Math Instructional Framework

Full Name	Math III Unit 1 Matrices Lesson 3
Time Frame	
Unit Name	
Learning Task/Topics/ Themes	**Candy? What Candy? Do We Get to Eat It?**
Standards **and** Elements	**MM3A5. Students will use matrices to formulate and solve problems.** a. Represent a system of linear equations as a matrix equation. b. Solve matrix equations using inverse matrices. c. Represent and solve realistic problems using systems of linear equations.
Lesson Essential Questions	How can a matrix be used to represent a system of linear equations? How is an inverse matrix used in solving a matrix equation? What type of realistic problems can be solved using system of linear equations?
Activator	**Learning task – Candy? What Candy? Do We Get to Eat It? Is located in Matrix unit on Georgia Math 3 frameworks.**
Work Session	Solve Matrix Equation – lesson examples worksheet
Summarizing/Closing/Formative Assessment	Practice and assessment problems can be found at Kutasoftware.com, easyworksheet.com, or textbook.

Solving Matrix Equations

A **matrix equation** is an equation in which a variable stands for a matrix.

You can solve the simpler matrix equations using matrix addition and scalar multiplication.

Examples: 1

Solve for the matrix *X*: $X + \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 7 & -1 \end{bmatrix}$

$$X + \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 7 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$$

$$X + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 6-3 & 3-2 \\ 7-1 & -1-0 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & 1 \\ 6 & -1 \end{bmatrix}$$

Examples: 2

Solve for the matrix *X*: $X - 3\begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 12 & -10 \end{bmatrix}$

$$X - \begin{bmatrix} -9 & -3 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 12 & -10 \end{bmatrix}$$

$$X - \begin{bmatrix} -9 & -3 \\ 6 & 0 \end{bmatrix} + \begin{bmatrix} -9 & -3 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 12 & -10 \end{bmatrix} + \begin{bmatrix} -9 & -3 \\ 6 & 0 \end{bmatrix}$$

$$X - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 4+(-9) & 0+(-3) \\ 12+6 & -10+0 \end{bmatrix}$$

$$X = \begin{bmatrix} -5 & -3 \\ 18 & -10 \end{bmatrix}$$

Solving systems of linear equations using matrices:

Matrix equations can be used to solve systems of linear equations by using the left and right sides of the equations.

Examples: 3

Solve the system of equations using matrices: $\begin{cases} 7x + 5y = 3 \\ 3x - 2y = 22 \end{cases}$

$$\begin{aligned} 7x+5y&=3 \\ 3x-2y&=22 \end{aligned} \quad \rightarrow \quad \begin{bmatrix} 7x+5y \\ 3x-2y \end{bmatrix} = \begin{bmatrix} 3 \\ 22 \end{bmatrix}$$

Write the matrix on the left as the product of coefficients and variables.

$$\underset{\substack{\uparrow \\ \text{coefficient} \\ \text{matrix}}}{\begin{bmatrix} 7 & 5 \\ 3 & -2 \end{bmatrix}} \underset{\substack{\uparrow \\ \text{variable} \\ \text{matrix}}}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{\substack{\uparrow \\ \text{constant} \\ \text{matrix}}}{\begin{bmatrix} 3 \\ 22 \end{bmatrix}}$$

First, find the inverse of the coefficient matrix. The inverse of $\begin{bmatrix} 7 & 5 \\ 3 & -2 \end{bmatrix}$ is

$$\frac{1}{7(-2)-(3)(5)} \begin{bmatrix} -2 & -5 \\ -3 & 7 \end{bmatrix} = -\frac{1}{29} \begin{bmatrix} -2 & -5 \\ -3 & 7 \end{bmatrix} = \begin{bmatrix} \frac{2}{29} & \frac{5}{29} \\ \frac{3}{29} & -\frac{7}{29} \end{bmatrix}$$

Next, multiply each side of the matrix equation by the inverse matrix. Since matrix multiplication is **not** commutative, the inverse matrix should be at the left on **each** side of the matrix equation.

$$\begin{bmatrix} \frac{2}{29} & \frac{5}{29} \\ \frac{3}{29} & -\frac{7}{29} \end{bmatrix} \begin{bmatrix} 7 & 5 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{2}{29} & \frac{5}{29} \\ \frac{3}{29} & -\frac{7}{29} \end{bmatrix} \begin{bmatrix} 3 \\ 22 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

The identity matrix on the left verifies that the inverse matrix was calculated correctly.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

The solution is (4, –5).