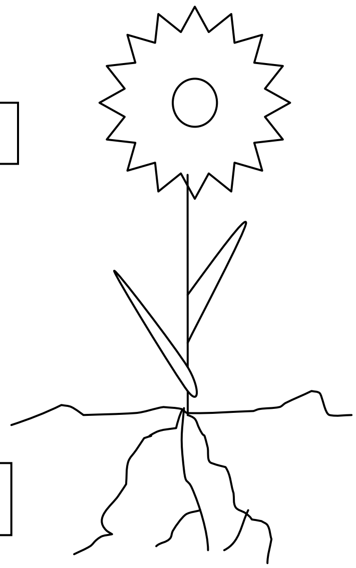
	Unit 3 – Lesson 1
Time Frame	
Unit Name	Logarithmic and Exponential Functions
Learning Task/Topics/ Themes	
Standards **and** Elements	MMA32. Logarithmic functions as inverses of exponential functions Element b. Extend to include properties of rational exponents
Lesson Essential Questions	How do I evaluate the n^{th} *root and use rational exponents?* How do I simplify expressions involving rational exponents?
Activator	Graphic Organizer/Foldable
Work Session	Properties/Rules of exponents (frameworks) Internet video resource http://www.montereyinstitute.org/courses/Elementary%20Algebra/course%20files/multimedia/lesson09/lessonp.html Vocabulary: ***Nth* roots:** The number that must be multiplied by itself n times to equal a given value. The *nth* root can be notated with radicals and indices or with rational exponents, i.e. $x^{1/3}$ means the cube root of x. **Rational exponents:** For $a > 0$, and integers m and n, with $n > 0$, $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$; $a^{m/n} = (a^{1/n})^m = (a^m)^{1/n}$. Problem set worksheet
Summarizing/Closing/ Formative Assessment	Assign pre-selected problems, have students work them and then students must write the exponent property they used to simplify each step.

Flower Power

Draw a single flower with its “dirt line”. Draw roots below the dirt line.

Use the explanation that a rational exponent is similar to flower power and roots are below the “dirt line”

The flower is the power



Dirt Line=→

Roots go below the dirt

$$\frac{3 \quad \text{(Power)}}{4 \quad \text{(Root)}}$$

$$X^{\frac{3}{4}}$$

Write each expression in radical form.

1) $7^{\frac{1}{2}}$

2) $4^{\frac{4}{3}}$

3) $2^{\frac{5}{3}}$

4) $7^{\frac{4}{3}}$

5) $6^{\frac{3}{2}}$

6) $2^{\frac{1}{6}}$

Write each expression in exponential form.

7) $(\sqrt{10})^3$

8) $\sqrt[6]{2}$

9) $(\sqrt[4]{2})^5$

10) $(\sqrt[4]{5})^5$

11) $\sqrt[3]{2}$

12) $\sqrt[6]{10}$

Write each expression in radical form.

13) $(5x)^{-\frac{5}{4}}$

14) $(5x)^{-\frac{1}{2}}$

15) $(10n)^{\frac{3}{2}}$

16) $a^{\frac{6}{5}}$

17) $(6v)^{1.5}$

18) $m^{-\frac{1}{2}}$

Write each expression in exponential form.

19) $(\sqrt[4]{m})^3$

20) $(\sqrt[3]{6x})^4$

21) $\sqrt[4]{v}$

22) $\sqrt{6p}$

23) $(\sqrt[3]{3a})^4$

24) $\frac{1}{(\sqrt{3k})^5}$

Simplify.

25) $9^{\frac{1}{2}}$

26) $343^{-\frac{4}{3}}$

27) $1000000^{\frac{1}{6}}$

28) $36^{\frac{3}{2}}$

29) $(x^6)^{\frac{1}{2}}$

30) $(9n^4)^{\frac{1}{2}}$

31) $(64n^{12})^{-\frac{1}{6}}$

32) $(81m^6)^{\frac{1}{2}}$

Write each expression in radical form.

1) $7^{\frac{1}{2}}$

$\sqrt{7}$

2) $4^{\frac{4}{3}}$

$(\sqrt[3]{4})^{4}$

3) $2^{\frac{5}{3}}$

$(\sqrt[3]{2})^{5}$

4) $7^{\frac{4}{3}}$

$(\sqrt[3]{7})^{4}$

5) $6^{\frac{3}{2}}$

$(\sqrt{6})^{3}$

6) $2^{\frac{1}{6}}$

$\sqrt[6]{2}$

Write each expression in exponential form.

7) $(\sqrt{10})^{3}$

$10^{\frac{3}{2}}$

8) $\sqrt[6]{2}$

$2^{\frac{1}{6}}$

9) $(\sqrt[4]{2})^{5}$

$2^{\frac{5}{4}}$

10) $(\sqrt[4]{5})^{5}$

$5^{\frac{5}{4}}$

11) $\sqrt[3]{2}$

$2^{\frac{1}{3}}$

12) $\sqrt[6]{10}$

$10^{\frac{1}{6}}$

Write each expression in radical form.

13) $(5x)^{-\frac{5}{4}}$

$\frac{1}{(\sqrt[4]{5x})^{5}}$

14) $(5x)^{-\frac{1}{2}}$

$\frac{1}{\sqrt{5x}}$

15) $(10n)^{\frac{3}{2}}$

$(\sqrt{10n})^{3}$

16) $a^{\frac{6}{5}}$

$(\sqrt[5]{a})^{6}$

17) $(6v)^{1.5}$

$(\sqrt{6v})^3$

18) $m^{-\frac{1}{2}}$

$\frac{1}{\sqrt{m}}$

Write each expression in exponential form.

19) $(\sqrt[4]{m})^3$

$m^{\frac{3}{4}}$

20) $(\sqrt[2]{6x})^4$

$(6x)^{\frac{4}{2}}$

21) $\sqrt[4]{v}$

$v^{\frac{1}{4}}$

22) $\sqrt{6p}$

$(6p)^{\frac{1}{2}}$

23) $(\sqrt[2]{3a})^4$

$(3a)^{\frac{4}{2}}$

24) $\frac{1}{(\sqrt{3k})^3}$

$(3k)^{-\frac{3}{2}}$

Simplify.

25) $9^{\frac{1}{2}}$

3

26) $343^{-\frac{4}{3}}$

$\frac{1}{2401}$

27) $1000000^{\frac{1}{6}}$

10

28) $36^{\frac{3}{2}}$

216

29) $(x^6)^{\frac{1}{2}}$

x^3

30) $(9n^4)^{\frac{1}{2}}$

$3n^2$

31) $(64n^{12})^{-\frac{1}{6}}$

$\frac{1}{2n^2}$

32) $(81m^6)^{\frac{1}{2}}$

$9m^3$

THE PLANET OF EXPONENTIA LEARNING TASK:

A new solar system was discovered far from the Milky Way in 1999. One of the planets in the system, Exponentia, has a number of unique characteristics. Scientists noticed that the radius of the planet has been increasing 500 meters each year.

When NASA scientists first spotted Exponentia, its diameter was approximately 40 km.

1. Make a table that lists the diameter and surface area of the planet from the years 1999 to 2009. (Leave your surface area answers in terms of π.)

	Diameter (in km)	Surface Area (in km^2)
1999	40	
2000		
2001		
2002		
2003		
2004		
2005		
2006		
2007		
2008		
2009		

a. Write a function rule that expresses the relationship between the radius of the planet and its surface area. What does it mean for the surface area to be a function of the radius (or diameter)? (Make sure you use proper function notation.)

b. Interchange the columns and create a second table so that surface area is the independent variable and diameter is the dependent.

	Surface Area (in km^2)	Diameter (in km)
1999		40
2000		
2001		
2002		
2003		
2004		
2005		
2006		
2007		
2008		
2009		

c. Graph the data from the first table. (Unless you are graphing on a calculator or computer, graphing every other point is sufficient.) How would a graph of the data from the second table look? How do you know?

d. If we wrote a rule (equation) for the new relationship in part (b), how would the new rule be related to the original? That is, how are the two rules related to each other? How do you know?

e. Using algebra, write a rule for the data in the second table. (Hint: We want an equation for the radius in terms of the surface area.)

f. What are the domain and range of the function in part (a)? What are the domain and range for the new relation in part (e)? What are the restrictions on the domain and range due to the context of the problem? Why are there restrictions?
Function in Part (a): Domain ______________ Range ______________________
Restrictions due to context:
Relation in Part (e): Domain ______________ Range ______________________
Restrictions due to context:

g. Is the new relation in part (e) also a function? How do you know? Explain two ways: using the graph of the original function in part (a) and using the graph of the unrestricted relation in part (e). (You may need to graph your equations on your graphing calculator or computer.)

2. Now let's consider how the volume of the planet changes.
a. Write a function rule that expresses the relationship between the radius of the planet and its volume.

b. Graph the function from part (a) over the interval $-10 \leq r \leq 10$. What part, if any, of this graph makes sense in the context of Exponentia's volume? Explain.

c. Using your exploration from part 1, consider the following.

i. If you wrote an equation with volume as the independent variable and the radius as the dependent variable, would this be a function? Explain.

ii. Describe how the graph of the new equation would look. Sketch this new graph with your graph in part (b). (Hint: How are the graphs of inverses related to each other?)

d. We want to write a rule for the second relation.

i. Explain how you knew to find the equation in part 1(e).

ii. We can use similar reasoning in for finding the equation in this problem. Let's start by solving for r^3.

iii. Now, to finish solving, we need the inverse of r^3. In number 1, to solve for r^2 by taking the square root of both sides of the equation. Likewise, we take the cube root of both sides of the equation in the above step. Solve for r.

When we take the square root of an expression or number, such as $x^2 = 4$, we must consider both the positive and negative roots of the expression or number, so x = □ 2. Do we need to consider both positive and negative roots when we take cube roots? Why or why not?

3. *nth Roots:*
The cube root of *b* is the number whose cube is b. Likewise, the *nth* root of *b* is the number that when raised to the *nth* power is *b*. For example, the 5th root of 32 is 2 because $2^5 = 32$. We can write the 5th root of 32 as $_{5}32$.
Another notation used to represent taking roots employs exponents. Instead of writing the 5th root of 32 as $_{5}$ 32 , we can write it as $_{51}32$. The cube root of 27 can be written as $_{31}27$. How do you think we would represent the *nth root* of a number *x?*
For the remainder of (3), consider $f(x) = x^n$ and $_{n}xxg_{1}$.

a. How do you think the graphs of *f(x)* and *g(x)* are related? Using graph paper and/or a calculator, test your conjecture, letting *n = 1, 2, 3, 4.*

b. Evaluate *f(g(x))* and *g(f(x))*. If you need to, use your examples of *n = 1, 2, 3, 4* to help you determine these compositions. What do the results tell you about *f(x)* and *g(x)*?

c. Explain how your work in problems 1 and 2 confirms your conclusions in parts (a) and (b) of this problem.

4. Using your investigations above and what you remember from Math 2, write a paragraph summarizing characteristics of inverses of functions, how to find inverses algebraically and graphically, and how to tell if inverses are functions