Acquisition Lesson Planning Form
Plan for the Concept, Topic, or Skill – Normal Distributions
Key Standards addressed in this Lesson: MM3D2
Time allotted for this Lesson:

Standard: MM3D2 Students will solve problems involving probabilities by interpreting a normal distribution as a probability histogram for a continuous random variable (z – scores are used for a general normal distribution).

- **a.** Determine intervals about the mean that include a given percent of data (Empirical Rule).
- **b.** Determine the probability that a given value falls within a specified interval.
- **c.** Estimate how many items in a population fall within a specified interval.

Essential Question:
What are the standard intervals for a normal distribution?

How are these intervals used to solve problems?

Activating Strategies:
Discussion topics:
What does it mean for you to score in the 95^{th} percentile?
How are the percentiles determined? How do colleges compare different test such as the ACT and the SAT?

Acceleration/Previewing: (Key Vocabulary)
Normal Distribution Normal Curve Standard normal distribution
Z – score

Task:
And You Believed That?!
Let's Be Normal

Distributed Guided Practice/Teaching Strategy:
Students will create a 3 – D foldable for the normal distribution intervals.
Use worksheet to work as a class to model how to find a normal distribution and z – score.

Extending/Refining Strategies:
Let's Be Normal

Summarizing Strategies:

Ticket Out The Door

If X is a normally distributed random variable and $X \sim N(\mu, \sigma)$, then the z-score is: $Z = \frac{x-\mu}{\sigma}$

The z-score tells you how many standard deviations that the value x is above (to the right of) or below (to the left of) the mean, μ. Values of *x* that are larger than the mean have positive z-scores and values of *x* that are smaller than the mean have negative z-scores. If *x* equals the mean, then *x* has a z-score of 0.

Example 1

Suppose $X \sim N(5, 6)$. This says that X is a normally distributed random variable with mean $\mu = 5$ and standard deviation $\sigma = 6$. Suppose $x = 17$. Then:

$$z = \frac{x-\mu}{\sigma} = \frac{17-5}{6} = 2$$

This means that x = 17 is *2 standard deviations* (2σ) above or to the right of the mean $\mu = 5$. The standard deviation is $\sigma = 6$.

Notice that: $5+2\cdot6=17$ (The pattern is $\mu+z\sigma=x$.)

Now suppose $x=1$. Then:

$$z = \frac{x-\mu}{\sigma} = \frac{1-5}{6} = -0.67 \text{ (rounded to two decimal places)}$$

This means that x = 1 is 0.67 standard deviations (- 0.67σ) below or to the left of the mean μ = 5. Notice that:

$5+(-0.67)(6)$ is approximately equal to 1 (This has the pattern $\mu+(-0.67)\sigma=1$

Summarizing, when z is positive, x is above or to the right of μ and when z is negative, x is to the left of or below μ.

Example 2

Some doctors believe that a person can lose 5 pounds, on the average, in a month by reducing his/her fat intake and by exercising consistently. Suppose weight loss has a normal distribution. Let *X* = the amount of weight lost (in pounds) by a person in a month. Use a standard deviation of 2 pounds. *X*~N(5, 2). Fill in the blanks.

Problem 1

Suppose a person *lost* 10 pounds in a month. The z-score when x = 10 pounds is z = 2.5 (verify). This z-score tells you that x = 10 is ________ standard deviations to the ________ (right or left) of the mean _____ (What is the mean?).

Problem 2

Suppose a person *gained* 3 pounds (a negative weight loss). Then *z* = __________. This z-score tells you that x = -3 is ________ standard deviations to the __________ (right or left) of the mean.

Suppose the random variables *X* and *Y* have the following normal distributions: *X* ~N(5, 6) and Y ~ N(2, 1). If x = 17, then *z* = 2. (This was previously shown.) If y = 4, what is *z*?

$$z = \frac{y-\mu}{\sigma} = \frac{4-2}{1} = 2$$

where μ=2 and σ=1.

The z-score for y = 4 is z = 2. This means that 4 is z = 2 standard deviations to the right of the mean. Therefore, x = 17 and y = 4 are both 2 (of *their*) standard deviations to the right of *their* respective means.

The z-score allows us to compare data that are scaled differently. To understand the concept, suppose *X* ~N(5, 6) represents weight gains for one group of people who are trying to gain weight in a 6 week period and *Y* ~N(2, 1) measures the same weight gain for a second group of people. A negative weight gain would be a weight loss. Since x = 17 and y = 4 are each 2 standard deviations to the right of their means, they represent the same weight gain *in relationship to their means*.

Unit 6 Lesson 2 Worksheet

1) The distribution of heights of adult American men is approximately Normal with mean 69 inches and standard deviation 2.5 inches. Use the 68 – 95 – 99.7 rule to answer the following questions.

   (a) What percent of men are taller than 74 inches?
   (b) Between what heights do the middle 95% of men fall?
   (c) What percent of men are shorter than 66.5 inches?
   (d) A height of 71.5 inches corresponds to what percentile of adult male American heights?

2) The distribution of weights of 9 –ounce bags of a particular brand of potato chips is approximately Normal with μ=9.12 ounces and standard deviation 0.15 ounce.

   (a) Draw an accurate sketch of the distribution of potato chip bag weights. Be sure to label the mean, as well as the points one, two and three standard deviations away from the mean on the horizontal axis.
   (b) A bag that weighs 8.97 ounces is at what percentile in this distribution?
   (c) What percent of 9 – ounce bags of this brand of potato chips weigh between 8.67 ounces and 9.27 ounces.

3) Scores on the Wechsler Adult Intelligence Scale (a standard IQ test) for the 20 to 34 age group are approximately Normally distributed with μ=110 and α = 25. For each part that follows, sketch an appropriate Normal distribution. Then show your work.

   (a) What percent of people aged 20 to 34 have IQ scores above 100?
   (b) What percent have scores above 150?
   (c) MENSA is an elite organization that admits as members people who score in the top 20% on IQ tests. What score on the Wechsler Adult Intelligence Scale would an individual have to earn to qualify for MENSA membership?

Ticket Out the Door

Unit 6 Lesson 2
Normal Distribution and Z Score

1. A distribution of test scores is approximately normal with a mean score of 72 and a standard deviation of 4.

a. Sketch the graph of the normal distribution curve. (Bell Curve)
b. What percent of the scores are between 68 and 76?
c. In what range do the middle 95% of all scores fall?

2. Find the z-score corresponding to a raw score of 132 from a normal distribution with mean 100 and standard deviation 15.

Ticket Out the Door KEY

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a. Sketch the graph of the normal distribution curve. (Bell Curve)
b. What percent of the scores are between 68 and 76?
c. In what range do the middle 95% of all scores fall?

a. self explanatory
b. 68%
c. Between 62 and 80 which is 95%

2. Find the z-score corresponding to a raw score of 132 from a normal distribution with mean 100 and standard deviation 15.

$$z = \frac{x - \mu}{\sigma}$$

We compute

$$z = \frac{132 - 100}{15} = 2.133$$