Math 3

Unit 4

Solving Equations and Inequalities

Georgia Performance Standards
High School Mathematics
Mathematics III: Advanced Algebra/Statistics

Georgia Performance Standards: Curriculum Map					
1st Semester			2nd Semester		
Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6
Modeling with Matrices	Conics	Logarithmic and Exponential Functions	Solving Equations and Inequalities	Polynomial Functions	Data Analysis
4 Weeks	7 Weeks	6 Weeks	5 Weeks	5 Weeks	4 Weeks
MM3A4 **MM3A5** **MM3A6** **MM3A7**	**MM3G1** **MM3G2** **MM3G3c**	**MM3A2a,b,c,e,f,g**	**MM3A2d,g** **MM3A3**	**MM3A1** **MM3G3a,b**	**MM3D1** **MM3D2** **MM3D3**
These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units. All units will include the Process Standards and indicate skills to maintain.					

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

Mathematics III – Unit 4
Solving Equations and Inequalities
(Polynomial, Exponential, Rational, and Logarithmic)

INTRODUCTION:

In unit 3, properties of exponents are studied with great detail and the logarithmic functions are introduced as an inverse of the function. Since Mathematics I, we have studied a variety of functions and have learned to solve them with a plethora of methods. This unit gives us a chance to bring it together and solve equations with known strategies and how to solve when those are not enough. A comprehensive understanding of work done in Mathematics I and Mathematics II will be necessary. Weaknesses of equation solving will be made evident in this unit, so there will need to be a readiness to address them.

ENDURING UNDERSTANDINGS:

- Solve exponential and logarithmic equations analytically and graphically

- Use characteristics of functions to understand graphic solutions

- Find and interpret solutions of higher order polynomials

- Importance of exponential and logarithmic models to interpret real phenomena.

KEY STANDARDS ADDRESSED:

MM3A2. Students will explore logarithmic functions as inverses of exponential functions.
      d. Understand and use properties of logarithms by extending laws of exponents.
      g. Explore real phenomena related to exponential and logarithmic functions including half-life and doubling time.

MM3A3. Students will solve a variety of equations and inequalities.
      a. Find real and complex roots of higher degree polynomial equations using the factor theorem, remainder theorem, rational root theorem, and fundamental theorem of algebra, incorporating complex and radical conjugates.
      b. Solve polynomial, exponential, and logarithmic equations analytically, graphically, and using appropriate technology.
      c. Solve polynomial, exponential, and logarithmic inequalities analytically, graphically, and using appropriate technology. Represent solution sets of inequalities using interval notation.
      d. Solve a variety of types of equations by appropriate means choosing among mental calculation, pencil and paper, or appropriate technology.

RELATED STANDARDS ADDRESSED:

MM3A1. Students will analyze graphs of polynomial functions of higher degree.

    d. Investigate and explain characteristics of polynomial functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior.

MM3P1. Students will solve problems (using appropriate technology).

    a. Build new mathematical knowledge through problem solving.
    b. Solve problems that arise in mathematics and in other contexts.
    c. Apply and adapt a variety of appropriate strategies to solve problems.
    d. Monitor and reflect on the process of mathematical problem solving.

MM3P2. Students will reason and evaluate mathematical arguments.

    a. Recognize reasoning and proof as fundamental aspects of mathematics.
    b. Make and investigate mathematical conjectures.
    c. Develop and evaluate mathematical arguments and proofs.
    d. Select and use various types of reasoning and methods of proof.

MM3P3. Students will communicate mathematically.

    a. Organize and consolidate their mathematical thinking through communication.
    b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
    c. Analyze and evaluate the mathematical thinking and strategies of others.
    d. Use the language of mathematics to express mathematical ideas precisely.

MM3P4. Students will make connections among mathematical ideas and to other disciplines.

    a. Recognize and use connections among mathematical ideas.
    b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
    c. Recognize and apply mathematics in contexts outside of mathematics.

MM3P5. Students will represent mathematics in multiple ways.

    a. Create and use representations to organize, record, and communicate mathematical ideas.
    b. Select, apply, and translate among mathematical representations to solve problems.
    c. Use representations to model and interpret physical, social, and mathematical phenomena.

<u>**Unit Overview:**</u>

Since Unit 3 addressed the properties of exponentials and logarithms as functions, Unit 4 will start with the properties of logarithms as an extension of rules of exponents (M3A2d). After looking at the properties, students will learn to solve application problems using the properties of logarithms and exponents. Since a main component of solving applications will be finding roots, students will then have the opportunity to discuss finding roots for other types of functions they have already studied, namely higher-degree polynomials. This will allow for the introduction and investigation of theorems for finding complex roots of polynomials. The goal is to weave the solving of inequalities into the unit in each sub-section since the unit could be broken into two parts, solving logs and exponentials, and then solving polynomials. One connection between the two different function sets is the logarithmic transformation of data to determine if a scatter plot represents exponential data or power function data. This represents a natural bridge between the two types of functions.

Historical Background: Discover logarithmic properties: A calculator driven exploration task, and eventually lead into an activity which asks: If $2^1 = 2$ and $2^2 = 4$, what power do we need for $2^x = 3$?

Potato Lab: Newton's Law of Cooling: Heat the potato in the microwave, take temperature afterwards, place potato in refrigerator, record readings in fridge and of potato, calculate the cooling coefficient and derive Newton's formula.

Is it Safe to Eat: With this lab, students will also use inequalities with an emphasis on food service industry. For example, at what temperature would the potato be too hot to serve in a restaurant? How long could the potato sit at room temperature before it cooled too much to be served?

Polynomial Root Task: In this task, students will revisit concepts of solving quadratics from Math 2 while taking the opportunity to extend their strategies with the concepts of synthetic division, the remainder theorem, and the rational root theorem. This task is intended to be used as guided instruction in the classroom.

Suitcase Design Task: In this culminating activity, the students will need to use their knowledge of polynomials to design a scale model suitcase that fits within certain criteria. In doing so, they will make use of techniques to solve equations and inequalities that have been emphasized in this unit.

Vocabulary and formulas:

Common logarithm: A logarithm with a base of 10. A common logarithm is the power, a, such that $10^a = b$. The common logarithm of x is written *log x*.
For example, *log 100* = 2 because $10^2 = 100$.

Exponential functions: A function of the form $y = a \cdot b^x$ where $a > 0$ and either $0 < b < 1$ or $b > 1$.

Logarithmic functions: A function of the form $y = \log_b x$, with $b \neq 1$ and b and x both positive. A logarithmic function is the inverse of an exponential function. The inverse of $y = b^x$ is $y = \log_b x$.

Higher order polynomials: A polynomial is considered to be higher order if the degree of the polynomial is greater than 2.
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Rational Root Theorem: A theorem that provides a complete list of possible rational roots of the polynomial equation $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 = 0$ where all coefficients are integers.
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Synthetic Division:. Synthetic division is a shortcut method for dividing a polynomial by a linear factor of the form (x − a). It can be used in place of the standard long division algorithm.

Remainder Theorem: An application of polynomial long division. It states that the remainder of a polynomial f(x) divided by a linear divisor (x - a) is equal to f(a).

Unit 4 – Solving Equations and Inequalities (5 weeks)

Unit Essential Questions:

How can I solve various equations and inequalities?

Lesson 1: **Applying the Properties of Logarithms and Exponential Functions**

Essential Question:

How do properties of logarithms and laws of exponents relate to real
phenomena?

Lesson 2: **Solving Equations**

Essential Questions:

How do I solve polynomial equations?
How do I solve logarithmic and exponential equations?

Lesson 3: **Solving Inequalities**

Essential Questions:

How do I solve polynomial inequalities?
How do I solve logarithmic and exponential inequalities?