Standard: MM3G3 – Students will investigate planes and spheres.

a. **Plot the point (x, y, z) and understand it as a vertex of a rectangular prism.**

b. **Apply the distance formula in 3-space.**

c. **Recognize and understand equations of planes and spheres.**

Essential Questions: How do I graph a point in 3-space? How do I determine the distance between two points in 3-space? What are the differences between the equations of a plane and sphere?

Activating Strategies: Graph a circle with endpoints of diameter at (-2,-6) and (2, -6) by finding midpoint of circle and distance formula. Write the equation for the circle.

Acceleration/Previewing: (Key Vocabulary) **three-dimensional coordinate system or 3-space standard form for equation of sphere $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$.**

Teaching Strategies: Extension of knowledge of 2-space into 3-space.
Discuss and demonstrate three-space being the intersection of an x-axis, a y-axis and a z-axis. The axes determine three coordinate planes: the xy-plane, the xz-plane, and the yz-plane. These planes divide the 3-sopace into eight octants. Each point in a 3-space is represented by an ordered triple (x, y, z).
distance in 3-space $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
Guided Practice - Model graphing a rectangular prism given the endpoints of the diagonal.
a. (3,0,5) and (0,4,0) Find the length of the diagonal. (see attachment)
Cooperative Pairs - Graph a rectangular prism given the endpoints of the diagonal. (0,-1,3) and (3,2,0). Find the length of the diagonal.
Discuss and demonstrate the equation of a plane. A linear equation in three variables is a plane.
Guided Practice – Model graphing the equation of a plane.
a. Graph the equation $2x – 3y + 9z = -36$. (see attachment)
Cooperative Paris – Graph the equation $2x + y – 5z = 10$.
Discuss and demonstrate the equation of a sphere.
Guided Practice – Model writing the equation of a sphere in standard form given center and radius. (see attachment)
a. Write the equation of a sphere in standard form with center (4,-2, 0) and radius r = 7.
Cooperative Pairs – Write the equation of a sphere in standard from with center (2, 0, 3) and r = 8.
Figures in 3- space worksheet. (see attachment)

Task:

Distributed Guided Practice:
see attachments

Extending/Refining Strategies:

Summarizing Strategies:
Determine whether the equation represents a plane or a sphere. If it is a plane, graph the equation.
a. $x^2 + y^2 + z^2 + 100 = 0$
b. $x – 2y + z + 14 = 0$

Guided Practice/Collaborative Pairs Lesson 2.5 (Using distance formula in 3-space)

Draw a rectangular prism having a diagonal with endpoints (3, 0, 5) and (0, 4, 0). Find the length of the diagonal. Step 1 – Draw the rectangular prism. Plot and connect the points (3, 0, 5) and (0, 4, 0). Because the difference of the x-coordinates is _____, the prism has a length of _____ unit(s). Because the difference of the y-coordinates is _____, the prism has a width of _____ unit(s). Because the difference of the z-coordinates is _____, the prism has a height of _____ unit(s). Draw the rectangular prism. Step 2 – Find the length of the diagonal. Let $(x_1, y_1, z_1) = (3, 0, 5)$ and $(x_2, y_2, z_2) = (0, 4, 0)$. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ $d = \sqrt{(\quad)^2 + (\quad)^2 + (\quad)^2}$ $d = \sqrt{(\quad)^2 + (\quad)^2 + (\quad)^2}$ $d =$ The length of the diagonal is _____units.	Draw a rectangular prism having a diagonal with endpoints (0, -1, 3) and (3, 2, 0). Find the length of the diagonal. Step 1 – Draw the rectangular prism. Plot and connect the points (0, -1, 3) and (3, 2, 0). Because the difference of the x-coordinates is _____, the prism has a length of _____ unit(s). Because the difference of the y-coordinates is _____, the prism has a width of _____ unit(s). Because the difference of the z-coordinates is _____, the prism has a height of _____ unit(s). Draw the rectangular prism. Step 2 – Find the length of the diagonal. Let $(x_1, y_1, z_1) = (0, -1, 3)$ and $(x_2, y_2, z_2) = (3, 2, 0)$. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ $d = \sqrt{(\quad)^2 + (\quad)^2 + (\quad)^2}$ $d = \sqrt{(\quad)^2 + (\quad)^2 + (\quad)^2}$ $d =$ The length of the diagonal is _____units.

Guided Practice/Collaborative Pairs Lesson 2.5 (Recognize and graph the equation of a plane)

Graph the equation $2x - 3y + 9z = -36$ The graph of a linear equation in three variables is a plane. In order to sketch a plane, complete the following steps. Step 1 – Find three points at which the plane intersects the axes. **Intersection of x-axis** The graph intersects the x-axis at (, ,). **Intersection of y-axis** The graph intersects the y-axis at (, ,). **Intersection of z-axis** The graph intersects the z-axis at (, ,). Step 2 – Plot and connect the points. Then shade the resulting region.	Graph the equation $2x + y - 5z = 10$ The graph of a linear equation in three variables is a plane. In order to sketch a plane, complete the following steps. Step 1 – Find three points at which the plane intersects the axes. **Intersection of x-axis** The graph intersects the x-axis at (, ,). **Intersection of y-axis** The graph intersects the y-axis at (, ,). **Intersection of z-axis** The graph intersects the z-axis at (, ,). Step 2 – Plot and connect the points. Then shade the resulting region.

Guided Practice/Collaborative Pairs Lesson 2.5 (Write an equation for a sphere)

Write an equation of a sphere in standard form with center((4, -2, 0) and a radius of 7 units. Step 1 – Write the standard form of an equation for a sphere. $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$ Step 2 – Identify x_0, y_0, and z_0. The center is at (4, -2, 0), so x_o = ____, y_0 = _____, and z_0 = _____. Step 3 – Find r^2. Because r = _____, r^2 = _____. The standard form of the equation is ( )^2 + ( )^2 + ( )^2 = _____	Write an equation of a sphere in standard form with center((2, 0, 3) and a radius of 8 units. Step 1 – Write the standard form of an equation for a sphere. $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$ Step 2 – Identify x_0, y_0, and z_0. The center is at (2, 0, 3), so x_o = ____, y_0 = _____, and z_0 = _____. Step 3 – Find r^2. Because r = _____, r^2 = _____. ( )^2 + ( )^2 + ( )^2 = _____

Worksheet Figures in 3-Space

Draw a rectangular prism having a diagonal with the given endpoints. Find the length of the diagonals.

1. (4, -5, 8) and (2, 2, 5) and (-2, 1, 0)
2. (9, 5, 1) and (12, 0, -1)
3. (0, 0, -8)

Graph the equation of the plane.

4. $2x - 4y - 3z = -15$
5. $3x + 7y - 3z + 42 = 0$

Write the equation of the sphere in standard form with the given center and radius.

6. center (8, -2, 0) r = 12
7. (10, -2, 3) $r = 3\sqrt{2}$
8. (-2, 7, 3) r = 7