### Decision One: Student Learning Map of Unit 2 Algebraic Investigations

**Key Learning(s):**

1. Algebraic equations can be identities that express properties of operations on real numbers.
2. Equivalence of algebraic expressions means that the expressions have the same numerical value for all possible values of the variable.
3. Equivalent expressions are useful tools in computation and problem solving.
4. It takes only one counterexample to show that a general statement is not true.

**Unit Essential Question(s):**

In what situations would it be helpful to represent a real life situation with an algebraic expression?

What could the algebraic expression help you determine about a real life situation?

**Optional Instructional Tools:** Graphing Calculator, Algebra Tiles or Algebra Blocks

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**Concept 1**

Many different expressions can represent the same quantity

**Lesson Essential Questions**

1. How can you justify different expressions representing the same quantity?
2. How would you know if an application can be modeled by a linear or quadratic expression/function?

**Vocabulary**

1. Theorem
2. Counterexample
3. Quadratic

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**Concept 2**

Polynomials

**Lesson Essential Questions**

1. How can you use patterns to show that the formulas for special products will hold true?
2. How is Pascal’s Triangle related to the Binomial Theorem for expanding binomials?
3. How do you solve and model real life problems using polynomial arithmetic?

**Vocabulary**

1. Difference of squares
2. Distributive property
3. Area model for polynomial arithmetic
4. Volume model for polynomial arithmetic
5. Expanded form
6. Zero Product Property
7. Monomial
8. Binomial
9. Trinomial

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**Concept 3**

Simplifying rational expressions

**Lesson Essential Questions**

1. How do you use your prior knowledge of performing operations with fractions to performing operations with rational expressions?
2. How can you apply your knowledge of simplifying rational expressions using special formulas such as \(d = rt\)?

**Vocabulary**

1. Rational expression

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**Concept 4**

Simplifying expressions with square roots

**Lesson Essential Questions**

1. How do you use prior knowledge of performing operations with variables to performing operations with square roots?
2. How do you rationalize a radical in the denominator?

**Vocabulary**

1. Radical expression
2. Rationalize the denominator
3. Simplify a radical
4. Radical
5. Index
6. Radicand
7. Radical conjugates
Notes:
1. Students should be able to show that two different expressions can represent the same quantity by:
   a. creating tables by substituting values for the variable(s)
   b. showing that when f(x)=g(x), then the solution is the intersection of the graph of y=f(x) and the graph of y=g(x)
   c. use algebraic properties to justify each step.
3. Emphasize reading and writing of mathematics.

Notes:
1. Factoring will not be taught in this unit, but will be taught in Unit 5.
2. When multiplying the special products listed in standard, model with area (algebra tiles or algeblocks) or volume (algeblocks) problems.
3. Use distributive property to explain multiplication of binomials before introducing FOIL.
4. Students should see the equivalency between the binomial factors and the product, thus their interchangeability.
5. Just point out that if $a \cdot b = 0$, then either $a=0$, $b=0$, or $a$ and $b=0$. Do not use the Zero Property to solve quadratic equations at this time. This is only an introduction.

Notes:
1. Use the interchangeability of the special products and their formulas to simplify rational algebraic expressions by canceling out special cases of 1.
2. Stress that denominators must be alike before you can add or subtract numerators.
3. Be sure to use d=rt formula.

Notes:
1. Exponential form of radicals will not be introduced until Math III.
2. Students should be able to compute with terms that include whole number exponents.
3. Exponents that are integers will be taught in Math II and rational exponents will be taught in Math III.
### Essential Question:

How would you justify that expressions represent the same quantity?

### Activating Strategies: (Learners Mentally Active)

<table>
<thead>
<tr>
<th>Warm up: Session 1 &amp; 2: Cell Phone Warm Up (see attachment)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warm up: Session 3: Linear vs. Quadratic Warm Up (see attachment)</td>
</tr>
</tbody>
</table>

Activating Strategy Session 1 & 2: Small group Activity: Tiling Learning Task #1 (a-d)

Activating Strategy Session 3: Write in words the differences observed from the Session 3 warm up. (see attachment)

### Acceleration/Previewing: (Key Vocabulary)

- Counterexample
- Quadratic

### Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

**Session 1 & 2:**

- Students will share their expressions from Task 1 (d).
- Class will discuss whether the expressions are equivalent.
- Students will complete graphic organizer covering “Techniques for Showing Two Expressions are Equivalent” (see attachment)
- Teacher will: (1) model with algebra tiles, (2) students will make a table of values for each expression; (3) Students will graph both functions as strategies to determine whether two expressions are equivalent; GO has one example and one common error (nonexample).
- Students will continue working on the Tiling Learning Task by completing #2 (a-h) in small groups.
- Using the three methods outlined in the graphic organizer, justify the equivalency of expressions from #2 (d & h)
- Discuss difference between the graphs of linear functions and those that are different. Introduce concept of quadratics.

**Session 3:**

- Students will work in small groups to complete Task #1, Problems 3 & 4
Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Session 1 & 2:
- Students will complete task 1 (e-h) in small groups.
- Teacher will gather and display student responses to Task 1, #1 (h).
- Students will provide justification that the expressions given by other groups are equivalent.
- Students will work in small groups to complete an investigation using Task 1, #2 (i & j).

Summarizing Strategies: Learners Summarize & Answer Essential Question
Session 1 & 2:
Students will answer the essential question using any technique learned to justify two expression are equivalent. (Ticket Out the Door) (see attachment)

Session 3: Distribute Ticket Out the Door from Sessions 1 & 2 to students to add explanation in words.

Session 4: Use Tiling Pools Learning Task #1-6 for the lesson assessment.
<table>
<thead>
<tr>
<th>Essential Question:</th>
</tr>
</thead>
<tbody>
<tr>
<td>How do you compare simplifying radical expressions to simplifying radical expressions with variables?</td>
</tr>
<tr>
<td>-Work a problem with Pythagorean Theorem</td>
</tr>
<tr>
<td>-Ladder task numbers 1 and 2(could possibly be used as a warm up or just do it completely in class)</td>
</tr>
<tr>
<td>How do you rationalize a denominator?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Activating Strategies: (Learners Mentally Active)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-Use Ti-83/84 calculator to graph ( y = \sqrt{x} )</td>
</tr>
<tr>
<td>Is it a function?</td>
</tr>
<tr>
<td>What's the domain? Range?</td>
</tr>
<tr>
<td>*If you need help using the graphing calculator for this activity, use the following link and look at the 3rd section which is graphing functions.</td>
</tr>
</tbody>
</table>

-Introduction to problem: say you have a 4m long pendulum and a 9m long pendulum; predict frequency of the pendulum (use a prediction guide). Add 6m (students will not have done square roots that are not perfect, this question should be answered in the summarizer)

-Pendulum problem:
Simple pendulums are interesting and important physical devices. You may have studied pendulum motion in science experiments by attaching a weight to a string and looking for patterns relating the weight, the length of the string, and the motion of the weight as it swings from side to side.
It turns out that the frequency of a pendulum (in swings per time unit) depends only on the length of the pendulum arm not the weight of the bob or the initial starting point of the swings. The function \( F = \frac{30}{\sqrt{L}} \) is a good model for the relationship between pendulum arm length \( L \) and frequency of swing \( F \) when length is measured in meters and frequency in swings per minute.

<table>
<thead>
<tr>
<th>Acceleration/Previewing: (Key Vocabulary)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radical expression</td>
</tr>
<tr>
<td>Simplify a radical</td>
</tr>
<tr>
<td>Radical</td>
</tr>
<tr>
<td>Index (pre-view cube roots/fourth roots, etc only)</td>
</tr>
<tr>
<td>Radicand</td>
</tr>
<tr>
<td><em>Previewing Vocabulary</em> - use graphic organizer day before class for students who need acceleration (those who may be in a math support class).</td>
</tr>
</tbody>
</table>

Rationalizing denominators
Radical conjugates

<table>
<thead>
<tr>
<th>Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-Begin a graphic organizer with vocabulary in lesson</td>
</tr>
<tr>
<td>-Direct Instruction on simplifying radical expressions without variables (max15 minute review)</td>
</tr>
<tr>
<td>Use paired heads (have 1's tell 2's ...); summarize review of simplifying radical expressions with variables.</td>
</tr>
<tr>
<td>-Direct instruction on simplifying radical expressions with variables (adding, subtracting, multiplying – do not cover division)</td>
</tr>
<tr>
<td>*This direct instruction may be split up with guided practice and distributed practice to allow breaks in instruction.</td>
</tr>
<tr>
<td>*all variables are non-negative integers in this lesson</td>
</tr>
<tr>
<td>-During direction instruction, once vocabulary words are taught, they should be brought over to</td>
</tr>
</tbody>
</table>

-Direct instruction: rationalizing denominators, radical conjugates
Begin with monomial radicals and move towards more difficult problems
-Summarizer Ex: Explain how to simplify \( 30/\sqrt{8} \)
*This direct instruction may be split up with guided practice and distributed practice to allow breaks in instruction.

-Direct Instruction: rationalizing denominators with conjugates
-Summarizer: Summarize how to do both.

**Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)**

How do you compare simplifying radical expressions to simplifying radical expressions with variables?
-Work a problem with Pythagorean Theorem
-Ladder task numbers 1 and 2 (could possibly be used as a warm up or just do it completely in class)

-Guided Practice on rationalizing denominators with monomial radicals
-Guided practice on rationalizing denominators with conjugates

**Summarizing Strategies: Learners Summarize & Answer Essential Question**

-Use Ti-83/84 calculator to graph $y = \sqrt{x}$
  Is it a function?
  What’s the domain? Range?
  *If you need help using the graphing calculator for this activity, use the following link and look at the 3rd section which is graphing functions.

Summarize Activating Strategy by answering prediction guide that was done.

Contact law enforcement agency and have them come talk with students about how radical expressions are used in law enforcement as a summarizer for this concept. (How they calculate speed from skid marks using radical expressions.) You could possibly simulate a problem and allow students to solve it (based on time available).

-School Resource Officers
-City Police
-County
-State Patrol

A very detailed organizer of this problem can be found at [http://www.harristechnical.com/articles/skidmarks.pdf](http://www.harristechnical.com/articles/skidmarks.pdf). You can take pieces of the 9 page document to use for this summary if there is not a law enforcement person available to give a presentation. (There are examples, definitions of terms, etc.)

Lesson Quiz over simplifying radicals with radicals
\[ Y = \sqrt{x} \quad y = \sqrt{(x) + 1} \]

<table>
<thead>
<tr>
<th>Word</th>
<th>Symbol/Example</th>
<th>Meaning</th>
<th>“What is it like”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radical Sign</td>
<td>[ Y = \sqrt{x} ]</td>
<td>The sign ( \sqrt{\cdot} )</td>
<td>( \sqrt{} )</td>
</tr>
<tr>
<td>Radicand</td>
<td>[ Y = \sqrt{x} ]</td>
<td>The term underneath the radical sign</td>
<td></td>
</tr>
<tr>
<td>Radical</td>
<td>( Y = \sqrt{x} ) (highlight or color radical)</td>
<td>An expression that contains a radical sign</td>
<td>( Y = \sqrt{49x} )</td>
</tr>
<tr>
<td>Radical</td>
<td>[ Y = \sqrt{(x) + 1} ]</td>
<td>An expression that contains one or more terms involving a radical</td>
<td>A number sentence that has ( \sqrt{\cdot} ) in it.</td>
</tr>
<tr>
<td>Conjugates</td>
<td>The conjugate of ( (3 - \sqrt{2}) ) is ( (3 + \sqrt{2}) )</td>
<td>The expression that when multiplied by another expression eliminates a radical</td>
<td>Two expressions that are multiplied together that get rid of a square root sign</td>
</tr>
<tr>
<td>Word</td>
<td>Symbol/Example</td>
<td>Meaning</td>
<td>“What is it like”</td>
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<td>Radical Expression</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Conjugates</td>
<td></td>
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</tbody>
</table>
Simplifying radicals

I. Simplify
   1) $\sqrt{36}$
   2) $\sqrt{72}$
   3) $\sqrt{200}$
   4) $\sqrt{v^6}$
   5) $\sqrt{200a^2b^2}$

II. Simplify
   1) $\sqrt{8}\cdot\sqrt{32}$
   2) $3\sqrt{81}\cdot\sqrt{81}$
   3) $\sqrt{10x^2}\cdot\sqrt{40y^3}$
   4) $(5\sqrt{7})^3$

III. Simplify
   1) $15\sqrt{9} - \sqrt{9}$
   2) $\sqrt{18} + \sqrt{3}$
   3) $3(\sqrt{27} + 1)$
   4) $3\sqrt{7} - \sqrt{28}$
Pendulum problem:
Simple pendulums are interesting and important physical devices. You may have studied pendulum motion in science experiments by attaching a weight to a string and looking for patterns relating the weight, the length of the string, and the motion of the weight as it swings from side to side.

It turns out that the frequency of a pendulum (in swings per time unit) depends only on the length of the pendulum arm not the weight of the bob or the initial starting point of the swings). The function $F = \frac{30}{\sqrt{L}}$ is a good model for the relationship between pendulum arm length $L$ and frequency of swing $F$ when length is measured in meters and frequency in swings per minute.

<table>
<thead>
<tr>
<th>Length of the pendulum</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Introduction: Which pendulum do you think will have the highest frequency (will swing the longest)? Explain.

Summary: Which pendulum actually swings the longest? How do you know?
## Acquisition Lesson
**Concept:** Simplifying Rational Expressions
**7 sessions (Sessions 1 – 4)\(^\text{1}\)**

### Essential Question:
How do you use your prior knowledge of performing operations with fractions to performing operations with rational expressions?

### Activating Strategies:
(Learners Mentally Active)

<table>
<thead>
<tr>
<th>Session</th>
<th>Topic</th>
<th>Warm – Up (5 min)</th>
<th>Discuss student answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session 1</td>
<td>Multiplication/Division of Rational Expressions</td>
<td>3 questions</td>
<td>Just Jogging #1</td>
</tr>
<tr>
<td>Session 2</td>
<td>Multiplication/Division of Rational Expressions with application</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Session 3</td>
<td>Adding/Subtracting of rational expressions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Session 4</td>
<td>Adding/Subtracting of rational expressions with application (end with summarizer assessment)</td>
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</tr>
</tbody>
</table>

### Acceleration/Previewing:
(Key Vocabulary)

- **Key Vocabulary:** Rational expression, expanded form, factored form, excluded values, complex fraction, simplest form, least common denominator
- **Graphing Organizers:** use attached graphing organizers “Rational expressions graphic organizers” (4 of them)

### Standard:
**MM1A2e:** Add, Subtract, multiply, and divide rational expressions.
Session 1: Multiplication/Division of Rational Expressions (30 minutes)

Whole group discussion

**Task:** Distribute graphic organizers and discuss

**Discuss:** Why can’t you divide a number by zero? What about 4/x; what value(s) can x not be? Why? What about 5/(x – 1) or 6/(2 + x)? What values can x not be? (7x)/(x^2 – 25)? What values can x not be? These are called excluded values. Discuss why the calculator says “error”.

**Guided Practice:**

1. \[ \frac{14}{c^2} \cdot \frac{c^5}{2c} \]

2. \[ \frac{3m^2}{2n} \cdot \frac{n^2}{12} \]

3. \[ \frac{x^2 - 4}{5}, \frac{x + 2}{x - 2} \]

4. \[ \frac{x + 1}{x + 5} \div \frac{2x + 2}{x + 4} \]

5. \[ \frac{5y + 15}{y + 6} \div \frac{2y + 6}{y - 3} \]

**Think/Pair/Share:** see attached and do even

Session 2: Multiplication/Division of Rational Expressions with application (end with summarizer assessment)

Groups of 4 – 6

**Task:** Students will be split up into groups of 4 – 6 and will rotate about 6 stations (approx. 7 minutes each). Teacher will preview and discuss example(s) before students rotate. See attached for station problems (Note: Teachers may add/subtract problems as they see fit)

**Guided Practice:**

1. The lengths of the sides of a right triangle can be expressed as x + 2 inches, x + 9 inches, and x + 10 inches. Find the lengths of the sides. (Hint: Pythagorean theorem)
   
   **Answer:** 5, 12, 13

2. The area of a triangle can be expressed as 4x^2 – 2x – 6 square meters. The height of the triangle is x + 1 meters. Find the length of the base of the triangle.
   
   **Answer:** (8x – 12) meters
**Session 3: Adding/Subtracting of rational expressions**

Whole group discussion

**Task:** Distribute graphic organizers and discuss

**Discuss:** Make sure to discuss the importance of LCD.

**Guided Practice:**

1. \( \frac{2p}{p+1} + \frac{2}{p+1} \)
2. \( \frac{3s}{s-5} - \frac{3s}{5-s} \)
3. \( \frac{m+3}{2m} + \frac{m-2}{m} \)
4. \( \frac{3x + 15}{x^2 - 25} + \frac{x}{x + 5} \)
5. \( \frac{5x}{3y^2} - \frac{2x}{9y} \)
6. \( \frac{a}{2a - 1} - \frac{2}{a + 3} \)

**Think/Pair/Share:** see attached and do even

**Session 4: Adding/Subtracting of rational expressions with application (end with summarizer assessment)**

**Session 1: Multiplication/Division of Rational Expressions**

Students finish problems they didn’t finish in think/pair/share (do odd)

**Session 2: Multiplication/Division of Rational Expressions with application**

Distribute 2 question quiz

**Session 3: Adding/Subtracting of rational expressions**

Students finish problems they didn’t finish in think/pair/share (do odd)

**Session 4: Adding/Subtracting of rational expressions with application (end with summarizer assessment)**

Distribute 2 question quiz
Session 1: Multiplication/Division of Rational Expressions

HOMEWORK  Ander and Alejandro were working on the following homework problem.

\[ \frac{n - 10}{n + 3} \cdot \frac{2n + 6}{n + 3} \]

Alejandro's Solution
\[
\frac{n - 10}{n + 3} \cdot \frac{2n + 6}{n + 3} = \frac{2n - 20}{n + 3}
\]

Ander's Solution
\[
\frac{n - 10}{n + 3} \cdot \frac{2n + 6}{n + 3} = \frac{2(n - 10)(n + 3)}{n + 3}
\]

Which student is correct? Explain.

Session 2: Multiplication/Division of Rational Expressions with application
Ticket out the door: summarizing quiz

Session 3: Adding/Subtracting of rational expressions

SWIMMING  Power Pools installs swimming pools. To determine the appropriate size of pool for a yard, they measure the length of the yard in meters and call that value \( x \). The length and width of the pool are calculated with the diagram below. Write an expression in simplest form for the perimeter of a rectangular pool for the given variable dimensions.
Session 4: Adding/Subtracting of rational expressions with application (end with summarizer assessment)

Ticket out the door: summarizing quiz
**Essential Question:**

How can you apply your knowledge of simplifying rational expressions using special formulas including \( d = rt \)?

### Session 5: Just Jogging Learning Task #6 - 8

**Warm – Up (5 min)**
Discuss student answers

**Activator (10 min)**
Just Jogging Learning Task #5

### Session 6: Just Jogging Learning Task #9 – 10

**Warm – Up (5 min)**

The inward flow and outward flow of a football player’s money (in millions of dollars during his ten year career from 1998 to 2007) can be modeled by the rational expressions in the table below, where \( x \) is the number of years since 1998.

<table>
<thead>
<tr>
<th>Income</th>
<th>Salary</th>
<th>Endorsements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{8(x + 1)(x + 2)}{(x + 1)(x + 5)} )</td>
<td>( \frac{x + 23}{x + 5} )</td>
</tr>
</tbody>
</table>

a. In which year did the player make 3 14/30 million dollars after expenses?

b. Suppose the player deposits an amount of money given by the sequence \( a_n = 1000n^2 \) in a retirement account each year, where \( n \) represents the nth year in his career. Write the amount of his first 5 deposits.

c. Is the players’ annual deposit ever greater than either expense listed in the label for that year? Explain.

### Session 7: Overall Assessment

**Warm – Up (5 min)**
Discuss student questions before assessment
### Acceleration/Previewing:
(Key Vocabulary)

**Key Vocabulary:** Rational expression, expanded form, factored form, excluded values, complex fraction, simplest form, least common denominator

### Teaching Strategies:
(Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

### Distributed Guided Practice/Summarizing Prompts:
(Prompts Designed to Initiate Periodic Practice or Summarizing)

### Summarizing Strategies:
Learners Summarize & Answer Essential Question

### Graphing Organizers:
use attached graphing organizers “Rational expressions graphic organizers” (4 of them)

### Session 5: Just Jogging Learning Task #6 - 8
- Guided Practice #6
- Independent Practice #7
- Pairs – Begin #8

### Session 6: Just Jogging Learning Task #8 – 10
- Groups of 3 – 4 complete #8 - 10

### Session 7: Overall Assessment
Distribute assessment (see attached)

### Session 5: Just Jogging Learning Task #1 - 5
- Modified KWL

### Session 6: Just Jogging Learning Task #6 – 10
- Ticket out the door – answer the EQs in your own words

### Session 7: Overall Assessment
Journal: Describe what you have learned about rational expressions. How are they used in “the real world”?
Session 1: Multiplication/Division of Rational Expressions

Think/Pair/Share:

State the excluded values for each rational expression.

1. \( \frac{2p}{p - 7} \)

2. \( \frac{4n + 1}{n + 4} \)

3. \( \frac{k + 2}{k^2 - 4} \)

4. \( \frac{3x + 15}{x^2 - 25} \)

5. \( \frac{y^2 - 9}{y^2 + 3y - 18} \)

6. \( \frac{b^2 - 2b - 8}{b^2 + 7b + 10} \)

Find each product.

1. \( \frac{14}{c^2} \cdot \frac{c^8}{2c} \)

2. \( \frac{3m^2}{2n} \cdot \frac{n^2}{12} \)

3. \( \frac{2a^2b}{b^2c} \cdot \frac{b}{a} \)

4. \( \frac{2x^2y}{3x^2y} \cdot \frac{3xy}{4y} \)

5. \( \frac{3(4m - 6)}{18n} \cdot \frac{9n^2}{2(4m - 6)} \)

6. \( \frac{4(n + 2)}{n(n - 2)} \cdot \frac{n - 2}{n + 2} \)

7. \( \frac{(y - 3)(y + 3)}{4} \cdot \frac{8}{y + 3} \)

8. \( \frac{(x - 2)(x + 2)}{x(x + 3)} \cdot \frac{2(8x + 3)}{x - 2} \)

9. \( \frac{(a - 7)(a + 7)}{a(a + 5)} \cdot \frac{a + 5}{a + 7} \)

10. \( \frac{4(b + 4)}{(b - 4)(b - 3)} \cdot \frac{b - 3}{b + 4} \)

11. \( \frac{x^2 - 4}{5} \cdot \frac{x + 2}{x - 2} \)

12. \( \frac{1 - c^2}{12} \cdot \frac{4}{1 - c} \)

13. \( \frac{y^2 - 36}{y^2 - 25} \cdot \frac{y + 5}{y - 6} \)

14. \( \frac{a + 2}{a^2 - a - 6} \cdot \frac{a - 3}{a + 1} \)

15. \( \frac{x + 4}{x} \cdot \frac{x^2}{x^2 + 5x + 4} \)

16. \( \frac{x^2 + x - 20}{x^2} \cdot \frac{x}{x + 5} \)
Find each quotient.

1. \( \frac{c^3}{d^3} \div \frac{d^3}{c^3} \)

3. \( \frac{6c^3}{4f^2} \div \frac{2c^2}{12f^2} \)

5. \( \frac{3b + 3}{b + 2} \div (b + 1) \)

7. \( \frac{c^2 - 4}{c} \div (c + 2) \)

9. \( \frac{x + 1}{x + 5} \div \frac{2x + 2}{x + 4} \)

2. \( \frac{x^2}{y^2} \div \frac{x^3}{y} \)

4. \( \frac{4m^3}{np^2} \div \frac{2m}{np} \)

6. \( \frac{x - 5}{x + 3} \div (x - 5) \)

8. \( \frac{b^2 - 25}{2b} \div (b - 5) \)

10. \( \frac{2n + 6}{n - 4} \div \frac{n + 3}{n - 4} \)

17. \( \frac{x^2 - x - 12}{6} \div \frac{x + 3}{x - 4} \)

19. \( \frac{m^2 + 2m + 1}{10m - 10} \div \frac{m + 1}{20} \)

21. \( \frac{b + 4}{b^2 - 8b + 16} \div \frac{2b + 8}{b - 8} \)

18. \( \frac{a^2 - 5a - 6}{3} \div \frac{a - 6}{a + 1} \)

20. \( \frac{y^2 + 10y + 25}{3y - 9} \div \frac{y + 5}{y - 3} \)

22. \( \frac{6x + 6}{x - 1} \div \frac{x^2 + 3x + 2}{2x - 2} \)
Session 3: Adding/Subtracting of rational expressions  

Think/Pair/Share:

Find each sum.

1. \( \frac{2y}{5} + \frac{y}{5} \)  
2. \( \frac{4r}{9} + \frac{5r}{9} \)

7. \( \frac{2q}{q + 2} + \frac{3}{q + 2} \)  
8. \( \frac{2p}{p + 1} + \frac{2}{p + 1} \)

Find each difference.

17. \( \frac{t + 3}{7} - \frac{t}{7} \)  
18. \( \frac{c + 8}{4} - \frac{c + 6}{4} \)

21. \( \frac{x}{x - 1} - \frac{1}{x - 1} \)  
22. \( \frac{3r}{r + 3} - \frac{r}{r + 3} \)

Find the LCM for each pair of expressions.

1. \( 4x^2y, 12xy^2 \)  
2. \( n + 2, n - 3 \)

3. \( 2r - 1, r + 4 \)  
4. \( t + 4, 4t + 16 \)

5. \( x^2 - 2x - 3, (x - 3)^2 \)  
6. \( c^2 + 2c - 3, c - 2 \)

Find each sum.

7. \( \frac{3}{y} + \frac{4}{y^2} \)  
8. \( \frac{3}{8c^2} + \frac{5}{2a} \)

9. \( \frac{m + 3}{2m} + \frac{m - 2}{m} \)  
10. \( \frac{5}{y + 2} + \frac{1}{y - 6} \)

11. \( \frac{b}{b - 1} + \frac{2}{b - 4} \)  
12. \( \frac{k}{k - 5} + \frac{k - 1}{k + 5} \)

13. \( \frac{3x + 15}{x^2 - 25} + \frac{x}{x + 5} \)  
14. \( \frac{x - 3}{x^2 - 4x + 4} + \frac{x + 2}{x - 2} \)
Find each difference.

15. \( \frac{5}{4r} - \frac{2}{r^2} \)

17. \( \frac{x}{x + 2} - \frac{4}{x - 1} \)

19. \( \frac{a}{2a - 1} - \frac{2}{a + 3} \)

21. \( \frac{6}{b^2 - 1} - \frac{b}{b + 1} \)

16. \( \frac{5x}{3y^2} - \frac{2x}{9y} \)

18. \( \frac{d - 1}{d - 2} - \frac{3}{d + 5} \)

20. \( \frac{-5}{s + 4} - \frac{-4}{s^2 + 4s} \)

22. \( \frac{2u}{u^2 + 3u - 4} - \frac{u - 1}{u^2 + 8u + 16} \)
Session 2: Multiplication/Division of Rational Expressions with application

Station 1: Multiplying Geometry

GEOMETRY A rectangular pyramid has a base area of $\frac{x^2 + 3x - 10}{2x}$ square centimeters and a height of $\frac{x^2 - 3x}{x^2 - 5x + 6}$ centimeters. Write a rational expression to describe the volume of the rectangular pyramid.

Station 2: Dividing Geometry

GEOMETRY A right triangle with an area of $x^2 - 4$ square units has a leg that measures $2x + 4$ units. Determine the length of the other leg of the triangle.

Station 3: Multiplying Problem Skills

\[
\frac{3m}{2n} \cdot \frac{n^3}{6} \quad \frac{x^2 - 4}{(x - 2)(x + 1)} \quad \frac{18}{2x - 6}
\]

Station 4: Dividing Problem Skills

\[
\frac{3x^2}{x + 2} \div \frac{3x}{x^2 - 4} \quad \frac{q^2 + 2q}{6q} \div \frac{q^2 - 4}{3q^2} \quad \frac{3x^2}{x + 2} \div \frac{3x}{x^2 - 4}
\]

Station 5: Salary Tasks

The inward flow and outward flow of a football player’s money (in millions of dollars during his ten year career from 1998 to 2007 can be modeled by the rational expressions in the table below, where x is the number of years since 1998.

<table>
<thead>
<tr>
<th>Income</th>
<th>Salary</th>
<th>Endorsements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{8(x + 1)(x + 2)}{(x + 1)(x + 5)}$</td>
<td>$\frac{x + 23}{x + 5}$</td>
</tr>
</tbody>
</table>

a. Simplify the salary expression
b. In which year did the salary income equal the endorsemeent income?
c. In what year was the salary equal to 2 ¼ million?
Session 3: Addition/Subtraction of Rational Expressions with application

Station 1: Salary Station

The inward flow and outward flow of a football player’s money (in millions of dollars during his ten year career from 1998 to 2007 can be modeled by the rational expressions in the table below, where x is the number of years since 1998.

<table>
<thead>
<tr>
<th>Income</th>
<th>Salary</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{8(x + 1)(x + 2)}{(x + 1)(x + 5)}$</td>
<td></td>
</tr>
<tr>
<td>Endorsements</td>
<td>$\frac{x + 23}{x + 5}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expenses</th>
<th>Charity</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{2x + 1}{3(x + 1)}$</td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td>$\frac{5(x^2 + 4)}{x^2 + 6x + 5}$</td>
<td></td>
</tr>
</tbody>
</table>

a. Find the sum of the salary and endorsement income.
b. Find the sum of charity and other expenses.

Station 2: Adding

**EGYPTIAN FRACTIONS** Ancient Egyptians used only unit fractions, which are fractions in the form $\frac{1}{n}$. Their mathematical notation only allowed for a numerator of 1. When they needed to express a fraction with a numerator other than 1, they wrote it as a sum of unit fractions. An example is shown below.

\[
\frac{5}{6} = \frac{1}{3} + \frac{1}{2}
\]

Simplify the following expression so it is a sum of unit fractions.

\[
\frac{5x + 6}{10x^2 + 12x} + \frac{2x}{8x^2}
\]

Station 3: Subtracting

**SERVICE** Members of the ninth grade class at Pine Ridge High School are organizing into service groups. What is the minimum number of students who must participate for all students to be divided into groups of 4, 6, or 9 students with no one left out?

Station 4: Adding/Subtracting Skill Problems

1. \(\frac{b + 5}{4b} + \frac{b - 2}{b}\)
2. \(\frac{p + 1}{p^2 + 3p - 4} + \frac{p}{p + 4}\)
3. \(\frac{y + 3}{y^2 - 16} + \frac{3y - 2}{y^2 + 8y + 16}\)
4. \(\frac{s + 1}{s^2 - 9} - \frac{2s + 3}{4s + 12}\)
5. \(\frac{b - 3}{b^2 + 6b + 9} - \frac{-3}{b - 3}\)
6. \(\frac{6p}{5x^2} - \frac{2p}{3x}\)
Station 6: Electricity Problem
Use the formula $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ to find the effective resistance of a 30-ohm resistor and a 20-ohm resistor that are connect in parallel. (Refer to page 569 in Algebra 2 Glencoe © 2001 as an introduction).
Questions 1 – 4, perform the steps necessary to complete each problem. Show all of your work to receive full credit. Each problem will count 10 points each.

1. Find \( \frac{x}{3} + \frac{3x}{4} \).

2. Find \( (12x^2 - 10x) ÷ 4x \).

3. Find \( \frac{x - 4}{x - 2} - \frac{2}{x + 6} \).

4. Find \( \frac{12}{x - 5} \cdot \frac{4(x - 3)}{8} \).

Questions 5 – 10, answer each question completely. Show all of your work to receive full credit. Each problem will count 10 points each.

5. A bathtub can be filled by the cold water faucet in 10 minutes and by the hot water faucet in 12 minutes. How long does it take to fill the tub if both faucets are open?

6. Power Pools installs swimming pools. To determine the appropriate size of pool for a yard, they measure the length of the yard in meters and call that value \( x \). The length and width of the pool are calculated with the, diagram below. Write an expression in simplest form for the perimeter of a rectangular pool for the given variable dimensions.

![Diagram of a rectangular pool with dimensions \( \frac{x}{4} \) m and \( \frac{2x}{5} \) m.]}
7. Susan drove 1500 miles to Daytona Beach for spring break. On the way back she averaged 10 miles per hour less, and the drive back took 5 hours longer. Find Susan’s average speed on the way to Daytona Beach.

8. Tamara bought 50 pounds of fruit consisting of Florida oranges and Texans grapefruit. She paid just as much for per pound for the grapefruit as for the oranges. If Tamara bought $12 worth of oranges and $16 worth of grapefruit, then how many pounds of each did she buy?

9. Mark, Connell, Zack, and Moses run the 4 by 400 meter relay together. Their average speeds were $s$, $s + 0.5$, $s - 0.5$, and $s + 1$ meters per second, respectively.

   a. What were their individual times for their own legs of the race?

   b. Write an expression for their time as a team. Write your answer as a ratio of two polynomials.

   c. If $s$ was 6 meters per second, what was the team’s time? Round your answer to the nearest second.

10. Harold runs to the local food mart to buy a gallon of soy milk. Because he is weighed down on his return trip, he runs slower on the way back. He travels $S_1$ feet per second on the way to the food mart and $S_2$ feet per second on the way back. Let $d$ be the distance he has to run to get to the food mart. Remember: distance = rate $\times$ time.

   a. Write an equation that gives the total time Harold spent running for this errand.

   b. What speed would Harold have to run if he wanted to maintain a constant speed for the entire trip yet take the same amount of time running?
How do you add or subtract rational algebraic expressions?

1. Factor the denominators and get a common denominator.
2. Determine the new numerators and combine them with the signs between the fractions.
3. Simplify the answer.

Example:

\[
\frac{3}{x - 1} - \frac{2}{x} + \frac{x + 3}{x^2 - 1}
\]

Your Turn:

\[
\frac{4x}{x^2 - y^2} - \frac{4y}{x^2 - y^2}
\]

\[
\frac{x - 1}{x + 1} - \frac{x^2 - x + 3}{x^2 - 1}
\]

\[
\frac{3}{x^2 + x - 6} - \frac{2}{2x^2 - 3x + 2}
\]

Graphic Organizer by Dale Graham and Linda Meyer
Thomas County Central High School; Thomasville GA
How do you add or subtract rational algebraic expressions?

Example

\[
\frac{3}{x-1} - \frac{2}{x^2 + x + 3} - \frac{x+3}{x^2 - 1}
\]

Your Turn

\[
\frac{4x}{x^2 - y^2} - \frac{4y}{x^2 - y^2}
\]

\[
\frac{x-1}{x+1} - \frac{x^2 - x + 3}{x^2 - 1}
\]

\[
\frac{3}{x^2 + x - 6} - \frac{2}{2x^2 - 3x + 2}
\]
How do you multiply or divide rational algebraic expressions?

For division problems, multiply by the reciprocal of the second fraction. Factor each expression completely and cancel common factors using the rules for exponents. Then multiply across.

Your Turn

\[
\frac{4x^3y^9}{9z^4} \div \frac{-2xy^3}{15z}
\]

\[
\frac{4x^3-12x^2}{x^3-27} \div \frac{x^2+3x+9}{-8x}
\]

\[
\frac{2t^2+4t}{4t^2-4t+1} \div \frac{2t^2-5t+2}{t^2-4}
\]

Example

\[
\frac{4x^3-xy^2}{9y^4} \div \frac{2x-y}{-xy}
\]

Graphic Organizer by Dale Graham and Linda Meyer
Thomas County Central High School; Thomasville GA
How do you multiply or divide rational algebraic expressions?

For division problems, multiply by the reciprocal of the second fraction. Factor each expression completely and cancel common factors using the rules for exponents. Then multiply across.

Your Turn

Example

\[
\frac{4x^3 - xy^2}{9y^4} \div \frac{2x - y}{-xy}
\]

\[
\frac{4x^3 y^9}{9z^4} \div \frac{-2xy^3}{15z}
\]

\[
\frac{4x^3 - 12x^2}{x^3 - 27} \div \frac{x^2 + 3x + 9}{-8x}
\]

\[
\frac{2t^2 + 4t}{4t^2 - 4t + 1} \div \frac{2t^2 - 5t + 2}{t^2 - 4}
\]

Graphic Organizer by Dale Graham and Linda Meyer
Thomas County Central High School; Thomasville GA
Concept: Simplifying Rational Expressions  
Standard: MM1A2e: Add, Subtract, multiply, and divide rational expressions

7 sessions (Sessions 1 – 4)

Essential Question:

How do you use your prior knowledge of performing operations with fractions to performing operations with rational expressions?

Activating Strategies: (Learners Mentally Active)

Session 1: Multiplication/Division of Rational Expressions
Warm – Up (5 min)
3 questions
Discuss student answers

Activating Strategy (10 min)
Just Jogging #1

Session 2: Multiplication/Division of Rational Expressions with application
Warm – Up (5 min)
Discuss student answers

Session 3: Adding/Subtracting of rational expressions
Warm – Up (5 min)
Discuss student answers

Session 4: Adding/Subtracting of rational expressions with application (end with summarizer assessment)
Warm – Up (5 min)
Discuss student answers

Acceleration/Previewing: (Key Vocabulary)

Key Vocabulary: Rational expression, expanded form, factored form, excluded values, complex fraction, simplest form, least common denominator

Graphing Organizers: use attached graphing organizers “Rational expressions graphic organizers” (4 of them)

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

Session 1: Multiplication/Division of Rational Expressions (30 minutes)
Whole group discussion

**Task:** Distribute graphic organizers and discuss

**Discuss:** Why can't you divide a number by zero? What about $4/x$; what value(s) can $x$ not be? Why? What about $5/(x - 1)$ or $6/(2 + x)$? What values can $x$ not be? $(7x)/(x^2 - 25)$? What values can $x$ not be? These are called excluded values. Discuss why the calculator says “error”.

**Guided Practice:**

1. \( \frac{14}{c^2} \cdot \frac{c^5}{2c} \)

2. \( \frac{3m^2}{2n} \cdot \frac{n^2}{12} \)

3. \( \frac{x^2 - 4}{5} \cdot \frac{x + 2}{x - 2} \)

4. \( \frac{x + 1}{x + 5} \div \frac{2x + 2}{x + 4} \)

5. \( \frac{5y + 15}{y + 6} \div \frac{2y + 6}{y - 3} \)

**Think/Pair/Share:** see attached and do even

**Session 2:** Multiplication/Division of Rational Expressions with application (end with summarizer assessment)

Groups of 4 – 6

**Task:** Students will be split up into groups of 4 – 6 and will rotate about 6 stations (approx. 7 minutes each). Teacher will preview and discuss example(s) before students rotate. See attached for station problems (Note: Teachers may add/subtract problems as they see fit)

**Guided Practice:**

1. The lengths of the sides of a right triangle can be expressed as $x + 2$ inches, $x + 9$ inches, and $x + 10$ inches. Find the lengths of the sides. (Hint: Pythagorean theorem)
   
   *Answer:* 5, 12, 13

2. The area of a triangle can be expressed as $4x^2 - 2x - 6$ square meters. The height of the triangle is $x + 1$ meters. Find the length of the base of the triangle.
   
   *Answer:* $(8x - 12)$ meters

**Session 3:** Adding/Subtracting of rational expressions

Whole group discussion

**Task:** Distribute graphic organizers and discuss

**Discuss:** Make sure to discuss the importance of LCD.

**Guided Practice:**

1. \( \frac{2p}{p + 1} + \frac{2}{p + 1} \)

2. \( \frac{3s}{s - 5} - \frac{3s}{5 - s} \)

3. \( \frac{m + 3}{2m} + \frac{m - 2}{m} \)

4. \( \frac{3x + 15}{x^2 - 25} + \frac{x}{x + 5} \)

5. \( \frac{5x}{3y^2} - \frac{2x}{9y} \)

6. \( \frac{a}{2a - 1} - \frac{2}{a + 3} \)
Think/Pair/Share: see attached and do even

Session 4: Adding/Subtracting of rational expressions with application (end with summarizer assessment)

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Session 1: Multiplication/Division of Rational Expressions
Students finish problems they didn’t finish in think/pair/share (do odd)

Session 2: Multiplication/Division of Rational Expressions with application
Distribute 2 question quiz

Session 3: Adding/Subtracting of rational expressions
Students finish problems they didn’t finish in think/pair/share (do odd)

Session 4: Adding/Subtracting of rational expressions with application (end with summarizer assessment)
Distribute 2 question quiz

Summarizing Strategies: Learners Summarize & Answer Essential Question

Session 1: Multiplication/Division of Rational Expressions
HOMEWORK Ander and Alejandro were working on the following homework problem.

\[
\frac{n - 10}{n + 3} \cdot \frac{2n + 6}{n + 3} = \frac{2(n - 10)(n + 3)}{(n + 3)(n + 3)} = \frac{2n - 20}{n + 3}
\]

Which student is correct? Explain.

Session 2: Multiplication/Division of Rational Expressions with application
Ticket out the door: summarizing quiz

Session 3: Adding/Subtracting of rational expressions
SWIMMING  Power Pools installs swimming pools. To determine the appropriate size of pool for a yard, they measure the length of the yard in meters and call that value $x$. The length and width of the pool are calculated with the diagram below. Write an expression in simplest form for the perimeter of a rectangular pool for the given variable dimensions.

![Diagram of a rectangular pool with dimensions $x/4$ m and $2x/5$ m]

**Session 4: Adding/Subtracting of rational expressions with application (end with summarizer assessment)**

Ticket out the door: summarizing quiz
ACTIVATING STRATEGY FOR MATH 1 UNIT 2, ESSENTIAL QUESTION #3

Fill in the missing numbers in Pascal's Triangle

1. Fill "1"s all the way down the left and right outer hexagons.
2. Fill in the remaining hexagons by adding the values of the two hexagons above it.

List 5 patterns that you notice:

_______________________________________________________________________________
________________________________________________________________________________
________________________________________________________________________________
________________________________________________________________________________
________________________________________________________________________________

Using a colored pencil, color in all even values. List something that you notice about the patterns that result. Based on coloring, predict what the pattern of the next row would be. Verify that this is true by finding the numerical values.

______________________________________________________________________________________
______________________________________________________________________________________
______________________________________________________________________________________
This is the pattern created by coloring the evens. As an extension of this project can be done for more advanced students in which more rows are completed. Blank patterns are available on the internet if you search for Pascal’s triangle. If more rows are completed the “evens” pattern looks like this.

Coloring multiples of other numbers produce different interesting patterns.
The Kevin Furniture Company has a current inventory of 40 benches, 70 chairs and 30 desks.

Write an expression representing this amount.

A school has placed an order for 30 benches, 120 chairs and 25 desks. Write an expression and use the appropriate operation to determine what furniture is needed to complete the order.

Suppose that the daily production for the factory is $15b + 40c + 20d$. Write and simplify expressions representing 3 days of production and 8 days of production.

Write an expression for ‘x’ days of production and simplify.

Write an expression for ‘x+2’ days of production and simplify.

If the factory produces $200b + 450c + 160d$ for the week (five days), find the daily average of production.
Kevin’s Furniture Company manufactures benches, chairs and desks for businesses and schools. The company factory is located in Valdosta, Georgia.

On Monday, KFC manufactured 60 benches, 80 chairs and 50 desks. If the variables “b, c and d” are used to represent the items produced, what algebraic expression can be written to represent the factory’s output?

\[ b + c + d \]

Suppose that the factory manufactured 40 benches, 70 chairs and 10 desks on Tuesday. Write an expression representing Tuesday’s production.

\[ 40b + 70c + 10d \]

How can you write an expression that shows the total combined output of the two days work?

\[ (60b + 80c + 50d) + (40b + 70c + 10d) \]

How can you express how many more products were produced Monday than Tuesday.

\[ 60b + 80c + 50d - (40b + 70c + 10d) \]

Suppose that the factory’s production for Wednesday, Thursday and Friday were the same as Tuesday’s. Write an expression for the final three days of the week. Are there any other ways that this production can be expressed?

\[ 3(40b + 70c + 10d) \]

Find the sum for the week’s production.

\[ (60b + 80c + 50d) + (40b + 70c + 10d) + 3(40b + 70c + 10d) \]

Colonel Sanders is the factory manager of KFC. He wants to find factory’s daily average of production. Write an expression that represents this daily average. Is there another way to represent this expression?

\[ \frac{60b + 80c + 50d + 40b + 70c + 10d + 3(40b + 70c + 10d)}{5} \]
Definition

Classification by Terms
1 Term -
2 Terms -
3 Terms -

Polynomial

<table>
<thead>
<tr>
<th>Degree</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$2x + 4$</td>
</tr>
<tr>
<td></td>
<td>$3x^2 - 4x + 1$</td>
</tr>
<tr>
<td></td>
<td>$-8x^3 + x$</td>
</tr>
</tbody>
</table>

What are some nonexamples?
### Definition
An algebraic expression with one or more terms.

### Classification by Terms
- **1 Term – Monomial**
  - 10, $3x^2$, $12ab$
- **2 Terms – Binomial**
  - $2x + 5$, $x - 2$
- **3 Terms – Trinomial**
  - $3x^2 + x - 5$
  - $5x^4 - 3x^2 + 1$

### Polynomial

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Degree</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5$</td>
<td>0</td>
<td>Constant</td>
</tr>
<tr>
<td>$2x + 4$</td>
<td>1</td>
<td>Linear</td>
</tr>
<tr>
<td>$3x^2 - 4x + 1$</td>
<td>2</td>
<td>Quadratic</td>
</tr>
<tr>
<td>$-8x^3 + x$</td>
<td>3</td>
<td>Cubic</td>
</tr>
</tbody>
</table>

### What are some nonexamples?
$x^3$, $x^{1/2}$
Essential Question:
How is Pascal’s triangle related to the binomial theorem for expanding polynomials?

Activating Strategies: (Learners Mentally Active)
Discovering patterns through exploration of Pascal’s triangle

Acceleration/Previewing: (Key Vocabulary)

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

Session 5:
1. Write identities discussed in sessions 3 & 4 on overhead and have students expand and compare answers to the rows of Pascal’s triangle. Ask students what do you notice about the first and last number in each row?
2. How many terms will there be in the expansion of \((x + y)^1\), \((x + y)^2\), \((x + y)^3\). Make a prediction about the number of terms in the expansion of \((x + y)^4\).
   Let’s generalize in how many terms will be in the expansion of \((x + y)^n\)
3. Now, let’s compare the coefficients in the expansions of \((x+y)^2\) and \((x +y)^3\) to the rows in Pascal’s triangle. Teacher will write expanded \((x + y)^4\) on the board and ask students how did he/she know that just by observing patterns in Pascal’s triangle.
4. Teacher will show students the following examples where coefficients of x and y terms are greater than 1:
   \((x + 4)^2\)
   \((2x + 3)^2\)
   \((2x + 3)^3\)
   \((3x – 2)^3\)

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Choose an appropriate task for expanding binomials for \(n = 2, 3\). Include error analysis on problems expanded incorrectly.

Summarizing Strategies: Learners Summarize & Answer Essential Question
Each student will write a journal entry to a student who missed class explaining how to use Pascal's triangle to expand (2x +5)^3.
Essential Question:

How can patterns be used to show that the formulas for special products will hold true?

Activating Strategies: (Learners Mentally Active)

Do “I've got your number” learning task (page 12)

Acceleration/Previewing: (Key Vocabulary)

Teacher will discuss the mathematical justification for the task (question from page 13 in “I've got your number”)

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

Session (3-4)

1. Use algebra tiles to model the following by calculating area.
   a. \((x + a)(x + b) = x^2 + (a + b)x + ab\) (learning task problem #1)
   b. \((x + y)^2 = x^2 + 2xy + y^2\) (learning task problem #4)
   c. \((x - y)^2 = x^2 - 2xy + y^2\) (learning task problem #7)
   d. The zero product property (learning task problem #11)

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Students will work in collaborative groups to discover the following identities using volume:
   e. \((x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3\)
   f. \((x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3\)
**Summarizing Strategies: Learners Summarize & Answer Essential Question**

Give students white boards, marker and an eraser. Students work the given problem on the board and show answer at teacher signal, then explain how to partner.

\[(x + 3)(x – 5)\]
Essential Question:
How do you model and solve real life problems using polynomials arithmetic?

Activating Strategies: (Learners Mentally Active)
Do Kevin’s Furniture Warehouse activity (10 mins)

Acceleration/Previewing: (Key Vocabulary)

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

Session (1 & 2)
a. Do Frayer model for polynomials
b. Use algebra tiles to model sums, differences and products of polynomials using the following examples:
   1. \((2x + 3) + (4x + 5)\)
   2. \((2x^2 - 3x + 1) - (-4x^2 + 2x + 1)\)
   3. \((2x + 1)(3x -4)\)

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Session (1 & 2):
1. Students will complete the Frayer Model
2. Students will perform operations using Algebra Tiles and then pair check their answers
Ticket out the door

a. Use polynomial operations to find the perimeter of a triangle with sides $(3x^2 - 5)$ ft, $(4x^2 - 1)$ ft, and $(5x^2 + 2)$ ft.

Find the area of a rectangle whose width is 5 in and length of $(4x - 1)$ in.
How would you justify that different expressions represent the same quantity? Choose one method discussed in sessions 1& 2. Explain in complete sentences how you would use this method to show the expressions are the same or different. Create an example and demonstrate using the method you chose.

Method: ____________________________
Explanation:_____________________________________________________________
_______________________________________________________________________
_______________________________________________________________________
_______________________________________________________________________
_______________________________________________________________________
_______________________________________________________________________

Demonstration:

Unit 2 Algebraic Expressions
Concept: Many different expressions represent the same quantity
Sessions 1,2&3: Ticket Out the Door
Linear vs. Quadratic

Graph: \[ y = 4x + 5 \]

Using a t-table, plug in values to create a graph.

\[ y = x^2 + 3 \]
### Techniques for Showing Two Expressions are Equivalent

<table>
<thead>
<tr>
<th>k + k - 1 = 2k - 1</th>
<th>k + k - 1 = k² - 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k + k - 1)</td>
<td>(k^2 - 1)</td>
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</table>

#### Algebra Tiles

<table>
<thead>
<tr>
<th>T-table and Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k + k - 1)</td>
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#### Algebraic Properties

Unit 2 Algebraic Expressions
Concept: Many different expressions can represent the same quantity
Session 1 & 2: Graphic Organizer
Cell Phone

You have a cell phone plan that charges you $33 a month and 5 cents for each minute that you use. The monthly bill $g(\text{in dollars})$ can be written as the function $g(x)=0.05x +33$ where $x$ is the number of minutes used.

1. What will your monthly bill be if you use 500 minutes, 1000 minutes, 15,000 minutes?
2. Graph the function with domain 0, 100, 200, and 300.
3. Is this a linear function?
Assessment and Rubric for Unit 2

Planning for the Prom Culminating Task

April, Eric, Jason, Nicole, Ryan, and Trina are junior class officers at James K. Polk High School. As junior class officers, they also serve as the Executive Committee for planning the prom and give final approval for all arrangements for the event. The Prom committee has arranged for the prom to be held at a county club in adjacent county.

1. One issue to be decided about prom arrangements is the size of the dance floor. The prom will be in the carpeted country club ballroom. For dances, the club places a parquet dance floor over part of the carpet. The dance floor is laid in interlocking sections so there are several sizes that can be chosen. The country club requires a final decision on the size of the floor a month before the date of the prom, so the committee wants to be prepared to base their decision on the latest possible information about the number of ticket sales.

   a) The minimum size dance floor that the Prom Committee is considering is 30' by 30'. They assume that this size will allow 100 couples to be on the dance floor at once. How many square feet of dance space per couple does this size allow?

   To allow for more dancers, the country club can increase the dimensions of the dance floor in 5 foot increments in either direction; however, the committee would like to maintain a square shape.

   b) Let \( n \) represent the number of 5' increases to the length of each side of the dance floor. Write and expression in terms of \( n \) for the additional square feet of dance floor. Show two different ways to find your answer. One method should involve adding areas and the other method should involve subtracting areas. Use algebraic identities to find a different but equivalent expression for the additional area.

   c) Write an expression that the committee could use to estimate the total number of couples that can be on the dance floor when the width and the length of the dance floor have been increasing using \( n \) additions of 5' each. Assume that each couple needs the same square feet of the dance space in part (a) and that the committee will round down to the nearest whole number of couples if the expression gives a fractional number of couples.

   d) Suppose the committee chooses to add enough 5' increments to increase the 30' side length of the dance floor by 50%. Find the total number of couples that can be on the dance floor with this increase in floor space. Calculate directly, and then verify that the expression in part (c) gives the same answer.
2. The Prom committee plans to have all the senior promenade before the Prom Queen and King are announced and to lay out a promenade aisle along one of the diagonals of the dance floor by rolling out a “red carpet” of vinyl sheeting just before the promenade. Therefore, they also need to plan ahead for the length of the sheeting they will need.

a) Write a radical for exact length of the diagonal of the 30' by 30' dance floor. Simplify the radical.

b) Write a radical for exact length of the diagonal on the 40' by 40' dance floor. Simplify the radical.

c) Write an expression for the length of the diagonal across the dance floor, in feet, as a function of the number $n$ of 5' extensions to the 30' by 30' square. Simplify the expression, if possible.

3. April, Eric, Jason, Nicole, Ryan, and Trina investigated getting a stretch limo to drive them and their dates on the evening of the prom. They found that a stretch limo for twelve people would be $150 per hour. They planned to meet at Trina's go out to dinner, then to the prom, and return to Trina's for the party after the prom. As members of the prom Executive Committee they planned to stay for the full length of the prom so they concluded that they should reserve the limo for at least six hours.

a. How much would it cost per couple if they rented a limo at $150 per hour for six hours?

Eric suggested that it might cost less and be more fun if they invited other members of the prom committee and their dates to join in and get a party bus together. He found that they could get a party bus that holds a maximum of 24 passengers at a cost $200 per hour as long as they rented the bus for at least six hours?

b) How much more would it cost per couple if they rented the party bus for six hours but did not get any other couples to share the cost?

c) Let $n$ denote a number of couples, in addition to the members of the Executive committee and their dates that could go on the party bus. What is the domain for $n$?
4. Nicole is wondering if six hours will be enough time to do all that they plan to do between the time the group leaves Trina's house and returns. She is especially concerned if they have estimated enough travel time for the trip from Trina's house to the restaurant, from the restaurant to the country club where the prom is being held, and from the prom back to Trina's house. In deciding on six hours for their transportation, they had roughly estimated an hour total for the travel.

Nicole decided to be more precise in making an estimate. She checked the distances and decided to make some reasonable guesses about how fast they would be able to go. Since the restaurant and country club are each very near interstate exits, she included their distances from the interstate with the interstate mileage.

She assumed that they could average 40 mph for the 4 miles from Trina's house to the nearest interstate and 60 mph for the additional 16 miles on interstate highways to get to the restaurant. Their route from the restaurant to the prom will take them back along part of their route to the restaurant and then near an arena where there is a major concert scheduled for the evening of the prom. We took the heavy traffic of concert-goers into consideration and assumed they could average 40 mph for this 8 miles of interstate. The concert will be over long before the prom, so she assumed that they will be able to go 60 mph along the 20 miles of interstate from the prom back to Trina's exit, late at night, will able to average 45 mph for the last 4 miles back to Trina's house.

(a) If all of their estimates of travel time are accurate how long will they spend traveling from one location to another on prom night? How good is the rough estimate of travel time?

* This is a 2 day test. It is suggested to delete 1c, 1d, 3b, and 3c for the 1 day test.

- The following vocabulary may need to be discussed/reviewed before administering the assessment: parquet floor and symbols for feet and inches, promenade, and domain.
- Differentiation: Provide drawings for problems as needed. Provide previous taught formulas such as the Pythagorean Theorem as needed.
<table>
<thead>
<tr>
<th>Task Value</th>
<th>Expectations</th>
<th>Points Earned</th>
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</table>
| 1. a) 2 points (1 point each) | 1. Find the area of the original dance floor  
2. Find the average square feet of dance floor needed per couple | |
| b) 12 pts (4 each) | 1. Write an expression for the additional square feet of dance floor using addition'  
2. Write an expression for the additional square feet of dance floor using subtraction.  
3. Use algebraic identities to write an equivalent expression for the additional area | |
| c) 5 points | 1. Write an expression to estimate the number of couples on the dance floor when the size is increased by n additions of 5' increments | |
| d) 3 points (2 points for #1 and 1 point for #2) | 1. Find the total number of couples that can be on the dance floor with a increase of floor space by 50%.  
2. Show verification that the expression in part (c) gives the same answer. | |
| 2. a) 5 points | 1. Write a radical expression for the exact length of the 30' by 30' diagonal and simplify. | |
| b) 5 points | 2. Write a radical expression for the exact length of the diagonal 40' by 40' diagonal and simplify. | |
| c) 2 points | 1. Write an expression for the length of the diagonal across the diagonal, in feet, as a function of the number n of 5' extensions and simplify | |
| 3) a) 1 point | 1. Compute the cost per couple for renting the limo | |
| b) 2 points | 1. Compute the additional cost per couple for renting the party bus. | |
| c) 1 point | 1. Find the domain for n, number of couples | |
| 4) a) 6 points (5 for #1 and 1 for #2) | 1. compute the total time spent traveling  
2. compare to the rough estimate | |