Acquisition Lesson Planning Form
Plan for the Concept, Topic, or Skill – Solving Polynomial Equations
Key Standards addressed in this Lesson: MM3A3a, b

Standard: MM3A3: Students will solve a variety of equations and inequalities

a. find real and complex roots of higher degree polynomial equations using the factor theorem, remainder theorem, rational root theorem, and fundamental theorem of algebra, incorporating complex and radical conjugates.

b. Solve polynomial, exponential, and logarithmic equations analytically, graphically, and using appropriate technology.

Essential Question: How do I solve polynomial equations? How do I solve logarithmic and exponential equations?

Activating Strategies:

Review factoring.

Acceleration/Previewing: (Key Vocabulary)
Polynomial functions, synthetic division.

Teaching Strategies:
Use graphic organizer to model synthetic division. Model the Remainder Theorem.
Use Descartes Rule of Signs, P/Q method to determine rules and solve.
Use Graphic organizer: Zeros of a polynomial function to model how to solve polynomial equations.

Task:
Historical Background
Potato Lab
Polynomial Root Task

Distributed Guided Practice:
Polynomial Function Review 1 Worksheet (individual or in pairs)
There are various worksheets provided for this unit created using kutasoftware website: Dividing polynomials, Log Equations, and Polynomial Functions
Extra worksheets can be found on www.kutasoftware.com

Extending/Refining Strategies:
Have students explain how the remainder theorem is used to determine if a polynomial is a factor of another polynomial.

Students will find examples of how polynomial equations are useful.

Summarizing Strategies:
Polynomial Functions: Ticket out the Door

How do you divide using

Synthetic Division

Procedure

1st — Set up the chart.

2n — Multiply and add.

3rd — Write your answer with the appropriate variables.

Example

$$\frac{2x^3 - 3x^2 - 10x + 7}{x - 3}$$

Your Turn

$$\frac{4x^4 + 8x^3 - 6x - 8}{x + 2}$$

Graphic Organizer by Dale Graham and Linda Meyer
Thomas County Central High School; Thomasville GA

How do you divide using

Synthetic Division

Procedure

1st

2nd

3rd

Example

$$\frac{2x^3 - 3x^2 - 10x + 7}{x - 3}$$

Your Turn

$$\frac{4x^4 + 8x^3 - 6x - 8}{x + 2}$$

Graphic Organizer by Dale Graham and Linda Meyer
Thomas County Central High School; Thomasville GA

How do you solve exponential equations?

???

Step	Example	Your Turn
	$3^{x-1}=27^{x}$	
Write each side with the same base. →		
Simplify the exponents. →		
Put the exponents equal each other and solve. →		

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How do I find the zeros for a polynomial function?

Example	Your Turn
$f(x) = 2x^3 + 3x^2 - 8x + 3$	$f(x) = 2x^4 - 2x^3 + 2x^2 - 6x - 12$

1st Find all the factors of the constant and of the leading coefficient.

2nd Form all rational numbers by making fractions with the numerator a factor of the constant and the denominator a factor of the leading coefficient.

3rd Using synthetic substitution, find one solution to the problem.

4th Rewrite the function as two factors.

5th Repeat this process as needed to get all your factors of the polynomial.

6th Set each factor equal to zero and solve the resulting equations.

Graphic Organizer by Dale Graham and Linda Meyer
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How do I find the zeros for a polynomial function?

Example	Your Turn
$f(x) = 2x^3 + 3x^2 - 8x + 3$	$f(x) = 2x^4 - 2x^3 + 2x^2 - 6x - 12$

1st

2nd

3rd

4th

5th

6th

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Algebra III
Sections 8.4 & 8.5 Review

For each function, state the number of positive real zeros, negative zeros, and imaginary zeros.

	Positive	Negative	Imaginary
1. $f(x) = 2x^8 - 4x^7 + 5x^5 - 6x + 4$			
2. $f(x) = 2x^4 - 2x^3 + 2x^2 - x - 1$			
3. $f(x) = 7x^6 + 4x^4 + 9x^2 + 2$			
4. $f(x) = 4x^5 - 3x^4 + 2x^3 - x^2 + 5x - 7$			

Given a function and one of the zeros, find all the zeros of the function.

5. $f(x) = x^3 - 6x^2 + 11x - 6$; 3
6. $f(x) = x^5 - x^4 - 16x + 16$; 1
7. $f(x) = x^3 - 7x^2 + 17x - 15$; $2 + i$
8. $f(x) = x^4 - 6x^3 + 12x^2 + 6x - 13$; $3 - 2i$

Find all the zeros of each function.

9. $f(x) = x^3 + 3x^2 - 6x - 8$

10. $f(x) = x^4 - 3x^3 - 11x^2 + 3x + 10$

11. $f(x) = x^4 - 4x^3 + x^2 + 16x - 20$

12. $f(x) = 2x^3 + 3x^2 + 5x + 2$

ID: 1

Math 3 Name________________________________

Dividing Polynomials

Date________________ Period____

Divide.

1) $(x^3 - 10x^2 + 28x - 23) \div (x - 3)$

2) $(r^3 + 10r^2 + 26r + 86) \div (r + 8)$

3) $(n^3 + 7n^2 + 10) \div (n + 7)$

4) $(b^3 - 5b^2 - 4b + 38) \div (b - 3)$

5) $(v^4 - 9v^3 + 13v^2 + 22v + 50) \div (v - 6)$

6) $(2x^4 + 18x^3 + 33x^2 - 31x - 11) \div (x + 4)$

7) $(n^4 - 4n^3 - 14n^2 + 22n - 59) \div (n - 6)$

8) $(a^4 - 9a^3 + a - 10) \div (a - 9)$

State if the given binomial is a factor of the given polynomial.

9) $(k^3 + 3k^2 + k - 1) \div (k + 1)$

10) $(p^3 + 5p^2 + 1) \div (p + 5)$

11) $(n^5 + 6n^4 + 4n^3 - 9n^2 + 30n + 8) \div (n + 4)$

12) $(9m^5 + 80m^4 + 59m^3 - 40m^2) \div (m + 8)$

13) $(4x^2 + 36x) \div (x + 9)$

14) $(r^4 - 5r^3 - 40r^2 - 106r + 50) \div (r - 10)$

15) $(x^3 - 3x^2 - 76x - 49) \div (x + 7)$

16) $(6n^3 + 69n^2 + 89n - 18) \div (n + 10)$

Ticket out the Door
Polynomial Functions

Find all the solutions for $x^4 - 6x^3 + 24x - 40$.

1) List the total number of solutions.

2) Find the possible positive, negative, and imaginary roots.

(+)	(-)	(I)

3) List all the possible solutions (p/q):

4) Find the solutions.

Ticket out the Door
Polynomial Functions

Find all the solutions for $x^4 - 6x^3 + 24x - 40$.

1) List the total number of solutions.

2) Find the possible positive, negative, and imaginary roots.

(+)	(-)	(I)

3) List all the possible solutions (p/q):

4) Find the solutions.

Math 3 Name________________________________ ID: 1

Dividing Polynomials

Date________________ Period____

Divide.

1) $(x^3 - 10x^2 + 28x - 23) \div (x - 3)$

$x^2 - 7x + 7 - \frac{2}{x - 3}$

2) $(r^3 + 10r^2 + 26r + 86) \div (r + 8)$

$r^2 + 2r + 10 + \frac{6}{r + 8}$

3) $(n^3 + 7n^2 + 10) \div (n + 7)$

$n^2 + \frac{10}{n + 7}$

4) $(b^3 - 5b^2 - 4b + 38) \div (b - 3)$

$b^2 - 2b - 10 + \frac{8}{b - 3}$

5) $(v^4 - 9v^3 + 13v^2 + 22v + 50) \div (v - 6)$

$v^3 - 3v^2 - 5v - 8 + \frac{2}{v - 6}$

6) $(2x^4 + 18x^3 + 33x^2 - 31x - 11) \div (x + 4)$

$2x^3 + 10x^2 - 7x - 3 + \frac{1}{x + 4}$

7) $(n^4 - 4n^3 - 14n^2 + 22n - 59) \div (n - 6)$

$n^3 + 2n^2 - 2n + 10 + \frac{1}{n - 6}$

8) $(a^4 - 9a^3 + a - 10) \div (a - 9)$

$a^3 + 1 - \frac{1}{a - 9}$

State if the given binomial is a factor of the given polynomial.

9) $(k^3 + 3k^2 + k - 1) \div (k + 1)$

Yes

10) $(p^3 + 5p^2 + 1) \div (p + 5)$

No

11) $(n^5 + 6n^4 + 4n^3 - 9n^2 + 30n + 8) \div (n + 4)$

Yes

12) $(9m^5 + 80m^4 + 59m^3 - 40m^2) \div (m + 8)$

Yes

13) $(4x^2 + 36x) \div (x + 9)$

No

14) $(r^4 - 5r^3 - 40r^2 - 106r + 50) \div (r - 10)$

No

15) $(x^3 - 3x^2 - 76x - 49) \div (x + 7)$

No

16) $(6n^3 + 69n^2 + 89n - 18) \div (n + 10)$

No

ID: 1

Math 3 Name________________________________

Log Equations

Date________________ Period____

Solve each equation.

1) $\log_2 7 + \log_2 2x^2 = 3$

2) $\log_8 2x^2 - \log_8 2 = 2$

3) $\log_7 9 - \log_7 (x-4) = \log_7 25$

4) $\log_2 (x^2+10) - \log_2 5 = 1$

5) $\log_2 6 - \log_2 (x-3) = 1$

6) $\log_7 -2x - \log_7 5 = \log_7 64$

7) $\log_4 x - \log_4 (x+6) = \log_4 32$

8) $\log_9 (x+6) + \log_9 x = \log_9 55$

9) $\log_3 (x+6) + \log_3 x = 3$

10) $\log_7 x + \log_7 (x+7) = \log_7 44$

11) $\log_4 (x^2+8) - \log_4 10 = 2$

12) $\log_5 3x^2 - \log_5 3 = 4$

13) $\log (x^2+9) - \log 2 = \log 29$

14) $\log_4 2x^2 - \log_4 2 = 1$

15) $\log_6 9 - \log_6 (x+3) = 1$

16) $\log_7 2x^2 - \log_7 8 = 4$

ID: 1

Math 3 Name________________________

Log Equations

Date________________ Period____

Solve each equation.

1) $\log_2 7 + \log_2 2x^2 = 3$

$\left\{\frac{2\sqrt{7}}{7}, -\frac{2\sqrt{7}}{7}\right\}$

2) $\log_8 2x^2 - \log_8 2 = 2$

$\{8, -8\}$

3) $\log_7 9 - \log_7 (x - 4) = \log_7 25$

$\left\{\frac{109}{25}\right\}$

4) $\log_2 (x^2 + 10) - \log_2 5 = 1$

$\{0\}$

5) $\log_2 6 - \log_2 (x - 3) = 1$

$\{6\}$

6) $\log_7 -2x - \log_7 5 = \log_7 64$

$\{-160\}$

7) $\log_4 x - \log_4 (x + 6) = \log_4 32$

No solution.

8) $\log_9 (x + 6) + \log_9 x = \log_9 55$

$\{5\}$

9) $\log_3 (x + 6) + \log_3 x = 3$

$\{3\}$

10) $\log_7 x + \log_7 (x + 7) = \log_7 44$

$\{4\}$

11) $\log_4 (x^2 + 8) - \log_4 10 = 2$

$\{2\sqrt{38}, -2\sqrt{38}\}$

12) $\log_5 3x^2 - \log_5 3 = 4$

$\{25, -25\}$

13) $\log (x^2 + 9) - \log 2 = \log 29$

$\{7, -7\}$

14) $\log_4 2x^2 - \log_4 2 = 1$

$\{2, -2\}$

15) $\log_6 9 - \log_6 (x + 3) = 1$

$\left\{-\frac{3}{2}\right\}$

16) $\log_7 2x^2 - \log_7 8 = 4$

$\{98, -98\}$

Math 3 | Name________________________ | ID: 1

Polynomial Functions

Date________________ Period____

State the number of complex zeros, the possible number of real and imaginary zeros, and the possible rational zeros for each function. Then find all zeros.

1) $f(x) = x^3 - 2x^2 - x + 2$

2) $f(x) = x^3 - 11x^2 + 17x + 21$

3) $f(x) = x^3 - 5x^2 - 42x - 54$

4) $f(x) = x^4 - 6x^2 + 5$

5) $f(x) = x^4 - 4x^2 - 32$

6) $f(x) = x^4 - 3x^2 - 4$

Math 3 Name________________________________ ID: 1

Polynomial Functions

Date________________ Period____

State the number of complex zeros, the possible number of real and imaginary zeros, and the possible rational zeros for each function. Then find all zeros.

1) $f(x) = x^3 - 2x^2 - x + 2$

# of complex zeros: 3
Possible # of real zeros: 3 or 1
Possible # of imaginary zeros: 2 or 0
Possible rational zeros: $\pm 1, \pm 2$
Zeros: $\{1, 2, -1\}$

2) $f(x) = x^3 - 11x^2 + 17x + 21$

# of complex zeros: 3
Possible # of real zeros: 3 or 1
Possible # of imaginary zeros: 2 or 0
Possible rational zeros: $\pm 1, \pm 3, \pm 7, \pm 21$
Zeros: $\{3, 4 + \sqrt{23}, 4 - \sqrt{23}\}$

3) $f(x) = x^3 - 5x^2 - 42x - 54$

# of complex zeros: 3
Possible # of real zeros: 3 or 1
Possible # of imaginary zeros: 2 or 0
Possible rational zeros:
$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm 27, \pm 54$
Zeros: $\{-3, 4 + \sqrt{34}, 4 - \sqrt{34}\}$

4) $f(x) = x^4 - 6x^2 + 5$

# of complex zeros: 4
Possible # of real zeros: 4, 2, or 0
Possible # of imaginary zeros: 4, 2, or 0
Possible rational zeros: $\pm 1, \pm 5$
Zeros: $\{\sqrt{5}, -\sqrt{5}, -1, 1\}$

5) $f(x) = x^4 - 4x^2 - 32$

# of complex zeros: 4
Possible # of real zeros: 4, 2, or 0
Possible # of imaginary zeros: 4, 2, or 0
Possible rational zeros:
$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32$
Zeros: $\{2\sqrt{2}, -2\sqrt{2}, 2i, -2i\}$

6) $f(x) = x^4 - 3x^2 - 4$

# of complex zeros: 4
Possible # of real zeros: 4, 2, or 0
Possible # of imaginary zeros: 4, 2, or 0
Possible rational zeros: $\pm 1, \pm 2, \pm 4$
Zeros: $\{-2, 2, i, -i\}$

Standard: MM3A3. Students will solve a variety of equations and inequalities.

c. Solve polynomial, exponential, and logarithmic inequalities analytically, graphically, and using appropriate technology. Represent solution sets of inequalities using interval notation.

Essential Questions: How do I solve polynomial inequalities? How do I solve logarithmic and exponential inequalities?

Activating Strategies: Solving quadratic/linear inequalities review.

Acceleration/Previewing: (Key Vocabulary) Review interval notation

Teaching Strategies:
Rewrite inequality as equation.
Apply Zero Product Property to Identify Zeros.
Graph zeros on number line.
Use Test Point method to determine solution set.
Use interval notation to write solution.

Examples from Purplemath

- **Solve $x^2 - 3x + 2 > 0$**

First, I have to find the x-intercepts of the associated quadratic, because the intercepts are where $y = x^2 - 3x + 2$ is *equal* to zero. Graphically, an inequality like this is asking me to find where the graph is above or below the x-axis. It is simplest to find where it actually *crosses* the x-axis, so I'll start there.

Factoring, I get $x^2 - 3x + 2 = (x - 2)(x - 1) = 0$, so $x = 1$ or $x = 2$. Then the graph crosses the x-axis at 1 and 2, and the number line is divided into the intervals (negative infinity, 1), (1, 2), and (2, positive infinity). Between the x-intercepts, the graph is either above the axis (and thus positive, or greater than zero), or else below the axis (and thus negative, or less than zero).

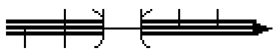
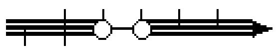
There are two different algebraic ways of checking for this positivity or negativity on the intervals. I'll show both.

1) Test-point method. The intervals between the x-intercepts are (negative infinity, 1), (1, 2), and (2, positive infinity). I will pick a point (any point) inside each interval. I will calculate the value of y at that point. Whatever the sign on that value is, that is the sign for that entire interval.

For (negative infinity, 1), let's say I choose $x = 0$; then $y = 0 - 0 + 2 = 2$, which is positive. This says that y is positive on the whole interval of (negative infinity, 1), and this interval is thus part of the solution (since I'm looking for a "greater than zero" solution).

For the interval (1, 2), I'll pick, say, $x = 1.5$; then $y = (1.5)^2 - 3(1.5) + 2 = 2.25 - 4.5 + 2 = 4.25 - 4.5 = -0.25$, which is negative. Then y is negative on this entire interval, and this interval is then not part of the solution.

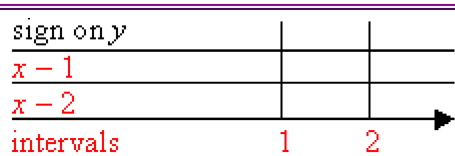
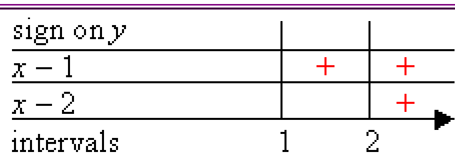
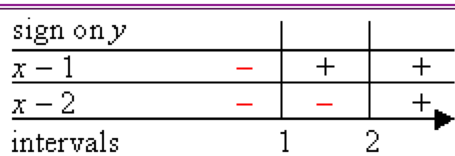
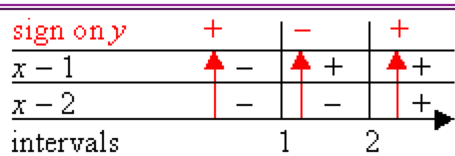
For the interval (2, positive infinity), I'll pick, say, $x = 3$; then $y = (3)^2 - 3(3) + 2 = 9 - 9 + 2 = 2$, which is positive, and this interval is then part of the solution. Then the complete solution for the inequality is $x < 1$ and $x > 2$. This solution is stated variously as:

$x < 1,\ \ x > 2$	**inequality notation**
$x \in (-\infty, 1) \cup (2, +\infty)$	**interval, or set, notation**
	**number line with parentheses (brackets are used for closed intervals)**
	**number line with open dots (closed dots are used for closed intervals)**

The particular solution format you use will depend on your text, your teacher, and your taste. Each format is equally valid.

2) Factor method. Factoring, I get $y = x^2 - 3x + 2 = (x - 2)(x - 1)$. Now I will consider each of these factors separately.

The factor $x - 1$ is positive for $x > 1$; similarly, $x - 2$ is positive for $x > 2$. Thinking back to when I first learned about negative numbers, I know that (plus)×(plus) = (plus), (minus)×(minus) = (plus), and (minus)×(plus) = (minus). So, to compute the sign on $y = x^2 - 3x + 2$, I only really need to know the signs on the factors. Then I can apply what I know about multiplying negatives.

First, I set up a grid, showing the factors and the number line.	
Now I mark the intervals where each factor is positive.	
Where the factors aren't positive, they must be negative.	
Now I multiply up the columns, to compute the sign of y on each interval.	

Then the solution of $x^2 - 3x + 2 > 0$ are the two intervals with the "plus" signs:

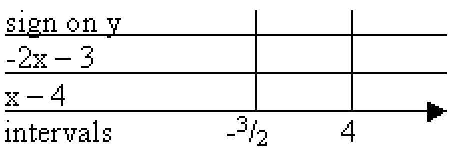
(negative infinity, 1) and (2, positive infinity).

- **Solve $-2x^2 + 5x + 12 \leq 0$.**

  First I find the zeroes, which are the endpoints of the intervals: $y = -2x^2 + 5x + 12 = (-2x - 3)(x - 4) = 0$ for $x = -{}^3/_2$ and $x = 4$. So the endpoints of the intervals will be at $-{}^3/_2$ and 4. The intervals are between the endpoints, so the intervals are (negative infinity, $^{-3}/_2$], [$^{-3}/_2$, 4], and [4, positive infinity). (Note that I use *brackets* for the endpoints in "or equal to" inequalities, instead of parentheses, because the endpoints will be included in the final solution.)

  To find the intervals where y is negative by the Test-Point Method, I just pick a point in each interval. I can use points such as $x = -2$, $x = 0$, and $x = 5$.

  To find the intervals where y is negative by the Factor Method, I just solve each factor: $-2x - 3$ is positive for $-2x - 3 > 0$, $-3 > 2x$, $-3/2 > x$, or $x < -{}^3/_2$; and $x - 4$ is positive for $x - 4 > 0$, $x > 4$. Then I fill out the grid:

  Then the solution to this inequality is all x's in

  (negative infinity, $^{-3}/_2$] and [4, positive infinity).

- **Solve $x^5 + 3x^4 - 23x^3 - 51x^2 + 94x + 120 \geq 0$.**

  First, I factor to find the zeroes:

  $$x^5 + 3x^4 - 23x^3 - 51x^2 + 94x + 120$$

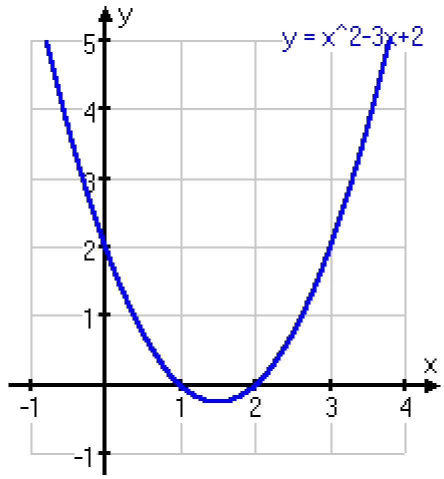
  $$= (x + 5)(x + 3)(x + 1)(x - 2)(x - 4) = 0$$

  ...so $x = -5, -3, -1, 2$, and 4 are the zeroes of this polynomial. (Review how to solve polynomials, if you're not sure how to get this solution.)

Then the solution (remembering to include the endpoints, because this is an "or equal to" inequality) is the set of x-values in the intervals **[–5, –3], [–1, 2], and [4, positive infinity]**.

Graphing Solutions to inequalities

There is another way to solve inequalities. You still have to find the zeroes (x-intercepts) first, but then you graph the function, and just look: wherever the graph is above the x-axis, the function is positive; wherever it is below the axis, the function is negative. For instance, for the first quadratic exercise, $y = x^2 - 3x + 2 > 0$, wx +e found the zeroes at $x = 1$ and $x = 2$. Now look at the graph:



On the graph, the solution is obvious: you would take the two intervals (but not the interval endpoints) w here the line is above the *x*-axis.

Solve Exponential and Logarithmic Inequalities

The same methods used to solve polynomial inequalities can be used to solve exponential and logarithmic Inequalities.

Example: Solve $4^{x+1} \geq 32$

Task: Is it Safe to Eat Task

At the end of the Unit: Culminating Task: Suitcase Design

Distributed Guided Practice:

Solve the inequality algebraically.

a. $2x^4 - 7x^2 > 4$ b. $x^3 - x^2 - 16x + 16 \geq 0$

Solve the inequality graphically.

a. $x^4 - 11x^2 + 18 \leq 0$ b. $3x^4 - 1 \leq 11$

Solving Logarithmic Inequalities Worksheet

Extending/Refining Strategies: Have the students create their own test questions showing work explanations and solutions.

Summarizing Strategies: Journal: Explain how to solve polynomial inequalities.

Name ______________________

Date______________________

Solving Logarithmic Inequalities

Solve each inequality and graph the solution on a number line.

1. $4^{x+2} \leq 36$

2. $\log_5 x \leq 4$

3. $\log_4 x + 9 > 12$

4. $\log_3 x - 1 > 3$

5. $\log_6 2x + 9 < 12$

6. $\log_9(x-8) \geq \frac{6}{4}$

7. $\log_4 x > 1$

8. $\log_5 x \leq 3$

9. $\log_6 x + 3 \geq 4$