Math Instructional Framework

Full Name	Math III Unit 3 Lesson 4
Time Frame	
Unit Name	Matrices and Linear programming
Learning Task/Topics/ Themes	**Linear Programming**
Standards **and** Elements	**MM3A6. Students will solve linear programming problems in two variables.** a. Solve systems of inequalities in two variables, showing the solutions graphically. b. Represent and solve realistic problems using linear programming.
Lesson Essential Questions	How to solve linear programming problems graphically? How to represent and solve realistic problems using linear programming?
Activator	Powerpoint lesson 4 systems of linear inequalites
Work Session	Linear programming application problems (attached in file)
Summarizing/Closing/Formative Assessment	Additional practice problems: textbook, notetaking guide book, kuta software, etc

Objective 1 – To understand the meaning of a "solution to a system of equations", to solve a system by graphing, and be able to identify by name the different types equation systems.

Reminder A system of equations may be consistent or inconsistent. An inconsistent system never intersects. If it is consistent, then the equations are either dependent (same line, infinitely many solutions) or independent (two lines that intersect at a single point; the solution is a single coordinate pair).

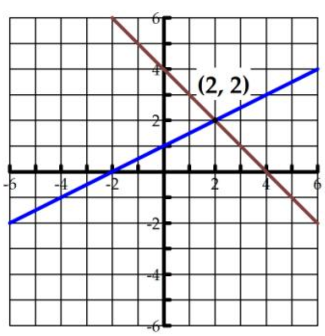
Example Graph each of the following systems of equations.

a) $\begin{Bmatrix} y = \frac{1}{2}x + 1 \\ x + y = 4 \end{Bmatrix}$ b) $\begin{Bmatrix} y = \frac{1}{2}x + 1 \\ y + 2 = \frac{1}{2}x \end{Bmatrix}$ c) $\begin{Bmatrix} y = \frac{1}{2}x + 1 \\ x - 2y = -2 \end{Bmatrix}$

Solution

a) Solve both equations for y

$y = \frac{1}{2}x + 1$ and graph
$y = -x + 4$



The system intersects at exactly one point (2, 2). Therefore, the system is consistent and independent. One equation does not depend on the other.

b) Solve the second equation for y

$$y + 2 = \frac{1}{2}x \Rightarrow y = \frac{1}{2}x - 2$$

Since both equations have the same slope but different y intercepts, the two lines are parallel. They will never cross (definition of parallel lines). There is no solution. This system is inconsistent.

c) Solve the second equation for y

$$x - 2y = -2 \Rightarrow -2y = -x - 2 \Rightarrow y = \frac{1}{2}x + 1$$

Since both equations are exactly the same. The two equations cross at every point on the line $y = \frac{1}{2}x + 1$. Since a line continues forever in each direction, there are infinitely many solutions to this system of equations. The system is consistent but dependent, because one line depends on the other.

Practice

Classify each of the following systems as either consistent or inconsistent. If consistent, further classify them and dependent or independent. Then, find the solution to the system of equations by graphing. Use graph paper.

1. $\begin{Bmatrix} x + y = -2 \\ x - y = 6 \end{Bmatrix}$ 2. $\begin{Bmatrix} y = 2x - 1 \\ 2x - y = -3 \end{Bmatrix}$ 3. $\begin{Bmatrix} -3x + 6y = -6 \\ x - 2y = 2 \end{Bmatrix}$

Objective 2 – To solve a system of linear equations by substitution

Reminder First obtain a single equation in one variable, which then can be solved. Then use that value for x or y, to solve for the other variable.

Example Solve these systems by substitution.

a) $\left\{\begin{array}{r} x = y - 3 \\ 2x + y = 18 \end{array}\right\}$ b) $\left\{\begin{array}{r} 3x - 4y = 17 \\ 2x + y = 4 \end{array}\right\}$

Solution

a) The first equation is already solved for x. Use this expression for x in terms of y in the second equation to find a numerical value for y. Second, substitute this value for y into the first equation to find the numerical value of x.

$$\begin{aligned} 2\ \ x\ \ + y &= 18 \\ 2\ \ y - 3\ \ + y &= 18 \\ 2y - 6 + y &= 18 \\ 3y - 6 &= 18 \\ 3y &= 24 \\ y &= 8 \end{aligned}$$

$$\begin{aligned} x &= y - 3 \\ x &= 8 - 3 \\ x &= 5 \end{aligned}$$

Solution: $5, 8$

b) Solve the second equation for y. Plug the resulting expression for y in terms of x into the first equation. Now solve the first equation for x. Plug that numerical value into the second equation to find y.

$$\begin{aligned} 2x + y = 4 \Rightarrow y &= -2x + 4 \\ 3x - 4y &= 17 \\ 3x - 4\ \ -2x + 4\ \ &= 17 \\ 3x + 8x - 16 &= 17 \\ 11x &= 33 \\ x &= 3 \end{aligned}$$

$$\begin{aligned} y &= -2x + 4 \\ y &= -2\ \ 3\ \ + 4 \\ y &= -6 + 4 \\ y &= -2 \end{aligned}$$

Solution: $3, -2$

Practice

Solve each of the following systems of equations using substitution. Check your solution.

4. $\left\{\begin{array}{r} y = x + 1 \\ x + y = 3 \end{array}\right\}$

5. $\left\{\begin{array}{r} x + y = 4 \\ 3x - 2y = 7 \end{array}\right\}$

6. $\left\{\begin{array}{l} y = 2x - 3 \\ x = 2y - 9 \end{array}\right\}$

7. $\left\{\begin{array}{r} 3x + 4y = 1 \\ x - 3y = -4 \end{array}\right\}$

8. $\left\{\begin{array}{r} 4x - y = 16 \\ 3x + 2y = 1 \end{array}\right\}$

9. $\left\{\begin{array}{r} 5x - 2y = 0 \\ x + 3y = 17 \end{array}\right\}$

10. $\left\{\begin{array}{r} 2x - 5y = -3 \\ 3y - 4x = -1 \end{array}\right\}$

11. $\left\{\begin{array}{r} 5x - 9y = 16 \\ x - 2 = 3y \end{array}\right\}$

12. $\left\{\begin{array}{l} \frac{1}{3}x - \frac{1}{2}y = 1 \\ -4y - 10 = -3x \end{array}\right\}$

Objective 3 – To solve a system of linear equations by elimination

Reminder Two lines having different slopes produce a consistent and independent system of equations. Therefore, the lines intersect at one point and that one point is the solution to the system.

To solve a system by elimination, eliminate one of the variables (x or y) by adding the two equations together thus "canceling out" one of the variables.

Example Solve this system by elimination.

$$\text{a)} \quad \begin{Bmatrix} 2x-5y=3 \\ 4y-3x=-1 \end{Bmatrix}$$

Solution

To solve by elimination means to multiply one or both equations by some number(s) which cause the elimination of one of the variables to occur when the two equations are added together to create a new equation.

First notice that the second equation is not in standard form. Rewrite the second equation in standard form (x term first, y term second, constant on right-hand-side of the equal sign).

$$\begin{Bmatrix} 2x-5y=3 \\ -3x+4y=-1 \end{Bmatrix}$$

Now chose to eliminate one of the variables. If we choose x, we must multiply the first equation by 3 and the second equation by 2.

$$\begin{Bmatrix} 3\ \ 2x-5y=3 \\ 2\ \ -3x+4y=-1 \end{Bmatrix} \Rightarrow + \begin{Bmatrix} 6x-15y=9 \\ -6x+8y=-2 \end{Bmatrix}$$

$$-7y=7$$

$$y=-1$$

Now substitute $y = -1$ into either of the two equations to find x.

$$\begin{aligned} 2x-5y&=3 \\ 2x-5\ \ -1\ &=3 \\ 2x+5&=3 \\ 2x&=-2 \\ x&=-1 \end{aligned}$$

The solution to this system is $-1,-1$.

Practice

Solve each of the following systems by elimination. Then, check your solution.

13. $\begin{Bmatrix} x-y=-3 \\ x+y=5 \end{Bmatrix}$

14. $\begin{Bmatrix} x+3y=6 \\ x-y=2 \end{Bmatrix}$

15. $\begin{Bmatrix} 7x-3y=32 \\ 2x+y=11 \end{Bmatrix}$

16. $\begin{Bmatrix} x+4y=8 \\ -2x+5y=23 \end{Bmatrix}$

17. $\begin{Bmatrix} 3x-2y=-4 \\ 4x+3y=-5 \end{Bmatrix}$

18. $\begin{Bmatrix} 8x+5y=-28 \\ -3x+2y=-5 \end{Bmatrix}$

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Solution

a) Solve both equations for y

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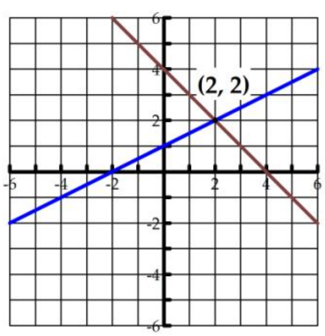

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Linear Programming Applications

1. A ski manufacturer makes two types of skis and has a fabricating department and a finishing department. A pair of downhill skis requires 6 hours to fabricate and 1 hour to finish. A pair of cross-country skis requires 4 hours to fabricate and 1 hour to finish. The fabricating department has 108 hours of labor available per day. The finishing department has 24 hours of labor available per day. The company makes a profit of $40 on each pair of downhill skis and a profit of $30 on each pair of cross-country skis.

   a) Write a system of linear inequalities
   b) Graph the feasible region
   c) Write the objective function for the profit, and find the maximum profit

2. Trenton, Michigan, a small community, is trying to establish a public transportation system of large and small vans. It can spend no more than $100,000 for both sizes of vehicles and no more than $500 per month for maintenance. The community can purchase a small van for $10,000 and maintain it for $100 per month. The large vans cost $20,000 each and can be maintained for $75 per month. Each large van carries a maximum of 15 passengers, and each small van carries a maximum of 7 passengers.

   a) Write a system of linear inequalities
   b) Graph the feasible region
   c) Write the objective function for the number of passengers, and find the maximum number of passengers

3. A tourist agency can sell up to 1200 travel packages for a football game. The package includes airfare, weekend accommodations, and the choice of two types of flights: a nonstop flight or a two-stop flight. The nonstop flight can carry up to 150 passengers, and the two-stop flight can carry up to 100 passengers. The agency can locate no more than 10 planes for the travel packages. Each package with nonstop flight sells for $1200, and each package with a two-stop flight sells for $900. Assume that each plane will carry the maximum number of passengers.

   a) Write a system of linear inequalities
   b) Graph the feasible region
   c) Write the objective function that maximizes the revenue for the tourist agency, and find the maximum revenue for the given constraints

4. A farmer has 90 acres available for planting millet and alfalfa. Seed costs $4 per acre for millet and $6 per acre for alfalfa. Labor costs are $20 per acre for millet and $10 per acre for alfalfa. The expected income is $110 per acre for millet and $150 per acre for alfalfa. The farmer intends to spend no more than $480 for seed and $1400 for labor.

a) Write a system of linear inequalities
b) Graph the feasible region
c) Write the objective function that maximizes the income, and find the maximum income for the given constraints

Math 3 Name ___________________________

Linear Programming #2

1. You are taking a test in which items of type A are worth 10 points and items of type B are worth 15 points. It takes 3 minutes to answer each item of type A and 6 minutes for each item of type B. The total time allowed is 60 minutes, and you may not answer more than 16 questions. Assuming all of your answers are correct, how many items of each type should you answer to get the highest score?

2. Mrs. Wood's biscuit Factory makes two types of biscuits, Biscuit Jumbos and Mini Mint Biscuits. The oven can cook at most 200 biscuits per day. Jumbos each require 2 ounces of flour. Minis each require 1 ounce of flour. There are 300 ounces of four available. The income from Jumbos is 10 cents each. The income from the Minis is 8 cents each. How many of each type should be baked to earn the greatest amount?

3. A company produces mopeds and bicycles. It must produce at least 10 mopeds per month. The company has the equipment to produce only 60 mopeds. It also can produce only 120 bicycles. The production of mopeds and bicycles cannot exceed 160. The profit on a moped is $134 and on a bicycle $20. How many of each should be manufactured per month to maximize profit?

4. There are 12 gallons of gas available to be distributed between the car and the scooter. The car's tank can hold at most 10 gallons. The scooter can hold at most 3 gallons. The car can go 20 miles on each gallon, and the scooter can go 100 miles on each gallon. How much should each vehicle get to achieve the greatest total miles?

5. You are about to take a test that contains questions of type A worth 4 points and of type B worth 7 points. You must answer at least 5 of type A and 3 of type B, but time restricts answering more than 10 of either type. In total, you can answer no more than 18. How many of each type of question must you answer, assuming all of your answers are correct, to maximize your score? What is the maximum score?

6. A man plans to invest up to $22,000 in Bank X or Bank Y, or both. He will invest at least $2,000, but no more than $14,000, in Bank X. He will invest no more than $15,000 in Bank Y. Bank X pays 6% simple interest and Bank Y pays 6.5%. How much should he invest in each to maximize income? What is the maximum income?

Accelerated Math 2 Name ____________________________
Linear Programming #2

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   12 A Q's, 4 B Q's for 180 points

2. Mrs. Wood's biscuit Factory makes two types of biscuits, Biscuit Jumbos and Mini Mint Biscuits. The oven can cook at most 200 biscuits per day. Jumbos each require 2 ounces of flour. Minis each require 1 ounce of flour. There are 300 ounces of four available. The income from Jumbos is 10 cents each. The income from the Minis is 8 cents each. How many of each type should be baked to earn the greatest amount?

   100 of each to earn 1800 cents = $18

3. A company produces mopeds and bicycles. It must produce at least 10 mopeds per month. The company has the equipment to produce only 60 mopeds. It also can produce only 120 bicycles. The production of mopeds and bicycles cannot exceed 160. The profit on a moped is $134 and on a bicycle $20. How many of each should be manufactured per month to maximize profit?

   60 mopeds and 100 bicycles

4. There are 12 gallons of gas available to be distributed between the car and the scooter. The car's tank can hold at most 10 gallons. The scooter can hold at most 3 gallons. The car can go 20 miles on each gallon, and the scooter can go 100 miles on each gallon. How much should each vehicle get to achieve the greatest total miles?

   9 gallons in the car and 3 gallons in the scooter

5. You are about to take a test that contains questions of type A worth 4 points and of type B worth 7 points. You must answer at least 5 of type A and 3 of type B, but time restricts answering more than 10 of either type. In total, you can answer no more than 18. How many of each type of question must you answer, assuming all of your answers are correct, to maximize your score? What is the maximum score?

   8 A's and 10 B's for a score of 102

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   Invest $14000 in Bank X
   Invest $8000 in Bank Y
   Max income of $1360