies:** **(h, k+ b)**

**Co-verticies:** **(h ± a, k)**

**III.**

**IV. Answer each of the following:**

1. $\frac{x^2}{25}+\frac{y^2}{9}=1$

Center:____________________

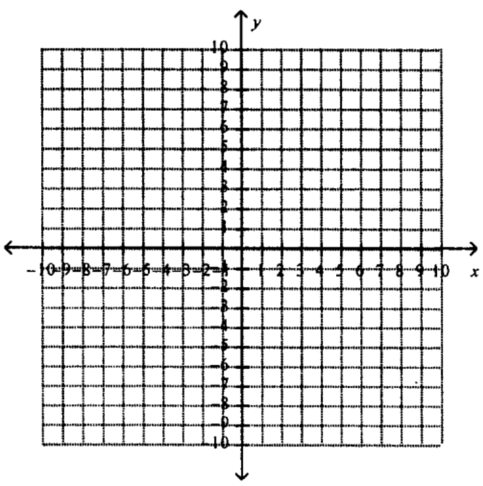
$a^2$ = _____ so $a$ = _____

$b^2$ = _____ so $b$ = _____

Vertex _______ and ___________

Co-vertex ________ and _________

c = ______ Foci _________ and _________

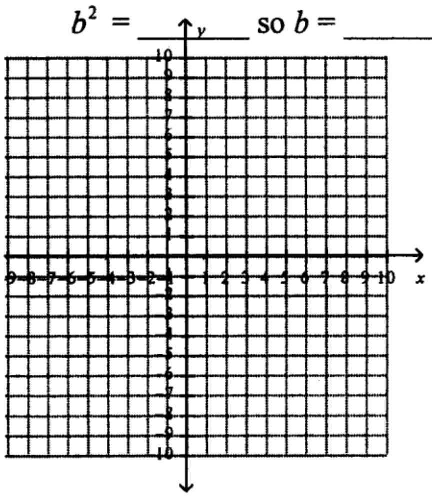


2. $\frac{(x-1)^2}{4}+\frac{(y+2)^2}{10}=1$

Center:____________________

$a^2$ = _______ so $a$ = ________

$b^2$ = _______ so $b$ = ________



Verticies_________

Co-Vertices ________

C = _____

Foci: ___________

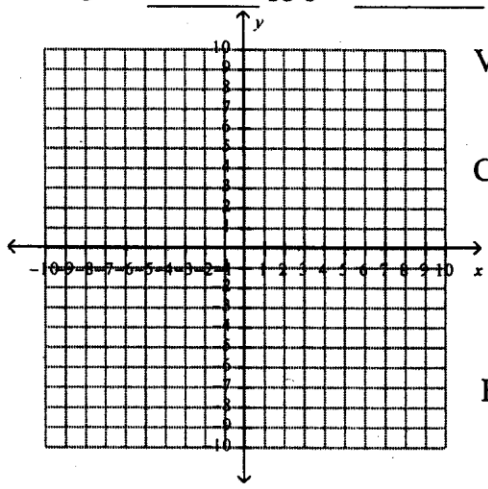
Get equal to 1 first.

3. $36x^2+81y^2=2916$

Center:____________________

$a^2$ = _______ so $a$ = ________

$b^2$ = _______ so $b$ = ________



Verticies _________

Co-Verticies ______

c = _____________

Foci ____________

4. $x^2+4y^2-2x+16y+1=0$

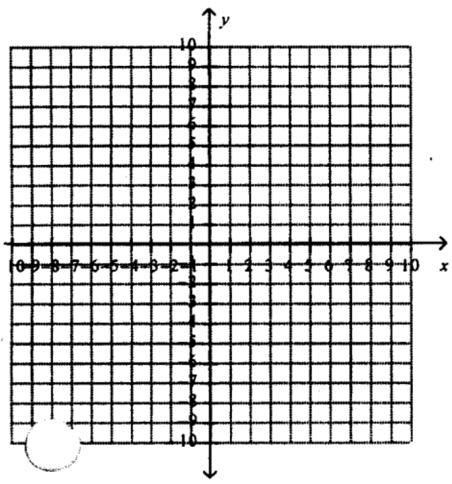
Standard form for the equation:

$X^2 - 2x +$ ___ $+ 4y^2 + 16y +$ ___ $= -1 +$ __ $+$ __

Center:____________________

$a^2$ = _______ so $a$ = ________

$b^2$ = _______ so $b$ = ________



Verticies_________

Co-Vertices ________

C = _____

Foci: ___________

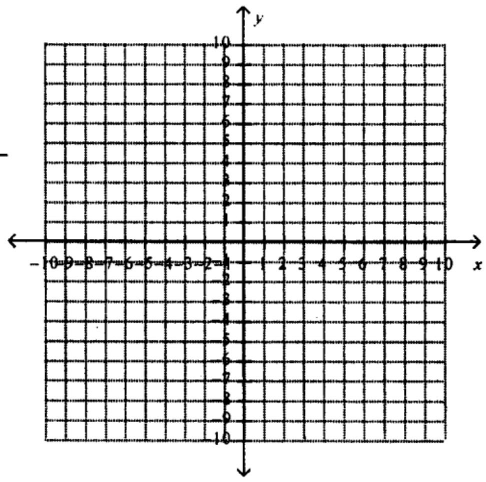
5. $x^2+9y^2-4x+54y+49=0$

Standard form for the equation:

Center:____________________

$a^2$ = _______ so $a$ = ________

$b^2$ = _______ so $b$ = ________



Verticies _________

Co-Verticies ______

c = _____________

Foci ____________

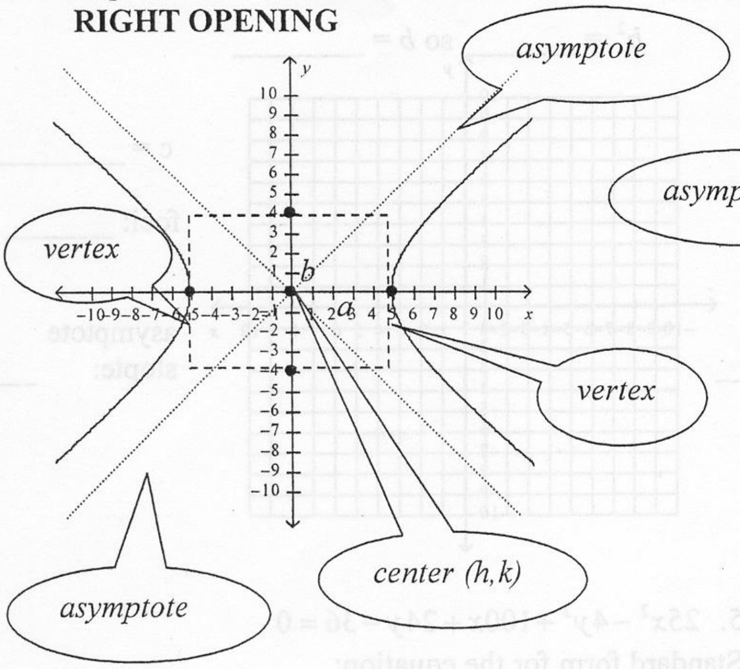
# 7-5 HYPERBOLAS

*NOTES WORKSHEET*

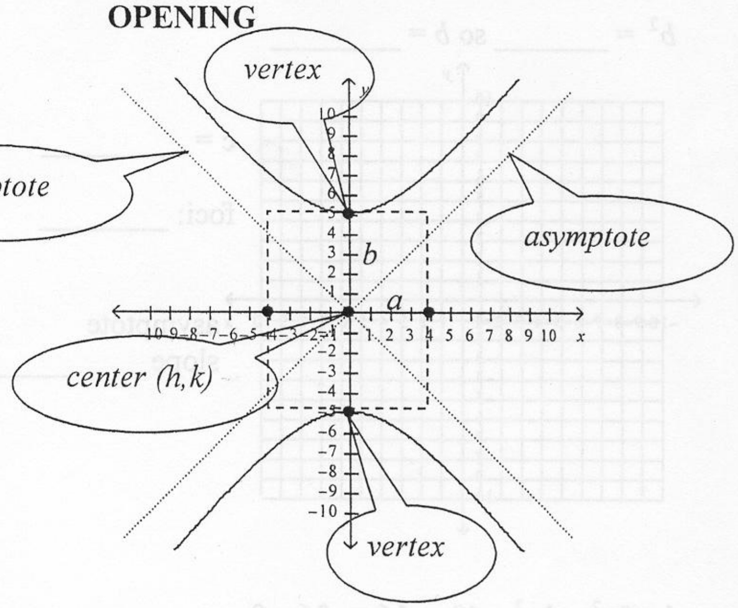
**NAME:** ______________________________________ **DATE:**__________________ **PERIOD:**___

**I.**

**EQUATION WITH LEFT AND RIGHT OPENING**




**EQUATION WITH UP AND DOWN OPENING**




**Notice $a^2$ is always positive**

| | | |
|---|---|---|
| **EQUATION:** | $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ | $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$ |
| **Transverse Axis** | **horizontal** | **vertical** |
| **Vertices** | **(h + a , k)** | **(h, k + a)** |
| **Foci** $c^2 = a^2 + b^2$ | **(h + c , k)** | **(h , k + c)** |
| **Asymptote slope** | $\pm\frac{b}{a}$**x** **remember** $\pm\frac{y}{x}$ | $\pm\frac{a}{b}$ **x** |

**III. Answer each of the following:**

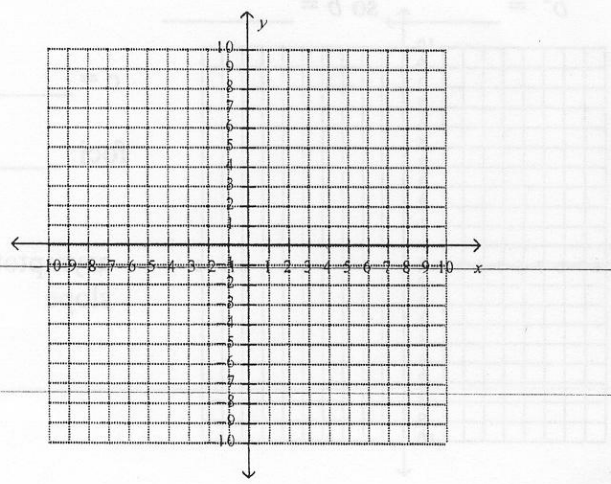
1. $\frac{x^2}{25}-\frac{y^2}{36}=1$

Center:__________ Vertices: ______________

$a^2$ = ________ so $a$ = _________

$b^2$ = ________ so $b$ = _________

c = _______ Foci:_______ Asymptotes _____

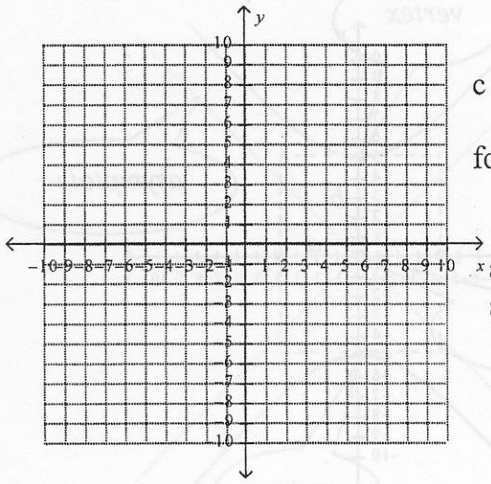


2. $\frac{(y+6)^2}{49} - \frac{(x-1)^2}{25} = 1$

Center:________ Vertices: ______________________

$a^2$ = _______ so $a$ = ________

$b^2$ = _______ so $b$ = ________



c = __________

foci: _________

asymptote slope ______

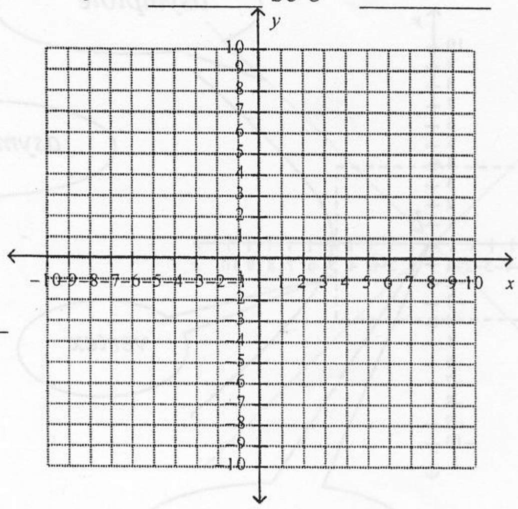
3. $6x^2 - 12y^2 = 108$

Standard form:

Center:________ Vertices: ____________________

$a^2$ = _______ so $a$ = ________

$b^2$ = _______ so $b$ = ________



c = __________

foci: _______

asymptote slopte: _____

4. $5x^2 - 4y^2 - 40x - 16y - 36 = 0$

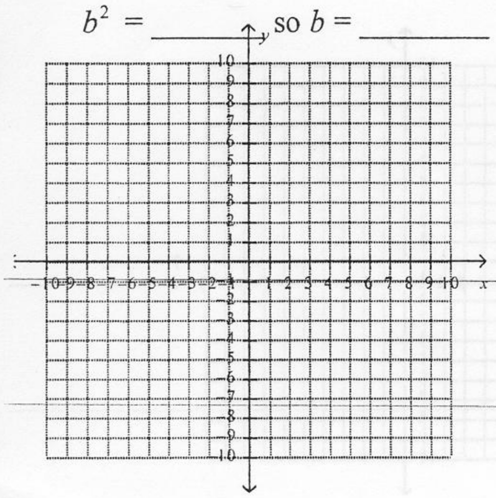
Standard form for the equation:

$5(x^2 - 8x + \_\_\_) - 4(y^2 + 4y + \_\_\_) = 36 + \_\_\_ - \_\_\_$

Center: _________ Vertices:________________

$a^2$ = _______ so $a$ = ________

$b^2$ = _______ so $b$ = ________



c = __________

foci: _________

asymptote slope ______

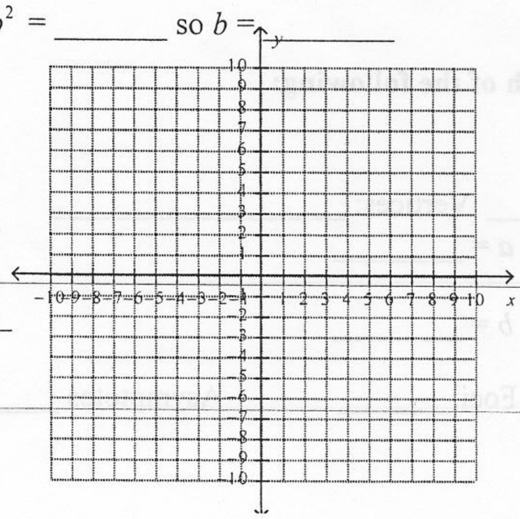
5. $25x^2 - 4y^2 + 100x + 24y - 36 = 0$

Standard form for the equation:

Center: _________ Vertices: __________________

$a^2$ = _______ so $a$ = ________

$b^2$ = _______ so $b$ = ________



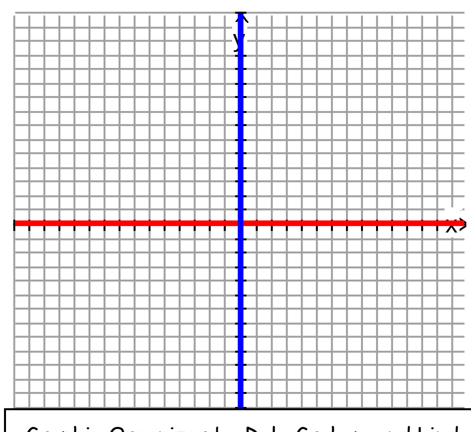
c = __________

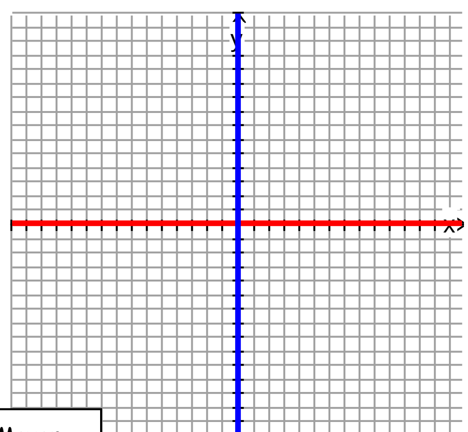
foci: _______

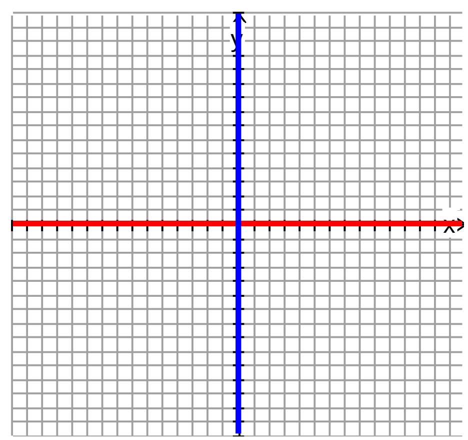
asymptote slopte: ____

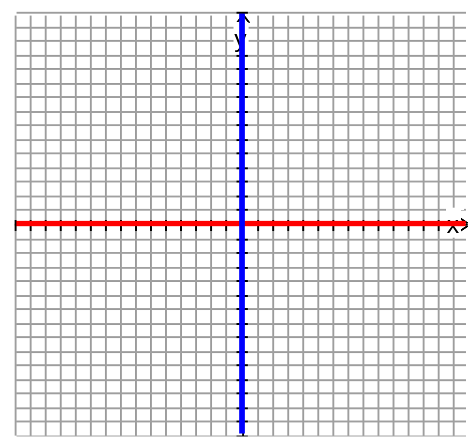
# How do you graph the four conic sections?

| | | | |
|---|---|---|---|
| Parabola<br>$y = a(x-h)^2 + k$ | Circle<br>$(x-h)^2 + (y-k)^2 = r^2$ | Ellipse<br>$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ | Hyperbola<br>$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ |
| Parabola<br>$y = -2(x-3)^2 + 5$ | Circle<br>$(x-3)^2 + (y-2)^2 = 20$ | Ellipse<br>$\frac{x^2}{9} + \frac{(y+2)^2}{16} = 1$ | Hyperbola<br>$\frac{(x-3)^2}{16} - \frac{y^2}{4} = 1$ |
| Vertex | Center | Center | Center |
| Axis of Symmetry | Radius | Length of Major Axis:<br><br>Length of Minor Axis: | Vertices |









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Thomas County Central High School; Thomasville GA

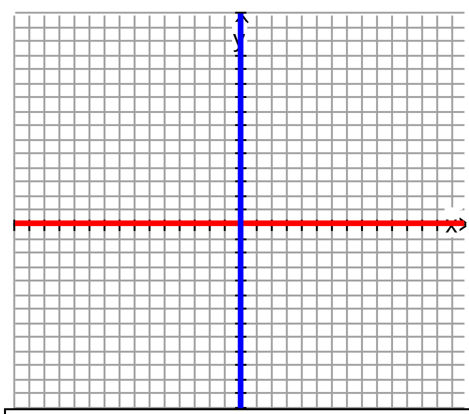
# How do you graph the four conic sections?

Parabola
$y = a(x-h)^2 + k$

Parabola

Vertex

Axis of Symmetry

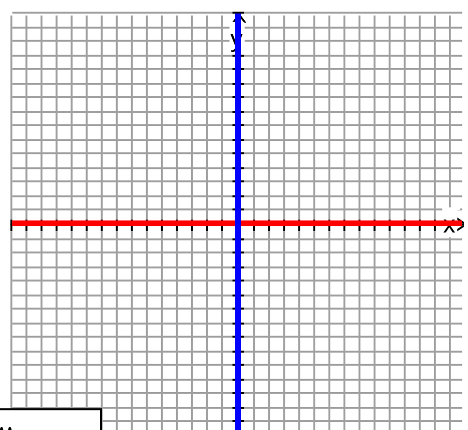


Circle
$(x-h)^2 + (y-k)^2 = r^2$

Circle

Center

Radius



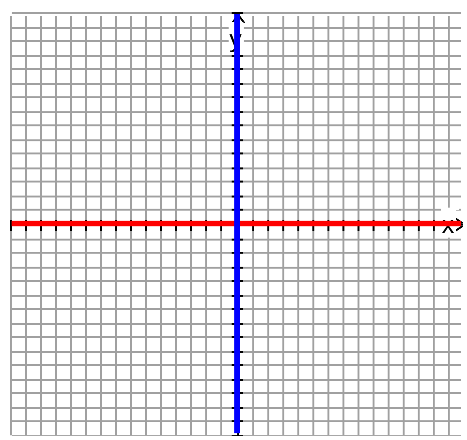
Ellipse
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Ellipse

Center

Length of Major Axis:

Length of Minor Axis:

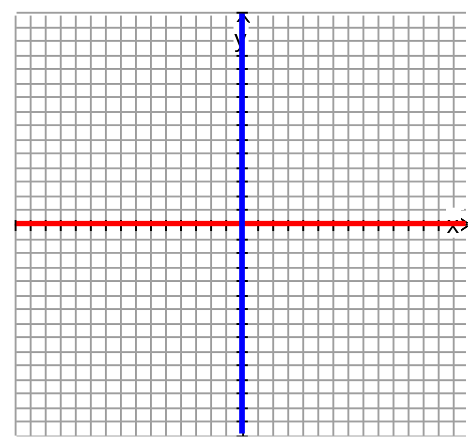


Hyperbola
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Hyperbola

Center

Vertices



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# Conic Sections

| Parabola | Circle | Ellipse | Hyperbola |
|---|---|---|---|
| Standard Equation:<br><br>Hint: | Standard Equations:<br><br>Hint: | Standard Equations:<br><br>Hint: | Standard Equations:<br><br>Hint: |
| Graph: | Graph: | Graph: | Graph: |

# Name That Conic - Vocabulary

1) Center of a circle – a fixed point in a plane where all radii of that circle commence.
2) Foci (singular focus) – a pair of special points used in describing conic sections. A circle and a parabola have one focus; an ellipse and a hyperbola have two foci.
3) Axis of symmetry – a line about which a figure is symmetric
4) Directrix – A line such that the distance from any point of a parabola to the focus is equal to the perpendicular distance from the point to that line.
5) Ellipse – the locus of all points in a plane such that the sum of the distances from two given points in the plane, called foci, is constant.
6) Eccentricity – the ratio of the distance between any point of a conic section and a fixed point to the distance between the same point of the conic section to a fixed line $0 \leq e \leq 1$
7) Circle – the locus of all points in a plane at a given distance, called a radius, from a fixed point on the plane, called the center
8) Parabola – the locus of all points in a given plane that are the same distance from a given point, called the focus, and a given line, called the directrix
9) Hyperbola - the locus of all points in the plane such that the absolute value of the difference of the distances from two given points in the plane, called foci, is constant
10) Minor axis - the axis of symmetry of an ellipse which does not contain the foci
11) Major axis – the axis of symmetry of an ellipse which contains the foci
12) Asymptotes – A line that a graph approaches but never intersects
13) Vertex of a conic section– a point at which a conic section intersects its axis of symmetry
14) Conjugate axis – the segment perpendicular to the transverse axis of a hyperbola through its center
15) Transverse axis - the line segment that has as its endpoints the vertices of a hyperbola

# Name That Conic

Materials:

1) Handout
2) Index Cards
3) Markers
4) Scissors
5) Glue Sticks or clear tape
6) Small Envelopes
7) Large Envelope

Procedure:

1) Using the markers, label index cards A1, A2, A3, A4, A5, B1, B2, B3, B4, B5, C1, C2, C3, C4, C5, …, H1, H2, H3, H4, H5.
2) Cut out the equations of the circles, ellipses, hyperbolas, and parabolas.
3) Paste them (or attach using clear tape) on the index cards.
4) Label the small envelopes A,B,C,D,E,F,G,H, using the markers.
5) Place the appropriate index cards in the small envelopes. Label each envelope with the appropriate letter.
6) Place all the small envelopes in the large envelope.

How to play the game:

1) Arrange your class in groups of 4. Hand out one small envelope to each group. Hand out one answer sheet to each group.
2) Explain the rules:
   a) Each group gets 5 cards and two minutes to "Name that Conic". They must write their answer on the corresponding line of the answer sheet and place the index cards back in the envelope.
   b) After two minutes the teacher will say "Rotate". Each group will pass their envelope clockwise and receive an envelope.
   c) The cycle repeats until each group has seen each envelope.
   d) Collect and score the answer sheets.
   e) The team with the most correct answers wins the game.

# Name That Conic

## General Form: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

| Conic Section | Standard Form of Equation | Variation of General Form of Conic Equations |
|---|---|---|
| Circle | $(x-h)^2 + (y-k)^2 = r^2$ | $A = C$ |
| Parabola | $(y-k)^2 = 4p(x-h)$ or $(x-h)^2 = 4p(y-k)$ | Either A or $C = 0$ |
| Ellipse | $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ or $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$ | A and C have the same sign and $A \neq C$ |
| Hyperbola | $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ or $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ | A and C have opposite signs |
| Hyperbola | $xy = k$ | $A = C = D = E = 0$ |

# Name That Conic

| Circles | Ellipses |
|---|---|
| $36x^2 + 36y^2 - 144 = 0$ | $9x^2 + 16y^2 - 144 = 0$ |
| $9x^2 + 9y^2 - 162 = 0$ | $16x^2 + y^2 - 16 = 0$ |
| $8x^2 + 8y^2 - 192 = 0$ | $49x^2 + 4y^2 - 196 = 0$ |
| $9x^2 + 9y^2 - 135 = 0$ | $9x^2 + 100y^2 - 900 = 0$ |
| $5x^2 + 5y^2 - 80 = 0$ | $10x^2 + 25y^2 - 250 = 0$ |
| $4x^2 + 4y^2 - 52 = 0$ | $25x^2 + 15y^2 - 375 = 0$ |
| $24x^2 + 24y^2 - 96 = 0$ | $2x^2 + y^2 - 4x - 4 = 0$ |
| $(1/4)x^2 + (1/4)y^2 - 16 = 0$ | $9x^2 + 4y^2 + 36x - 24y + 36 = 0$ |
| $9x^2 + 9y^2 - 441 = 0$ | $36x^2 + 16y^2 - 25x + 22y + 2 = 0$ |
| $x^2 + y^2 - 4x - 2y - 4 = 0$ | $12x^2 + 20y^2 - 12x + 40y - 37 = 0$ |
| | |

| Hyperbolas | Parabolas |
|---|---|
| $36x^2 - 4y^2 - 144 = 0$ | $x^2 - 4y = 0$ |
| $12y^2 - 25x^2 - 300 = 0$ | $8y^2 - x = 0$ |
| $36x^2 - 9y^2 - 324 = 0$ | $y + 5x^2 = 0$ |
| $y^2 - 81x^2 - 81 = 0$ | $-6x + y^2 = 0$ |
| $36y^2 - 4x^2 - 9 = 0$ | $-12x + y^2 = 0$ |
| $16y^2 - 36x^2 + 9 = 0$ | $x^2 - 2y = 0$ |
| $4y^2 - 81x^2 - 324 = 0$ | $y^2 - 4x = 0$ |
| $25y^2 - 4x^2 - 100 = 0$ | $x^2 + 4y = 0$ |
| $36x^2 - 10y^2 - 360 = 0$ | $x^2 - 4y = 0$ |
| $100x^2 - 81y^2 - 8100 = 0$ | $y^2 + 4x = 0$ |

# Name That Conic

| | |
|---|---|
| A1 | E1 |
| A2 | E2 |
| A3 | E3 |
| A4 | E4 |
| A5 | E5 |
| B1 | F1 |
| B2 | F2 |
| B3 | F3 |
| B4 | F4 |
| B5 | F5 |
| C1 | G1 |
| C2 | G2 |
| C3 | G3 |
| C4 | G4 |
| C5 | G5 |
| D1 | H1 |
| D2 | H2 |
| D3 | H3 |
| D4 | H4 |
| D5 | H5 |