

	Unit 3 - Lesson 3
Time Frame	
Unit Name	<u>MM3A2</u> - Logarithmic Functions and Inverses of exponential functions
Learning Task/Topics/ Themes	
Standards and Elements	<p>MM3A2e – Investigate characteristics: domain and range, asymptotes, zeros, intercepts, intervals of increase and decrease, rate of change.</p> <p>MM3A2f – Graph of functions as transformations</p>
Lesson Essential Questions	<p>What are the characteristics of exponential functions?</p> <p>What are the characteristics of logarithmic functions?</p> <p>How do we graph transformations of exponential functions?</p> <p>How do we graph transformations of logarithmic functions?</p>
Activator	<p>KWL – Have students list what they know about exponential functions. As a class, discuss what each student wrote down and add it to the teacher’s list.</p> <p>With a partner, discuss what you want to learn about exponential functions.</p> <p>Have students make a foldable with 4 sheets of notebook paper. Once they have their foldable, have them label each tab :</p> <ul style="list-style-type: none"> • Convert from radical to rational notations (examples) • Convert from rational to radical notations (examples) • Graph the parent graph of exponential and logarithmic functions (exponential on the left and logarithmic on the right) • Transformations of exponential (left side) and logarithmic (right side) • Inverse of exponential (left side) and logarithmic

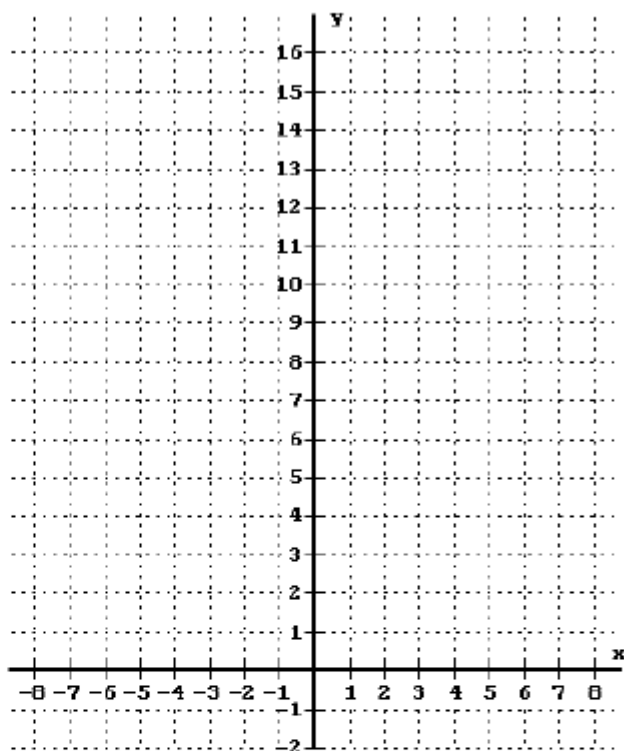
	<p>(right side)</p> <ul style="list-style-type: none"> • Characteristics of exponential (left side) and logarithmic (right side) • Half-time • Doubling time
Work Session	<p><u>Vocabulary</u>: domain, range, asymptotes, zeros, intercepts, intervals of increase and decrease, rate of change</p> <p>PowerPoint – Exponential Functions</p> <p>Use the PowerPoint and worksheet “Graph the Exponential Function” to guide the lesson. Explain how to find the graph the exponential function. Then, discuss how to find the domain and range, intercepts, and intervals of increase and decrease.</p> <p>Use the attached worksheets titled “Transformations of Exponential Functions” to guide the lesson on transformations.</p> <p><u>Independent Practice</u>: Practice worksheets for graphing exponential functions and logarithmic functions on the web using Kuta Software.</p>
Summarizing/Closing/Formative Assessment	<p>Prompt: What are the characteristics of exponential functions?</p> <p>What are the characteristics of logarithmic functions?</p> <p>How do we graph transformations of exponential functions?</p> <p>How do we graph transformations of logarithmic functions?</p>

Graph the Exponential Function: $f(x) = 2^x$

1. Complete the Table.

x	$y = 2^x$
5	$2^5 = 32$
4	
3	
2	
1	
0	
-1	$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$
-2	
-3	
-4	
-5	
10	
-10	

2. Graph the function.



3. Where does the graph of $y = 2^x$ cross the y -axis?

That is, find the y -intercept.

4. Where does the graph of $y = 2^x$ cross the x -axis?

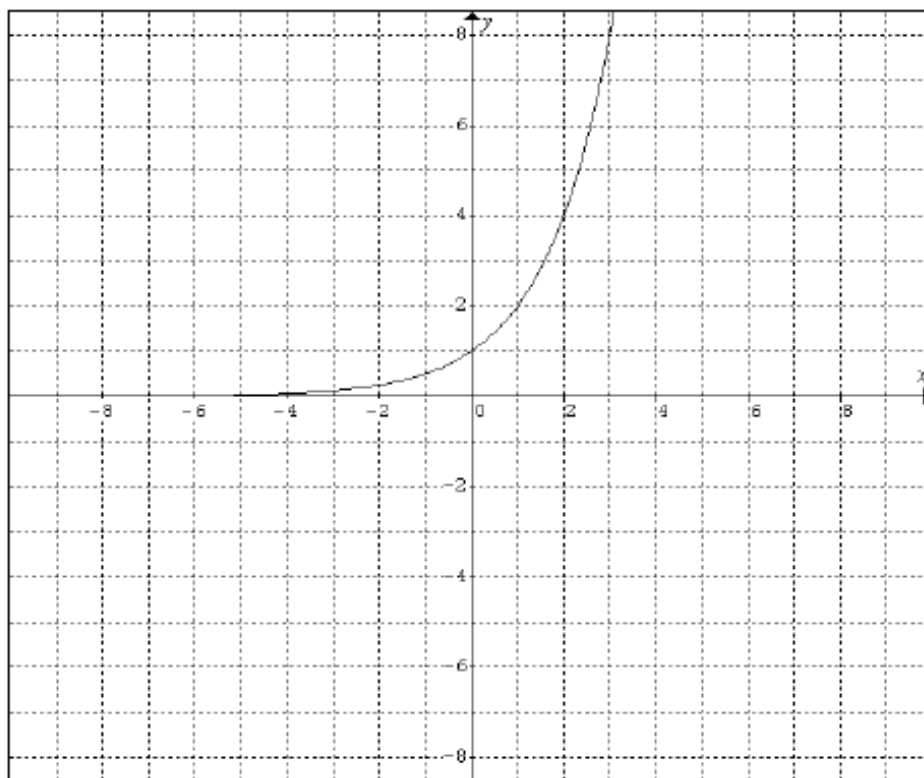
That is, find the x -intercept(s).

5. State the Domain

6. State the Range

Transformations of Exponential Functions

A. Given the graph of the exponential function $f(x) = 2^x$:



B. Graph the following functions on the specified graphs. Describe the transformations applied to the graph of $f(x)$ to obtain the graph of each of the $g(x)$, $h(x)$, $m(x)$, $n(x)$, $p(x)$, $s(x)$ and $t(x)$ functions.

$$g(x) = \frac{1}{3}(2^x)$$

Transformation:

$$h(x) = -\frac{1}{3}(2^x)$$

Transformation:

$$m(x) = 2^{3x}$$

Transformation:

$$n(x) = 2^{-3x}$$

Transformation:

$$p(x) = 2^x - 5$$

Transformation:

$$s(x) = 2^{x-5}$$

Transformation:

$$t(x) = -3(2)^{x-5} - 4$$

Transformation:

C. Summarize your observations in the table below. Given the function,

$$g(x) = a f(b(x - c)) + d$$

describe the meaning of each of the a, b, c, and d and the corresponding transformations of the graph of f(x) .

a	
b	
c	
d	

Practice:

1. Identify a, b, c and d and describe what transformations the graphs of the 4^x function has undergone in each of the following cases.

$$g(x) = 4^{x-1}$$

$$h(x) = 5(4^{x-4})$$

$$m(x) = 4^x + 4$$

$$n(x) = \frac{4^x}{2}$$

$$r(x) = 3(4^{-x}) + 1$$

$$s(x) = -4^{-x} - 2$$

2. Sketch the graph of the relation by hand. Start with the graph of $f(x) = 3^x$ and use the appropriate transformations:

$$g(x) = \frac{3^x}{3}$$

$$h(x) = -3^x$$

$$m(x) = 3^x + 4$$

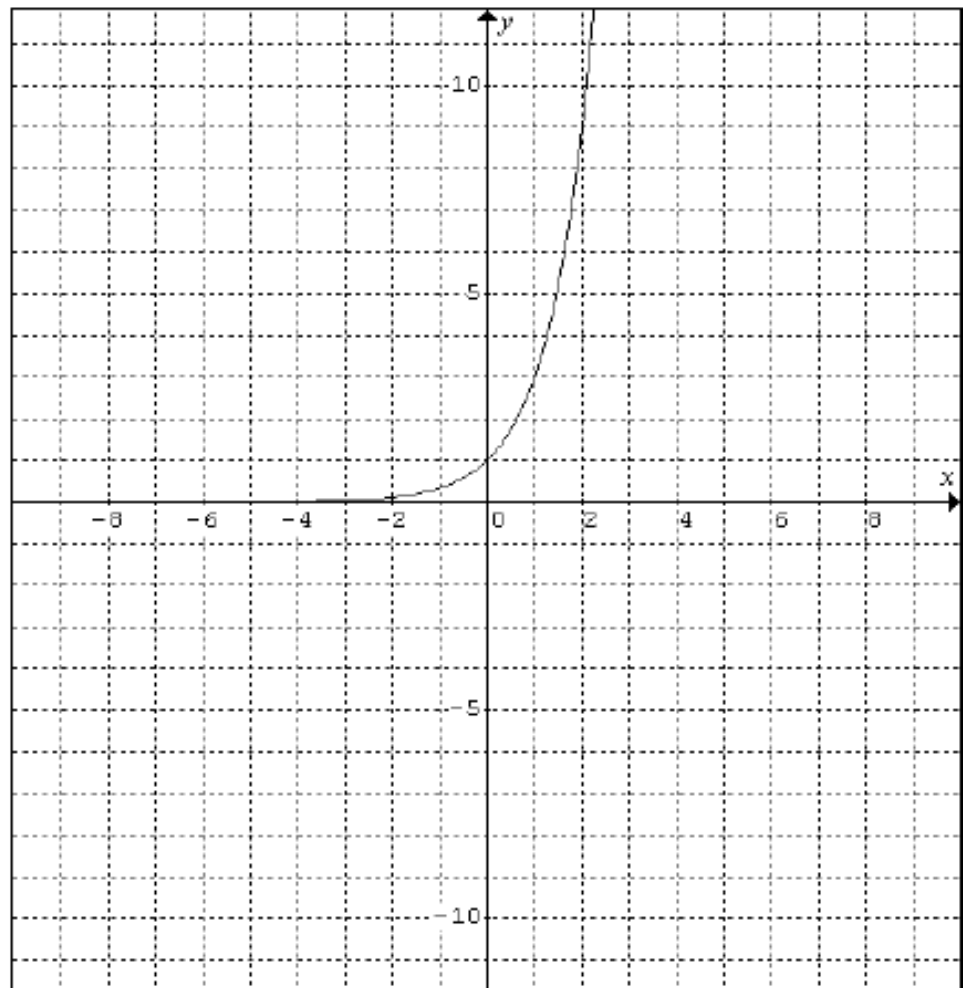
$$n(x) = -2(3^{4(x-1)}) + 1$$

$$p(x) = -(3)^{-(x-3)}$$

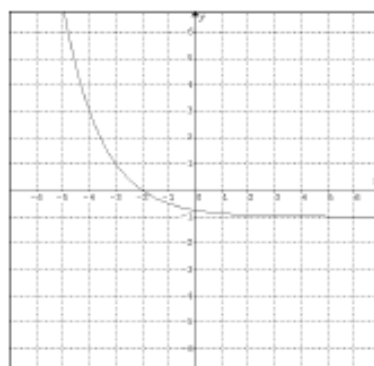
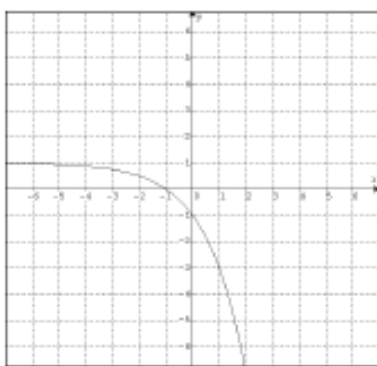
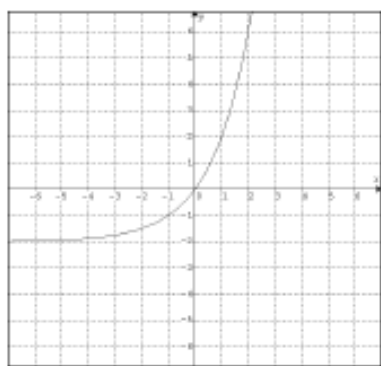
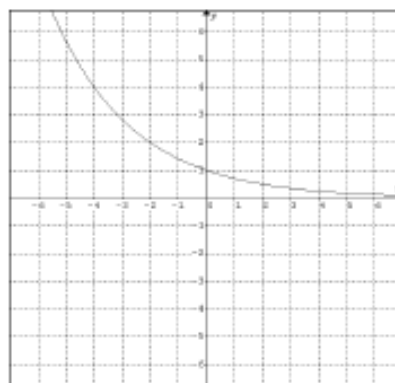
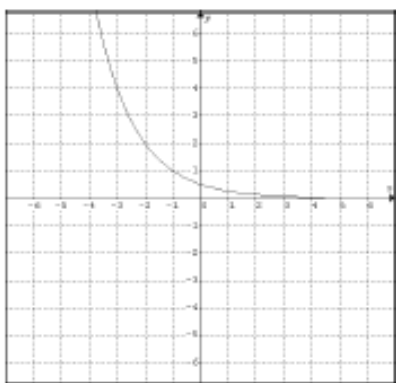
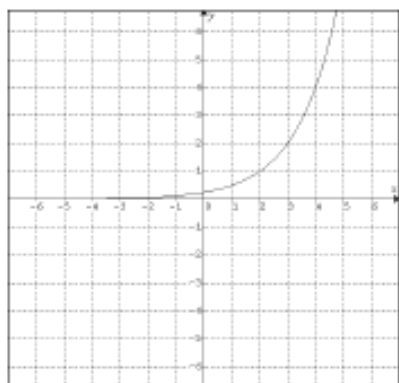
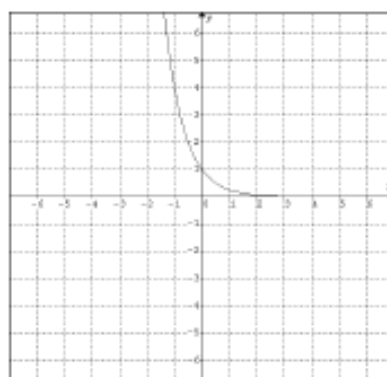
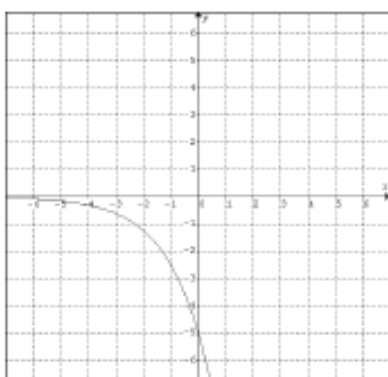
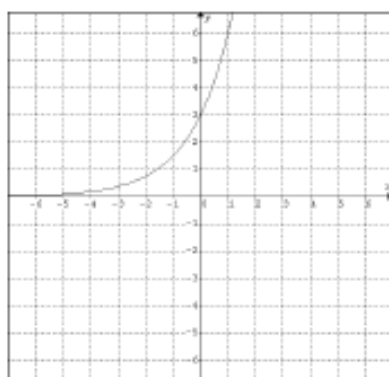
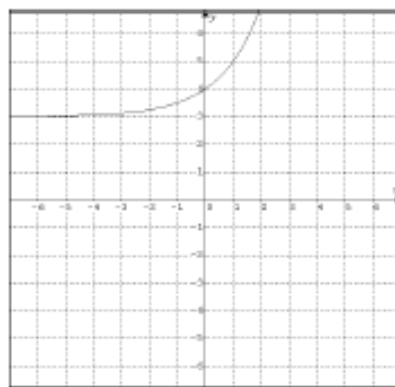
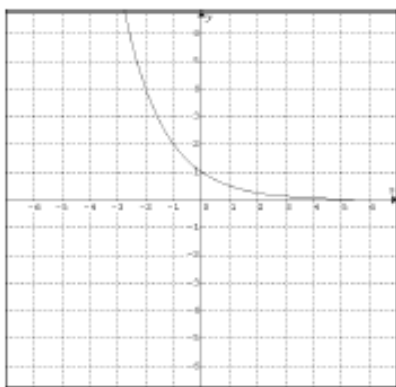
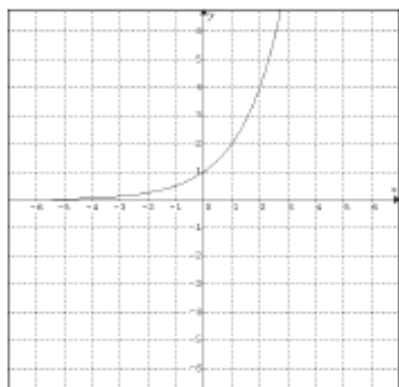
$$s(x) = 2 - (3)^{4-x}$$

$$t(x) = \frac{1}{3}3^{-3x-9} + 5$$

$$v(x) = 3^{-3x}$$



4. State the equation for each of the following graphs:

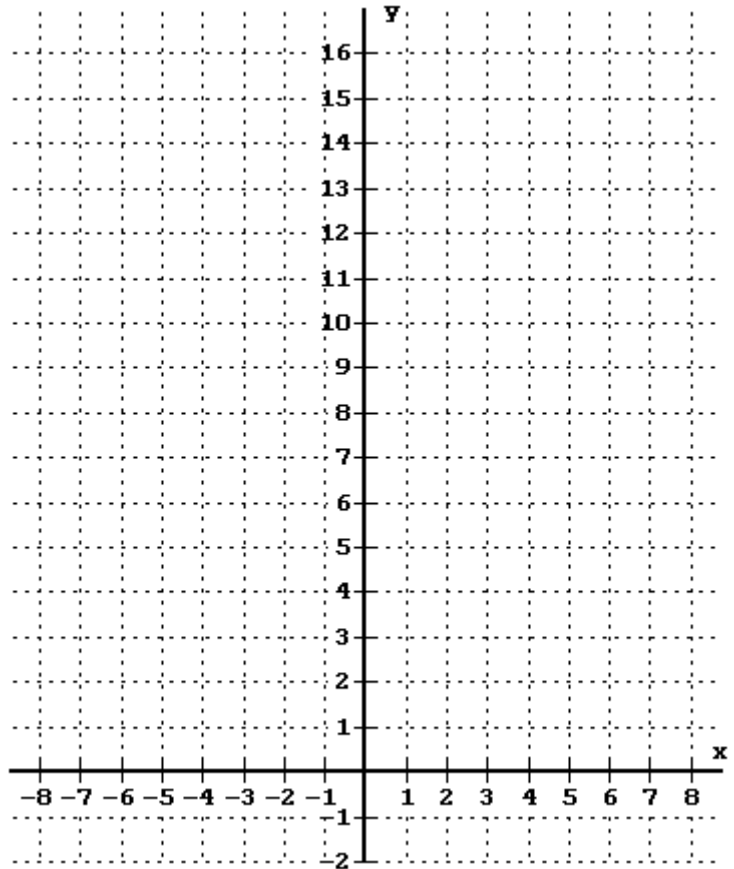


Graph the Exponential Function: $f(x) = 2^x$

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That is, find the x-intercept(s).

5. State the Domain

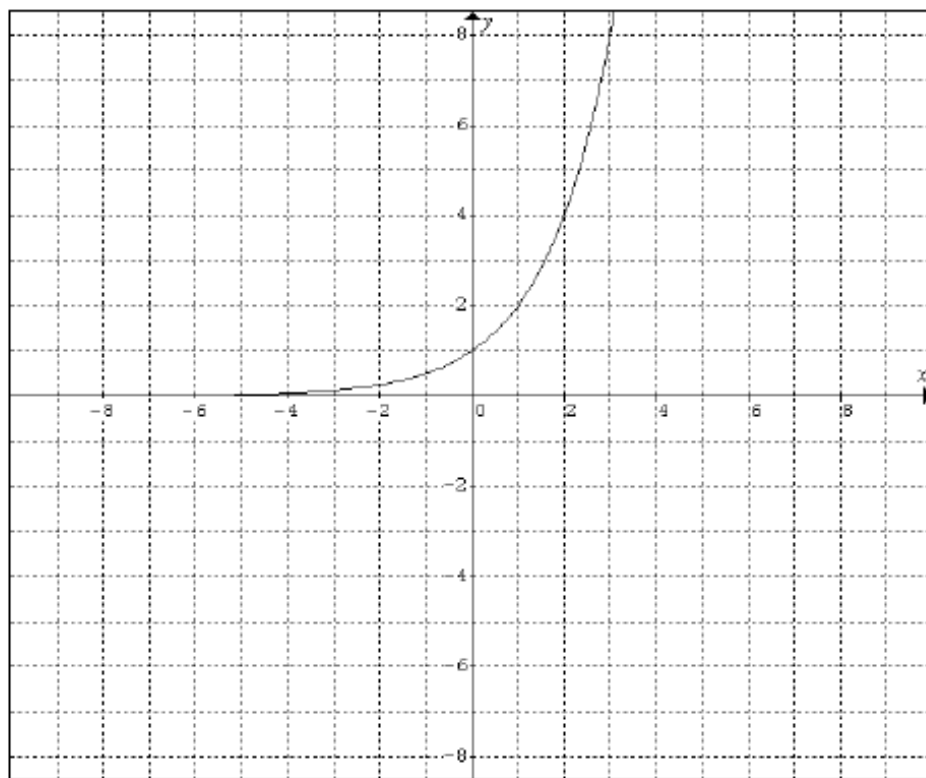
6. State the Range

Transformations Involving Exponential Functions

Transformation	Equation	Description
Horizontal Translation	$g(x) =$	*Shifts the graph of $f(x)$ to the left c units if $c < 0$. *Shifts the graph of $f(x)$ to the right c units if $c > 0$.
Vertical Stretching or shrinking		Multiplying y-coordinates of $f(x)$ *Stretches the graph of $f(x)$ if $a > 1$. *Stretches the graph of $f(x)$ if $0 < a < 1$.
Reflecting		*Reflects the graph of $f(x)$ about the x -axis. *Reflects the graph of $f(x)$ about the y -axis.
Vertical Translation		*Shifts the graph of $f(x)$ upward c units if $c > 0$. *Shifts the graph of $f(x)$ downward c units if $c < 0$.

Transformations of Exponential Functions

A. Given the graph of the exponential function $f(x) = 2^x$:



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Transformation:

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2. Sketch the graph of the relation by hand. Start with the graph of $f(x) = 3^x$ and use the appropriate transformations:

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$$h(x) = -3^x$$

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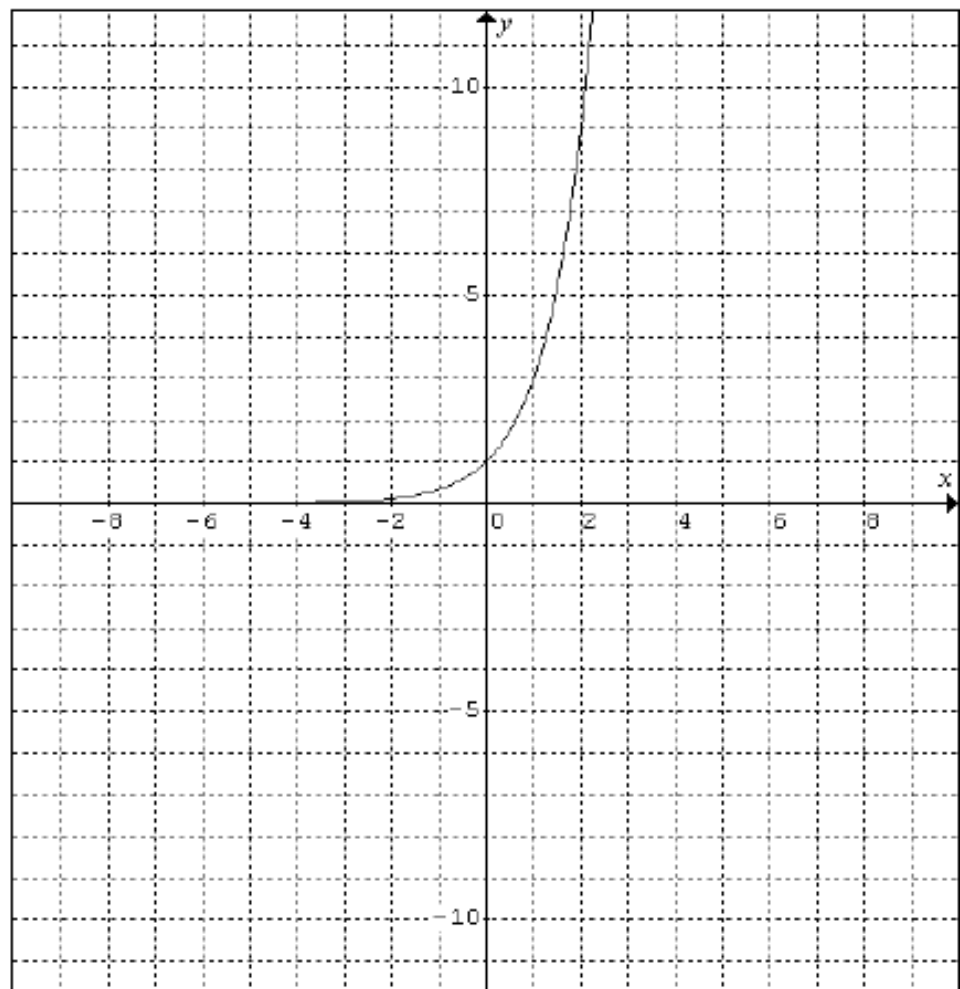
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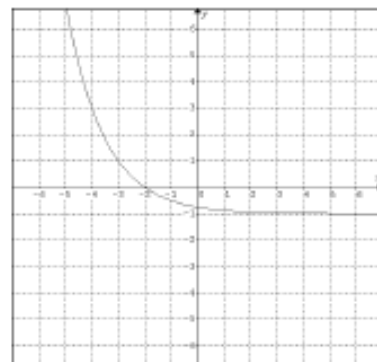
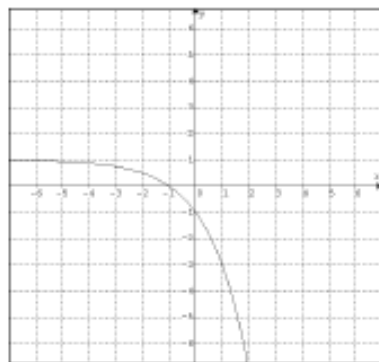
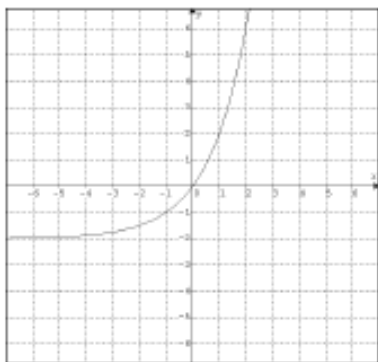
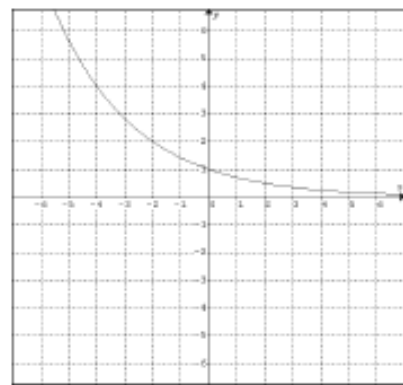
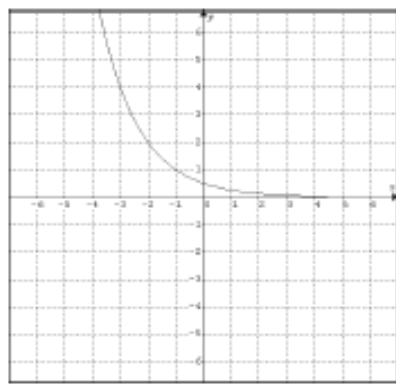
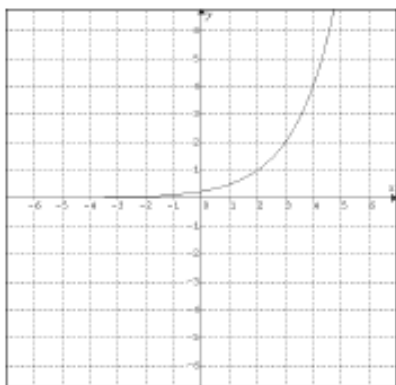
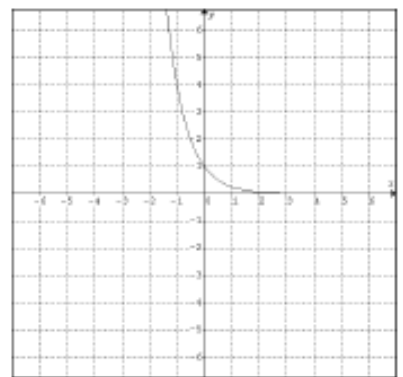
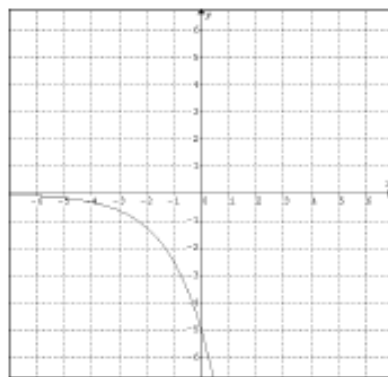
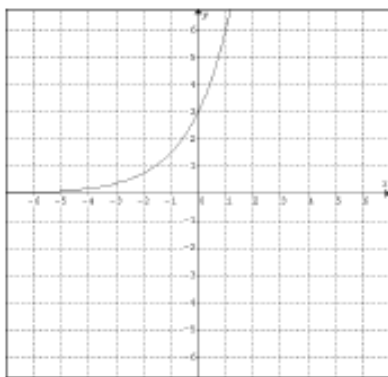
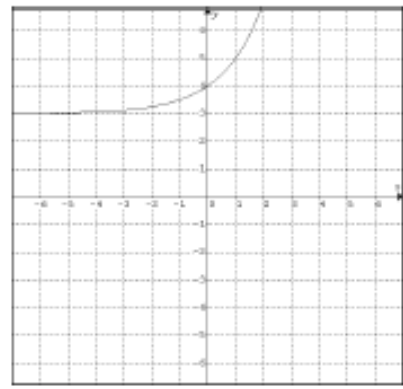
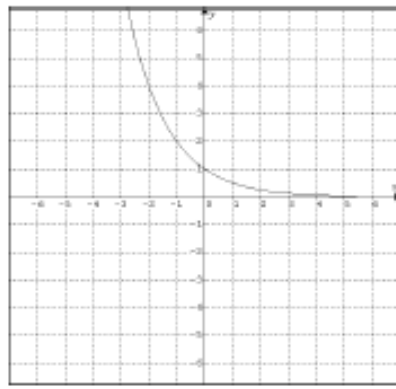
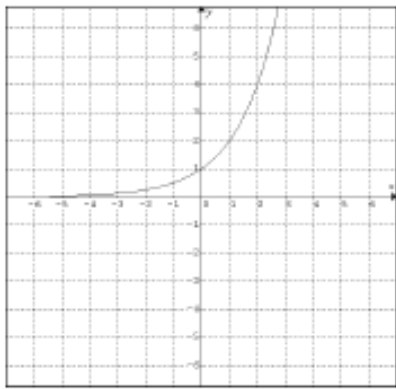
$$s(x) = 2 - (3)^{4-x}$$

$$t(x) = \frac{1}{3}3^{-3x-9} + 5$$

$$v(x) = 3^{-3x}$$



4. State the equation for each of the following graphs:



Analyze Graphs

1) Equation of Graph

$$y = 2^x$$

2) Calculator Notation

$$Y1 = 2 \wedge X$$

3) Classify / Type of Graph

4) Function? yes/no

5) One-to-One Function? yes/no

6) State any Symmetry:

7) Domain:

8) Range:

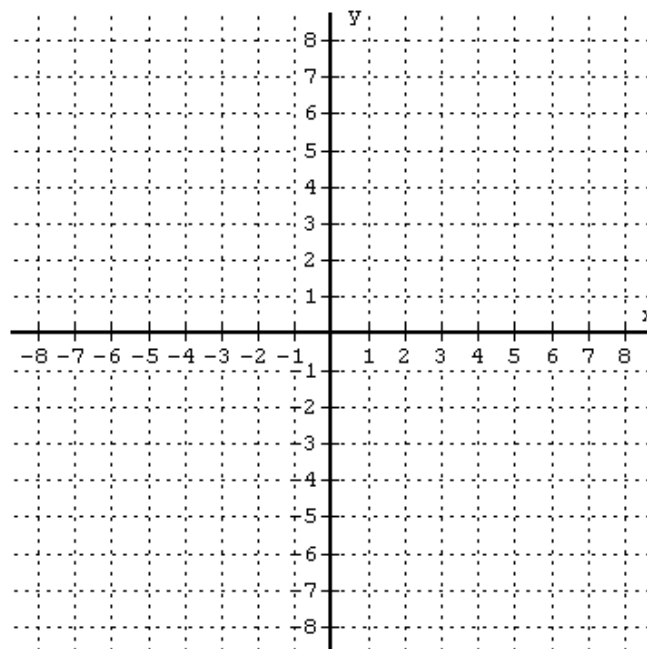
9) x -intercept(s):

10) y -intercept(s):

11) Where is $f(x) < 0$?
State the values of x using interval notation.

12) Where is $f(x) \geq 0$?
State the values of x using interval notation.

13) Where does $f(x) = 0$?



14) Where is $f(x)$ increasing?
State the values of x using interval notation.

15) Where is $f(x)$ decreasing?
State the values of x using interval notation.

16) Absolute Maximum value:

17) Absolute Minimum value:

18) Equation(s) of any Asymptote(s).